2. Explain the difference between exponential and hyperbolic discounting. How can hyperbolic discounting by individuals give rise to self-control problems? Illustrate with an example.

3. Consider the following portfolio design problem. An investor has initial wealth of W, and invests a fraction s of this wealth in a risky asset, and a fraction 1-s of this wealth in a safe asset. The safe asset pays a return of zero. The risky asset pays a return of r (where $0 < r < 1$) with probability p, and a return of $-r$ with probability 1-p. The investor has a utility function, $u(c) = \frac{c^{1-\theta}}{1-\theta}$, $\theta > 0$, where c is the final wealth (and consumption) of the investor.

(a) Derive the expected utility of the investor, V, as a function of s, and the parameters of the problem, W, p and r. (15 marks)

(b) Compute the first derivative of V with respect to s. (15 marks)

(c) Using your answer to (b), find conditions on W, p and r such that the investor will invest a share s strictly between 0 and 1 in the risky asset. (15 marks)

(d) Assuming that the conditions in (c) hold, find the value of s between 0 and 1 that maximizes the investor's expected utility. (15 marks)

(e) How does this value of s change with W, p, and r? Give an economic intuition for your findings. (15 marks)

(f) Prove the general result that an investor choosing between a safe and a risky asset will invest a positive amount in the risky asset if and only if the expected return on the risky asset is greater than the return on the safe asset. Explain how this result generalizes to the case of n risky assets. (25 marks)

4. There are two rice farmers, Alice and Ben. In state of the world 1, Alice can grow $1+a$ units of rice, and in state of the world 2, her crop fails and she can grow nothing. In state of the world 2, Ben can grow $1+b$ units of rice, and in state of the world 1, his crop fails and he can grow nothing. Both farmers believe that state of the world 1 will occur with probability π, $1 > \pi > 0$.

In this question, it is permissible to use the following mathematical result without proof: the solution to the problem of maximizing $\pi \ln x_1 + (1-\pi)\ln x_2$ subject to $p_1 x_1 + p_2 x_2 = m$ is

$$x_1 = \pi \frac{m}{p_1}, \quad x_2 = (1-\pi)\frac{m}{p_2}.$$

(Question (4) continued overleaf)