a) $\text{proj}_{\vec{u}}(\vec{v}) = \frac{2}{3} < 1, 2, 2 >$

b) $\vec{v} - \text{proj}_{\vec{u}}(\vec{v}) = \frac{1}{3} < 4, -7, 5 >$

c) scalar projection of $\vec{v}$ onto $\vec{u}$, $\left|\text{proj}_{\vec{u}}\vec{v}\right| = 2$

Summary

- A vector can be written as

$$\vec{V} =< v_1, v_2, v_3 >= v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$$

- Direction angle

$$\cos\alpha = \frac{v_1}{\left\|\vec{v}\right\|}, \quad \cos\beta = \frac{v_2}{\left\|\vec{v}\right\|}, \quad \cos\gamma = \frac{v_3}{\left\|\vec{v}\right\|},$$

satisfying the relation $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

- If $\vec{u} =< u_1, u_2, u_3 >$, $\vec{v} =< v_1, v_2, v_3 >$
- Dot Product

$$\vec{u} \bullet \vec{v} = u_1v_1 + u_2v_2 + u_3v_3$$
$$\vec{u} \bullet \vec{v} = \left|\vec{u}\right|\left|\vec{v}\right|\cos\theta$$

- Orthogonal Projections
A vector $\vec{v}$ can be decomposed into two vectors, one vector parallel to $\vec{u}$

$$\text{proj}_{\vec{u}}\vec{v} = \frac{\vec{v} \bullet \vec{u}}{\left\|\vec{u}\right\|^2}\vec{u}$$

and another vector perpendicular to $\vec{u}$

$$\vec{v} - \text{proj}_{\vec{u}}\vec{v}$$