3. COMPLETING THE SQUARE

If the quadratic equation is of the form $ax^2 + bx + c = 0, \ where \ a \neq 0$ and the quadratic expression is not factorable, try completing the square.

<u>Example:</u> $x^2 + 6x - 11 = 0$

****Important: If $a \neq 1$, divide all terms by "a" before proceeding to the next steps.**

Move the constant to the right side $x^2 + 6x = 11$

Find half of b, which means $\frac{b}{2}$: $\frac{6}{2} = 3$

Find $\left(\frac{b}{2}\right)^2$: $3^2 = 9$

Add $\left(\frac{b}{2}\right)^2$ to both sides of the equation $x^2 + 6x + \underline{9} = 11 + \underline{9}$

Factor the quadratic side $(x+3)(x+3) = 20$

(which is a perfect square because you just made it that way!)

Then write in perfect square form $(x+3)^2 = 20$

Take the square root of both sides $\sqrt{(x+3)^2} = \pm\sqrt{20}$

$x + 3 = \pm\sqrt{20}$

Solve for x

$x = -3 \pm \sqrt{20}$ *Simplify the radical*

$x = -3 \pm 2\sqrt{5}$

This represents the exact answer.

Decimal approximations can be found using a calculator.

4. QUADRATIC FORMULA

Any quadratic equation of the form $ax^2 + bx + c = 0, \ where \ a \neq 0$ can be solved for both real and imaginary solutions using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

<u>Example:</u> $x^2 + 6x - 11 = 0 \quad (a = 1, \quad b = 6, \quad c = -11)$

Substitute values into the quadratic formula:

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-11)}}{2(1)} \rightarrow x = \frac{-6 \pm \sqrt{36 + 44}}{2} \rightarrow x = \frac{-6 \pm \sqrt{80}}{2} \quad simplify \ the \ radical$$

$$x = \frac{-6 \pm 4\sqrt{5}}{2} \rightarrow x = -3 \pm 2\sqrt{5} \ \ This \ is \ the \ final \ simplified \ \ EXACT \ answer$$