PART A – PHYSICS

1. A uniform cylinder of length L and mass M having cross - sectional area A is suspended, with its length vertical, from a fixed point by a massless spring such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is :

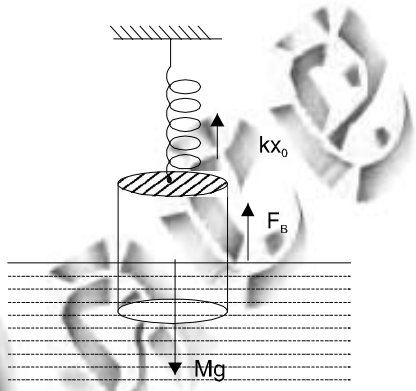
(1) $\frac{Mg}{k}$ (2) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{M}\right)$ (3) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{2M}\right)$ (4) $\frac{Mg}{k}\left(1+\frac{LA\sigma}{M}\right)$

Sol. $kx_0 + F_B = mg$

$kx_0 + \sigma\frac{L}{2}Ag = Mg$

$$x_0 = \frac{Mg - \frac{\sigma LAg}{2}}{k}$$

$$= \frac{Mg}{k}\left(1-\frac{\sigma LA}{2M}\right)$$

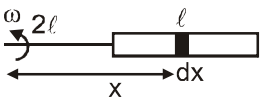


Ans (3)

2. A metallic rod of length 'l' is tied to a string of length 2l and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field 'B' in the region, the e.m.f. induced across the ends of the rod is:

(1) $\frac{2B\omega l^2}{2}$ (2) $\frac{3B\omega l^2}{2}$ (3) $\frac{4B\omega l^2}{2}$ (4) $\frac{5B\omega l^2}{2}$

Sol. $e = \int_{2\ell}^{3\ell}(\omega x)Bdx = B\omega\frac{[(3\ell)^2-(2\ell)^2]}{2}$



$$= \frac{5B\ell^2\omega}{2}$$

Ans. (4)

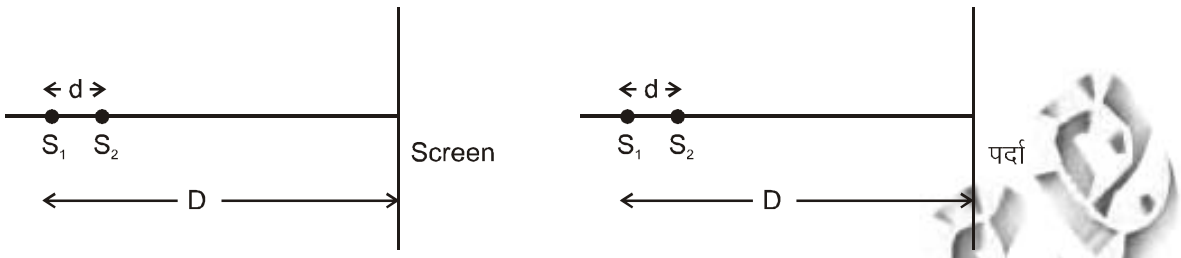
Sol. $q = CV\ (1 - e^{t/\tau})$

at $t = 2\tau$

$q = CV\ (1 - e^{-2})$

Ans (3)

21. Two coherent point sources S_1 and S_2 are separated by a small distance 'd' as shown. The fringes obtained on the screen will be :



(1) points (2) straight lines

(3) semi-circles (4) concentric circles

Sol. It will be concentric circles

Ans (4)

22. The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is :

(1) 3V/m (2) 6V/m (3) 9V/m (4) 12 V/m

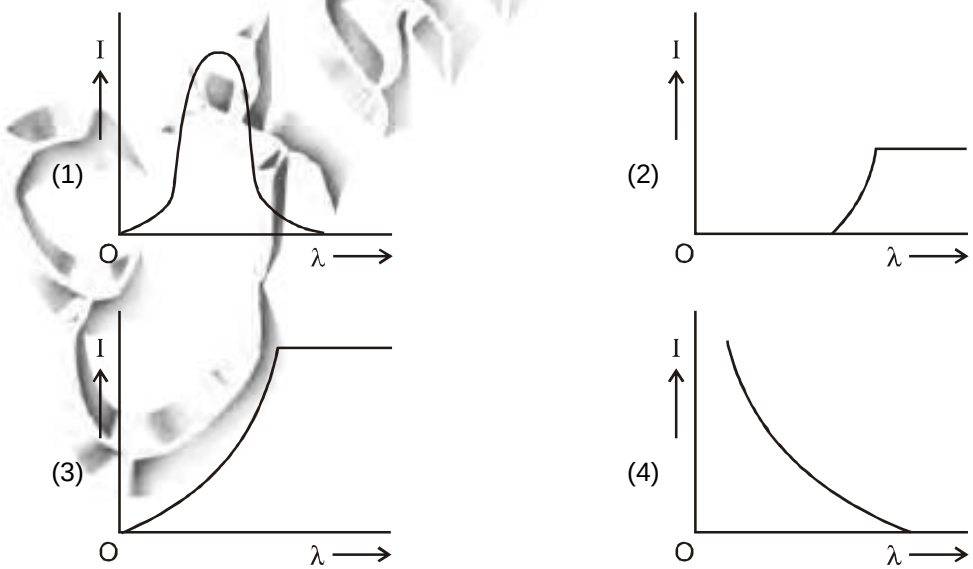
Sol. $\vec{E} = \vec{B} \times \vec{C}$

$|\vec{E}| = |\vec{B}| . |\vec{C}| = 20 \times 10^{-9} \times 3 \times 10^{8}$

$= 6$ V/m.

Ans (2)

23. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows :



Sol. As λ is increased, there will be a value of λ above which photoelectrons will be cease to come out so photocurrent will become zero. Hance (4) is correct answer.

Ans (4)

PART B – CHEMISTRY

31. Which of the following complex species is not expected to exhibit optical isomerism ?

(1) $[Co(en)_3]^{3+}$ (2) $[Co(en)_2 Cl_2]^+$

(3) $[Co(NH_3)_3 Cl_3]$ (4) $[Co(en)(NH_3)_2 Cl_2]^+$

Ans. (3)

Sol. $[Co(NH_3)_3Cl_3]$ show facial as well as meridional isomerism. But both contain plane of symmetry.

So, the answer is (3).

32. Which one of the following molecules is expected to exhibit diamagnetic behaviour ?

(1) C_2 (2) N_2 (3) O_2 (4) S_2

Ans. **(1) and (2)**

Sol. N_2 and C_2 both are diamagnetic

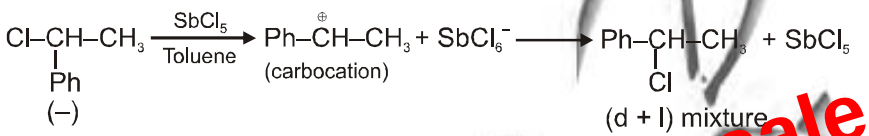
Ans is (1) and (2).

33. A solution of (–) – 1 – chloro–1–phenylethane in toluene racemises slowly in the presence of a small amount of $SbCl_5$, due to the formation of :

(1) carbanion (2) carbene

(3) carbocation (4) free radical

Ans. (3)

Sol. 

34. Given : $E^0_{Cr^{3+}/Cr} = -0.74$ V ; $E^0_{MnO_4^-/Mn^{2+}} = 1.51$ V

$E^0_{Cr_2O_7^{2-}/Cr^{3+}} = 1.33$ V ; $E^0_{Cl_2/Cl^-} = 1.36$ V

Based on the data given above, strongest oxidising agent will be :

(1) Cl (2) Cr^{3+} (3) Mn^{2+} (4) MnO_4^-

Ans. (4)

Sol. Higher the SRP, better is oxidising agent

Hence MnO_4^- is strongest oxidising agent.

35. A piston filled with 0.04 mol of an ideal gas expands reversibly from 50.0 mL to 375 mL at a constant temperature of 37.0°C. As it does so, it absorbs 208 J of heat. The values of q and w for the process will be: (R = 8.314 J/mol K) (ln 7.5 = 2.01)

(1) q = +208 J, w = – 208 J (2) q = –208 J, w = – 208 J

(3) q = –208 J, w = + 208 J (4) q = +208 J, w = + 208 J

Ans. (1)

Sol. The process is isothermal expansion Hence, q = - w $\Delta u = 0$

q = + 208 J

w = – 208 J(expansion work)

Alternate

Statement-II : $(p \to q) \leftrightarrow (\sim q \to \sim p)$

$\sim q \to \sim p$ is contrapositive of $p \to q$ hence $(p \to q) \leftrightarrow (p \to q)$ will be a tautology

statement-II $(p \wedge \sim q) \wedge (\sim p \wedge q)$

p	q	P ^ ~ q	~p ^ q	(p ^ ~ q) ^ (~ p ^ q)
T	T	F	F	F
T	F	T	F	F
F	T	F	T	F
F	F	F	F	F

$\therefore$ It is a fallacy

74. If $\int f(x)dx = \psi(x)$, then $\int x^5 f(x^3)dx$ is equal to

(1) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^2\psi(x^3)dx\right] + C$

(2) $\frac{1}{3}x^3\psi(x^3) - 3\int x^3\psi(x^3)dx + C$

(3) $\frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)dx + C$

(4) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^3\psi(x^3)dx\right] + C$

Sol. **(3)**

$\int f(x)dx = \psi(x)$

$I = \int x^5 f(x^3)dx$

put $x^3 = t \quad \Rightarrow \quad x^2dx = \frac{dt}{3}$

$= \frac{1}{3}\int tf(t)dt$

$= \frac{1}{3}\left[t\psi(t) - \int \psi(t)dt\right]$

$= \frac{1}{3}\left[x^3\psi(x^3) - 3\int x^2\psi(x^3)dx\right] + c$

$= \frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)dx + c$

75. $\lim\limits_{x\to 0} \frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to

(1) $-\frac{1}{4}$ (2) $\frac{1}{2}$ (3) 1 (4) 2

Sol. **(4)**

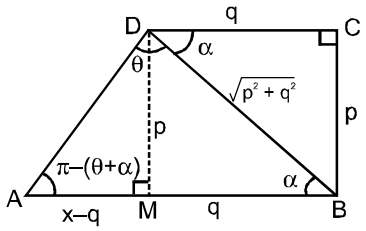
$I = \lim\limits_{x\to 0} \frac{(1-\cos 2x)}{x^2}\frac{(3+\cos x)}{1}.\frac{x}{\tan 4x}$

$= \lim\limits_{x\to 0} \frac{2\sin^2 x}{x^2}.\frac{3+\cos x}{1}.\frac{x}{\tan 4x}$

$= 2.4.\frac{1}{4} = 2$

Sol. **(1)**

Let AB = x



$\tan(\pi-\theta-\alpha)=\frac{p}{x-q} \Rightarrow \tan(\theta+\alpha)=\frac{p}{q-x}$

$\Rightarrow$ $q-x=p\cot(\theta+\alpha)$

$\Rightarrow$ $x=q-p\cot(\theta+\alpha)$

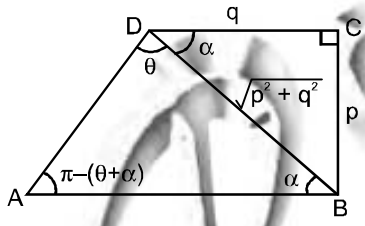
$$=q-p\left(\frac{\cot\theta\cot\alpha-1}{\cot\alpha+\cot\theta}\right)$$

$$=q-p\left(\frac{\frac{q}{p}\cot\theta-1}{\frac{q}{p}+\cot\theta}\right)=q-p\left(\frac{q\cot\theta-p}{q+p\cot\theta}\right)=q-p\left(\frac{q\cos\theta-p\sin\theta}{q\sin\theta+p\cos\theta}\right)$$

$\Rightarrow$ $x=\frac{q^2\sin\theta+pq\cos\theta-pq\cos\theta+p^2\sin\theta}{p\cos\theta+q\sin\theta} \Rightarrow AB=\frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}$.

Alternative

From Sine Rule



$$\frac{AB}{\sin\theta}=\frac{\sqrt{p^2+q^2}}{\sin(\pi-(\theta+\alpha))}$$

$$AB=\frac{\sqrt{p^2+q^2}\sin\theta}{\sin\theta\cos\alpha+\cos\theta\sin\alpha}$$

$$=\frac{(p^2+q^2)\sin\theta}{q\sin\theta+p\cos\theta} \quad \left(\because \cos\alpha=\frac{q}{\sqrt{p^2+q^2}}\right)$$

$$=\frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}.$$

85. If $P=\begin{bmatrix}1 & \alpha & 3\\ 1 & 3 & 3\\ 2 & 4 & 4\end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A|=4$, then α is equal to :

(1) 4 (2) 11 (3) 5 (4) 0

Sol. **(2)**

$|P| = 1(12-12)-\alpha(4-6)+3(4-6)$

$= 2\alpha-6$

$|P| = |A|^2 = 16$

$2\alpha-6=16$

$\alpha = 11.$

86. The intercepts on x-axis made by tangents to the curve, $y=\int_0^x |t|dt, x\in R$, which are parallel to the line $y = 2x$, are equal to :

(1) ±1 (2) ±2 (3) ±3 (4) ±4

Sol. **(1)**

$\frac{dy}{dx} = |x| = 2$

$x = \pm 2$

points $y=\int_0^{\pm2}|t|dt = \pm 2$

$\therefore$ equation of tangent is

$y-2=2(x-2)$ or $y+2=2(x+2)$ $\Rightarrow$ x - intercept = ± 1.