$(D-m_1)(D-m_2)\ldots(D-m_n)y=0$, $m_1, m_2, m_3, \ldots$ Auxilliary Eq^n (A.E)

→ (Before it, see diff operator)

→ Finding P.I by method of variation of Parameters -

• So, let,

$$y''+py'+qy=X \quad (1)$$ [p, q, X are f(x)]

let y_1, y_2 be sol^n of it & c_1 & c_2 are C.F

$c_1y_1+c_2y_2$ → CF of (1)

y_1 & y_2 got by AF.

Replace $\frac{dy}{dx}=m$, $\frac{d^2y}{dx^2}=m^2$

If $m=\pm ai$ ... $c_1 \sin ax + c_2 \ldots$

let $c_1=u(x)$ & $c_2=v(x)$ (unknowns).

$$y=uy_1+vy_2$$

$y'=uy_1'+vy_2'$ ... cos u.

$u'y_1'+v'y_2'+uy_1''+vy_2''$

→ got u & v here

or

$$\therefore u'y_1'+v'y_2'=X$$

y_1 & y_2 got

~~Let W=~~

$$\therefore u'=\frac{-y_2X}{y_1y_2'-y_1'y_2}=-\frac{y_2X}{W}$$

$$v'=\frac{y_1X}{y_1y_2'-y_1'y_2}=\frac{y_1X}{W}$$

W → Wronskian of y_1 & y_2.

if $m=\pm 1$

$c_1e^t+c_2e^{-t}$

$$\therefore u=-\int\frac{y_2X}{W}$$

$$v=-\int\frac{y_1X}{W}$$

→ Homogeneous Linear Diff eq^n - (reduced to linear ~~[illegible]~~ diff eq^n with const. coeff)

$$x^n \frac{d^n y}{dx^n} + a_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + a_2 x^{n-2} \frac{d^{n-2} y}{dx^{n-2}} + \text{---}$$
$$\text{---} + a_n y = X(x).$$

(Cauchy's)

Subs. $x = e^t$ $\therefore \frac{dy}{dx} = \frac{1}{x} \frac{dy}{dt}$

$$\frac{dy}{dt} = x \frac{dy}{dx} = Dy \quad \left(\frac{d}{dt}\right)$$

IIly, $x^2 \frac{d^2 y}{dx^2} = D(D-1)y$

$x^3 \frac{d^3 y}{dx^3} = D(D-1)(D-2)y$ and so on.

→ Ex Diff eq^{n}s solving using Differential Operator -
We know that,

$$(D^n + a_1 D^{n-1} + a_2 D^{n-2} + \text{---} + a_n) y = X$$
$$\therefore f(D) . y = X$$

↳ Differential operator

So, Complementary Funct^n (CF) $\Rightarrow f(D).y = 0$
So,

$$(D-m_1)(D-m_2)(D-m_3) \text{---} (D-m_n) y = 0$$

So, Auxilliary Eq^n is

$$f(m) = 0 \Rightarrow m^n + a_1 m^{n-1} + \text{--} + a_n = 0$$

$(f(m) = 0)$

④ Higher Ratio Test (Raabe's Test) -

- $\lim_{n\to\infty} n\left(\frac{u_n}{u_{n+1}} - 1\right) = K$, $K>1$, cgt. $K<1$, dgt.

⑤ Logarithmic Test -

- $\lim_{n\to\infty} n\left(\log\left(\frac{u_n}{u_{n+1}}\right)\right) = K$, $K>1$, cgt. $K<1$, dgt.

⑥ Cauchy Integral Test -

- for $x \geq 1$, $f(x)$ is non-ve, monotonic f^n of x

$$f(n) = u_n.$$

So, for all vals of n, Σu_n & $\int_1^\infty f(x)\,dx$ cgt. or dgt. together.

- If $\int_1^\infty f(x)\,dx$ = fixed/finite (cgt ✓)
- If $\int_1^\infty f(x)\,dx = \pm\infty$, (dgt ✓)

→ Alternating series -

- ∞ series, terms are alternatively +ve & -ve
- $u_1 - u_2 + u_3 - u_4 + \ldots$

→ Leibniz test -

- Alt. series is cgt if -
- $u_1 > u_2 > u_3$ or, $|u_{n+1}| < |u_n|$ $\qquad \lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| < 1$
- $\lim_{n\to\infty} |u_n| = 0$.

- if not alt, then osc. (no case of dgt)

- Absolute cgt -
- $\Sigma |u_n| = |u_1| + |u_2| + |u_3| + \dots$ is cgt. & leibnitz also ✓✓
- conditionally cgt -
- leibnitz ✓✓ but • $\Sigma |u_n| = |u_1| + |u_2| - \dots$ ✗ cgt.

Successive Differentiation

→ $y = f(x),\ y' = f'(x),\ \dots\ y^n = f^n(x)$.

① $y = e^{ax}$, $y^n = a^n e^{ax}$ //

② $y = (ax+b)^m$, $y^n = \dfrac{(ax+b)^{m-n} \times m! \times a^n}{(m-n)!}$ //

• If $m = n$,

$y^n = n!\, a^n$ //

⑤ If $m < n$, $y^n = 0$

If $m > n$ ✓

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③ If $m = -1$,

$$y = (ax+b)^{-1} = \frac{1}{(ax+b)}$$

$$\therefore y^n = \frac{a^n \times (-1)^n \times n!}{(ax+b)^{n+1}}$$

④ $y = \log(ax+b) \rightarrow y_1 = \dfrac{a}{ax+b}$

$$y^n = \frac{a^n \times (-1)^{n-1} \times (n-1)!}{(ax+b)^n}$$

$(n-1)$ of ③

$\rightarrow$ cop. with 3rd rep. of β

$$\star \rightarrow \int_0^{\pi/2} \sin^p\theta \cos^q\theta \, d\theta = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\,\Gamma\left(\frac{p+q+2}{2}\right)}$$

Reduction Formula

- connects an Integral, with other of same type integral, but of lower order.

(by ILATE)

$$\text{I. } I_n = \int \sin^n x \, dx = \int \overset{I}{\sin^{(n-1)}x} \; \overset{II}{\sin x} \, dx$$

$$= \sin^{(n-1)}x \times (-\cos x) - \int (n-1)\sin^{(n-2)}x \times -\cos x \times \cos x \, dx$$

$$(\cos^2 x = 1 - \sin^2 x)$$

$$I_n = -\sin^{(n-1)}x \cos x + (n-1) I_{n-2} - (n-1) I_n$$

$$\therefore I_n = -\frac{1}{n}\sin^{(n-1)}x \cos x + \left(\frac{n-1}{n}\right) I_{n-2}$$

So,

$$\int_0^{\pi/2} \sin^n x \, dx \Rightarrow \left[I_n\right]_0^{\pi/2} = \left(\frac{n-1}{n}\right)\left[I_{n-2}\right]_0^{\pi/2}$$

$$\star \text{ So, } \left[I_n\right]_0^{\pi/2} = \int_0^{\pi/2} \sin^n x \, dx = \int_0^{\pi/2} \cos^n x \, dx = \begin{cases} \dfrac{(n-1)(n-3)\ldots 3.1}{n(n-2)(n-4)\ldots 4.2} \times \dfrac{\pi}{2} & \rightarrow n \text{ is even} \\[2ex] \dfrac{(n-1)(n-3)\ldots 4.2}{n(n-2)\ldots 3.1} \times 1 & \rightarrow n \text{ is odd} \end{cases}$$