Q No. 6 $x^2-y^2, x+y, \frac{x+y}{x-y}, \ldots$ is $\frac{x+y}{(x-y)^9}$

Here $a_1 = x^2-y^2$

$r = \frac{x+y}{x^2-y^2} = \frac{\cancel{x+y}}{(x-y)\cancel{(x+y)}} = \frac{1}{x-y}$

$n = ?$, $a_n = \frac{x+y}{(x-y)^9}$

Since $a_n = a_1 r^{n-1}$

$\Rightarrow \frac{x+y}{(x-y)^9} = (x^2-y^2)\left(\frac{1}{x-y}\right)^{n-1}$

$\Rightarrow \frac{\cancel{(x+y)}}{(x-y)^9} = \cancel{(x+y)}(x-y)\frac{1}{(x-y)^{n-1}}$

$\Rightarrow \frac{1}{(x-y)^9} = \frac{1}{(x-y)^{n-1-1}}$

$\Rightarrow \frac{1}{(x-y)^9} = \frac{1}{(x-y)^{n-2}}$

$\Rightarrow \left(\frac{1}{x-y}\right)^9 = \left(\frac{1}{x-y}\right)^{n-2}$

$\Rightarrow 9 = n-2 \Rightarrow 9+2 = n$

$\Rightarrow \boxed{n = 11}$ Answer

Q No. 7 Since a, b, c, d are in G.P

therefore $\frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

So $\frac{b}{a} = \frac{c}{b} \Rightarrow b^2 = ac$ — (i)

$\frac{c}{b} = \frac{d}{c} \Rightarrow c^2 = bd$ — (ii)

$\frac{b}{a} = \frac{d}{c} \Rightarrow bc = ad$ — (iii)

i) To show $a-b, b-c, c-d$ are in G.P

Let $r = \frac{b-c}{a-b}$ — (1)

Also $r' = \frac{c-d}{b-c}$

$= \frac{c-d}{b-c} \cdot \frac{a-b}{a-b}$

$= \frac{ac-ad-bc+bd}{(b-c)(a-b)}$

$= \frac{b^2-bc-bc+c^2}{(b-c)(a-b)}$ $\because ac = b^2$, $ad = bc$, $bd = c^2$

$= \frac{b^2-2bc+c^2}{(b-c)(a-b)}$

$= \frac{(b-c)^{\cancel{2}}}{\cancel{(b-c)}(a-b)} = \frac{b-c}{a-b}$ — (2)

from (1) & (2)

$r = r'$

therefore $a-b, b-c, c-d$ are in G.P.

ii) To show $a^2-b^2, b^2-c^2, c^2-d^2$ are in G.P.

Let $r = \frac{b^2-c^2}{a^2-b^2}$ — (1)

Also

$r' = \frac{c^2-d^2}{b^2-c^2}$

$= \frac{c^2-d^2}{b^2-c^2} \cdot \frac{a^2-b^2}{a^2-b^2}$

$= \frac{a^2c^2-a^2d^2-b^2c^2+b^2d^2}{(b^2-c^2)(a^2-b^2)}$

$= \frac{(ac)^2-(ad)^2-(bc)^2+(bd)^2}{(b^2-c^2)(a^2-b^2)}$

$= \frac{(b^2)^2-(bc)^2-(bc)^2+(c^2)^2}{(b^2-c^2)(a^2-b^2)}$ from (i) (ii) & (iii)

$= \frac{b^4-2b^2c^2+c^4}{(b^2-c^2)(a^2-b^2)}$

$= \frac{(b^2-c^2)^{\cancel{2}}}{\cancel{(b^2-c^2)}(a^2-b^2)}$

$= \frac{b^2-c^2}{a^2-b^2}$ — (2)

From (1) and (2)

$r = r'$

Hence $a^2-b^2, b^2-c^2, c^2-d^2$ are in G.P.

iii) Do yourself

Hint:- $r = \frac{b^2+c^2}{a^2+b^2}$ — (1)

Also $r' = \frac{c^2+d^2}{b^2+c^2} = \frac{c^2+d^2}{b^2+c^2} \cdot \frac{a^2+b^2}{a^2+b^2}$

(Same as ii)