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The expression state that the probability is the value that tends to $\frac{a}{n}$ as n approaches infinity. For example, if we toss a coin for ten different time times, it will not be compulsory we get 5 heads and 5 tails. However, the more number of time we carry out same experiment.the accurate the result will become. This means that the probability of head and tail will tend to 0.5 respectively which indicate 50% of each events.

In conclusion, one can attempt to choose a close estimate of P(A) by making “n” sufficiently large, for this reason, modern probability is undefined concept the same as point and line are undefined in geometry.

## AXIOMATIC APPROACH TO PROBABILITY

A.N. Rolmogorov, a russian mathmatician, in 1933, introduced the axiomatic approach to probability. This approach explains that probability have no precise definition, rather they gives axioms or postulates on which probability calculations are based. A set function was introduced and considered as classic. There are three axioms that govern the field of probability theory for finite sample spaces. These axioms are ;



i) the probability of an event ranges from zero(0) to one (1). If the event cannot take place, its probability shall be zero,and if it is certain to occur, its probability shall be one (1).

ii) the probability of an entire sample space is one(1) ie P(S) = 1.

Iii) if A and B are mutually exclusively (or disjoint) events, then the probability of occurance of either A or B donated by P$(A \cup B)$ shall be given by

P$(A \cup B) = P(A) + P(B)$

## **SUBJECTIVE APPROACH TO PROBABILITY**

This approach to probability was the first introduce in a book tittled “Foundation of mathmatics” written by frank Ramsey in 1926. This approach involve the application of present beliefs or concepts to statistical problems. This necessarily occur when a random experiment is conducted. For events to be exhausive, the sumation of their probabilities must be equal to a unity (1).

$$P( E_{1,} + E_2 +_{,} E_{3,} + E_4 +_{\dots\dots\dots} + E_n) = 1$$

If the two dices are thrown, the total number of outcomes is donated by

$6^2 = 36$ since two dices are involved in the experiment.

The exhausive events can bre presented in the table bellow

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1,1 | 1,2 | 1,3 | 1,4 | 1,5 | 1,6 |
| 2 | 2,1 | 2,2 | 2,3 | 2,4 | 2,5 | 2,6 |
| 3 | 3,1 | 3,2 | 3,3 | 3,4 | 3,5 | 3,6 |
| 4 | 4,1 | 4,2 | 4,3 | 4,4 | 4,5 | 4,6 |
| 5 | 5,1 | 5,2 | 5,3 | 5,4 | 5,5 | 5,6 |
| 6 | 6,1 | 6,2 | 6,3 | 6,4 | 6,5 | 6,6 |



The value of the table possesses the total number of outcome.

Example 1; what is the probability of obtaining numbers that will be sum up to nine,

Solution; since two dices are thrown, then the possible values that will be sum up to nine (9), are { (3,6), (6,3), (5,4), (4,5)}

$$P(A) = \frac{a}{n}$$

Where a = possible set of number that can be sum up to 9

a= 4

therefore $P(A) = \frac{4}{36} = \frac{1}{9}$

Example 2: supposing the probability of hitting a target with an arrow is $\frac{2}{7}$, *then the* probability of miss.

$$P(A) + P(B) = 1$$

Where $P(A) = probability\ of\ hit = \frac{2}{7}$,

$$P(B) = \text{probability of miss}$$

$$P(B) = 1 - P(A)$$

$$P(B) = \ 1 - \frac{2}{7} = \frac{5}{7}.$$

The above solution interpretes that P(B) is the probability of the event A not occuring and it is expressed as A[!].

Example 3: given that the probability of (A or C) is $\frac{3}{6}$ and that of (B or C) is $\frac{2}{6}$, then calculate the probability of C.

A or C is given as $P(A \cup C) = P(A) + P(C) = \frac{5}{6}$

B or C is given as $P(B \cup C) = P(B) + P(C) = \frac{2}{6}$



$$P(A) + P(B) + P(C) = 1$$

$$P(A) = 1 - (P(B \cup C))$$

$$= 1 - \frac{2}{6} = \frac{4}{6}$$

$$P(A) + P(C) = \frac{3}{6}$$

$$therefore\ P(C) = \frac{5}{6} - P(A) \qquad P(A) = \frac{4}{6}$$

$$P(C) = \frac{5}{6} - \frac{4}{6} = \frac{1}{6}$$

Therefore P(C) is given as $\frac{1}{6}$

$$A \times B \times C$$

**FACTORIAL**

Factorial n, written as n! or $\sqrt{n},$ is defined mathematically as $n! = n \times (n-2)(n-3) \ldots \ldots \ldots .3\ 2\ 1$. We are ask to calculate 5! It will be

$$5 \times 4 \times 3 \times 2 \times 1 = 120$$

Is also used to express the number of ways a given set can be arrange.

e.g. show the possible ways we can arrange four different books in a shelf of four different row

Solution

$$n = 4 \times 3 \times 2 \times 1 = 24$$

Therefore, there are 24 different ways to arrange those books. What if a given set is arranged in r ways, the permutation will be represented by

$$n! = (n-r)!$$

Example: How many ways can we filled a vacant spot of 5 columns with 10 different book

$$nPr = \frac{n!}{(n-r)!}$$

$$10P5 = \frac{10!}{(10-5)!} \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5!}$$



The above expression, expressed the fact that the number of permutation or arrangement (n) take r at a time supposing the sets are unlike (not the same).

In a situation whereby in a given set, there exist the occurrence of same set of numbers e.g. (1, 2, 3, 3, 4,5).

The permutation can be calculated by first calculating for 3 which occurred twice

$n = 2!$

$= number\ of\ ways\ 3\ can\ be\ arrange\ while\ the\ rest\ figure\ (1,2,3,4,5)\ is\ replaced\ h\ x.$

$$with\ the\ total\ number\ of\ sets\ is\ six (6)$$

Therefore, $616 = 6!$

$$x = \frac{6!}{2!} = \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!} = 360$$

The probability of those passing the exam at below 15 years is

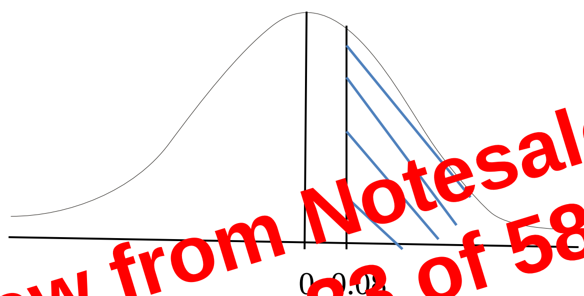
$P(x < -z) = 0.5 - t_z = 0.5 - 0.1915 = 0.3085 \times 20{,}000 = 6{,}170.$

Therefore 6,170 students below the age of 15 are expected to pass the examination.

ii) Above twenty two years.

$$x = 22 \qquad \mu = 21 \qquad \sigma = 12$$

$$z = \frac{x-\mu}{\sigma} = \frac{22-21}{12} = \frac{1}{12} = 0.08.$$






$P(x > z) = 0.5 - T_z = 0.5 - 0.0319 = 0.4681$

$$0.4681 \times 20{,}000 = 9{,}362$$

Therefore 9,362 students above the age of 22 are likely to pass the examination.

iii) Above twenty years.

$$x = 20 \qquad \mu = 21 \qquad \sigma = 12$$

$$z = \frac{x-\mu}{\sigma} = \frac{20-21}{12} = -\frac{1}{12} = -0.08$$

$\therefore\ P = 4{,}500 - 3{,}000 = \$1{,}500$

### For Business C

$$P = TR - TC$$

$$TR = \in (Xp)p = Pp \times Xp = 4{,}800 \times 0.7$$

$$= 3{,}360$$

$$TC = \in (Xl) = Pl \times l = 1{,}500 \times 0.2 = 300$$

$$\therefore \cap = 3{,}360 - 300 = 3060$$

The businessman will stand to gain $2,200 if he picks-up "A business", $1,500 if he pick-up "B business" and $3060, if he pick-up "C business". Since the maximum gain is $3060 which can only be achieve with business C, therefore, the businessman is advice to pick-up business C, for maximum profit.



**STATISTICS AND DECISION MAKING AS RELATED TO PROBABILITY**

We have threw little light on this while discussing mathematical expectation. At this topic, we will discuss how certain statement made can be proven either true or false, which will guide our decision making. Statement initially made can be either true or false until it is proven otherwise. This will lead us to what is called a ***hypothesis.***

Hypothesis is a supposition or a presumption that needs to be subjected to several tests before it will be accepted. It is a tentative statement made by a scientist or a researcher which is yet to be proven. If proven right, it will become a theory.

Statiscians just like scientist make hypothesis about data distribution, which is tested under certain specification, with the knowledge of probability.

hypothesis testing is of great importance to statistical work, and there are two basic tests for mean

This test involve

Large samples $x > 30$

Small samples $x < 30$

## Large samples

We may be asked to test for the difference between two given means of samples that has its size greater than 30. At this point, we should bear in mind that, we will carry out test for the hypothesis of large samples. It is mathematically expressed as.



$$\frac{Difference}{S.E}$$

$$S.E = \frac{\sigma}{\sqrt{n}}$$

$$\sigma = standard\ deviation\ of\ the\ distribution$$

$$n = size\ of\ the\ sample$$

**Example**:- Someone estimated the mean score of a given class of 56 students to be 43%, when the pass mark is 45% therefore he concluded that they failed judging by average performance. It was later founded out that the mean score was 47%. Is there any significant difference between the estimate mean and the actual mean at 99% confident level, at a given standard deviation of 12%?.

**Step l**

$H_o \rightarrow \hat{o} = \mu,\ \hat{o} - \mu = 0$

$$\hat{o} = estimated\ mean$$

iii) P(-1.7 < x < +2.3).

2) An amount of money is given to villagers residing in a particular village. If the total amount is \$20million and they are paid according to their age, with a mean amount given to villagers of 25years with a standard deviation of 12years. What will be the amount given to those villagers that falls under,

i) Above 20 years

ii) Below 28 years

iii) Above 26 years,

iv) Between 27 and 29 years,

v) Between 24 and 26 years, will be paid.

3) calculate the mathematical expectation of a trader whose business has a probability of 0.02, and a profit of \$2million dollars.

4) A coin is tossed 600 times and 325 heads turned up, can we say a die is unbiased at 99% confident level.

5) Town A has 2500 youths, with 700 youths as smokers, town B has 2900 youths with 1100 youths as smokers, can we say there is a significant difference in the smoking habits of youths of both towns, as the proportion of smoking is concerned?(5% significant level).



## Answers to mathematics exercises

| | | |
|---|---|---|
| 1)i) $\frac{1}{3}$ | ii) $\frac{3}{4}$ | iii) $\frac{1}{12}$ |
| 2)i) $\frac{1}{4}$ | ii) $\frac{3}{80}$ | iii) $\frac{2}{5}$ |
| 3)i) $\frac{4}{18}$ | ii) $\frac{10}{18}$ | iii) $\frac{4}{102}$ |