Example 1: Differentiate each of the following:

(a) $y = 3x^2 - 5x + 8$

(b) $y = x^2 e^x$

(c) $y = \ln x / x$

(d) $y = (x^3 + x - 1)^4$

(e) $y = \sqrt{x^2 + 1}$

(f) $y = \sin(x^2)$

(g) $y = \sin^2 x$

(h) $y = e^{\tan x}$

The **second partial derivatives** of f come in four types:

	Notation
■ Differentiate f with respect to x twice. (That is, differentiate f with respect to x; then differentiate the result with respect to x again.)	$\frac{\partial^2 f}{\partial x^2}$ or f_{xx}
■ Differentiate f with respect to y twice. (That is, differentiate f with respect to y; then differentiate the result with respect to y again.)	$\frac{\partial^2 f}{\partial y^2}$ or f_{yy}

Techniques of Indefinite Integration

Integration by substitution. This section opens with integration **by substitution**, the most widely used integration technique, illustrated by several examples. The idea is simple: Simplify an integral by letting a single symbol (say the letter u) stand for some complicated expression in the integrand. If the differential of u is left over in the integrand, the process will be a success.

Differential equations are among the linchpins of modern mathematics which, along with **matrices**, are essential for analyzing and solving **complex problems in engineering**. The emergence of low-cost, high-speed computers has spawned new techniques for solving differential equations, which allows problem solvers to model and solve complex problems based on **systems of differential equations**.

If there is **more than one independent variable** and **partial derivatives** appear in the equation, the equation is called a **partial differential equation**. Some common examples are the heat equation

$$k\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \tag{3}$$

the wave equation

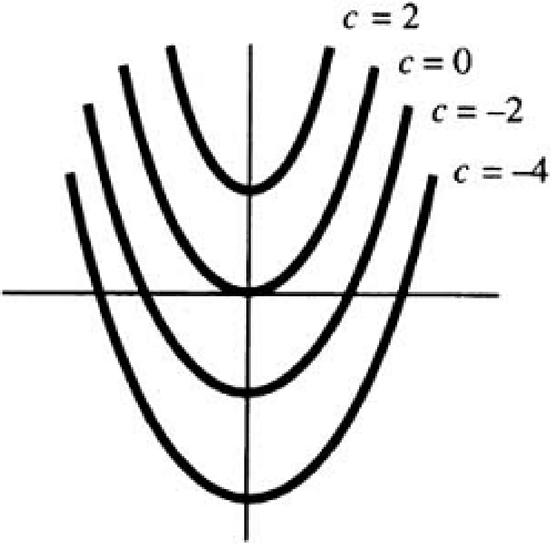
$$\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \tag{4}$$

the laplace's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0. \tag{5}$$

It is obvious in these cases which is the dependent variable and which are the independent variables.

Differential Equations: Basic Definitions



and the integral of the right-hand side is evaluated using integration by parts:

$$\int -x\cos x\,dx = -\int x\cos x\,dx$$

$$= -\left[x\sin x - \int \sin x\,dx\right]$$

$$= -(x\sin x + \cos x + c)$$

The solution of the differential equation is therefore

$$\cos y = x\sin x + \cos x + c \qquad \blacksquare$$