Example 2 – Composing Functions

Find *f(g(x))* and *g(f(x))*. State the domain.

a. $f(x) = x^2 - 1; g(x) = \dfrac{1}{x-1}$

You start with the outside function and replace the inside function wherever you see an x.

$K(x) = f(g(x)) = \left(\dfrac{1}{x-1}\right)^2 - 1$ $D_K : \{x : x \neq 1\}$ or $(-\infty, 1) \cup (1, \infty)$

The resulting function can be simplified but, we are going to leave it the way it is for now.

$N(x) = g(f(x)) = \dfrac{1}{(x^2-1)-1} = \dfrac{1}{x^2-2}$ $D_N : \{x : x \neq \pm\sqrt{2}\}$ or $(-\infty, -\sqrt{2}) \cup (-\sqrt{2}, \sqrt{2}) \cup (\sqrt{2}, \infty)$

Because the parentheses are not really needed, we combine the negative ones. To find your domain, set $x^2 - 2 = 0$ and solve for x.

b. $f(x) = x^3; g(x) = \sqrt[3]{1-x^3}$

You start with the outside function and replace the inside function wherever you see an x.

$H(x) = f(g(x)) = \left(\sqrt[3]{1-x^3}\right)^3 = 1 - x^3$ $D_H : (-\infty, \infty)$

When you cube a cube root, you get just the part under the radical. Because the cubic function has no restrictions on x, the domain is all real numbers

$B(x) = g(f(x)) = \sqrt[3]{1-(x^3)^3} = \sqrt[3]{1-x^9}$ $D_B : (-\infty, \infty)$

Remember that $1 - x^9$ is a quantity, you cannot take the cube root of it.

In example 2, two functions were composed to form a new function. There are times in calculus when we need to reverse the process. This is called decomposing functions.

Example 3 – Decomposing Functions

Find *f(x)* and *g(x)* so that the function can be described as y = *f(g(x))*.

In other words, y is the answer we got after we substituted $g(x)$ into $f(x)$.
We need to find out what $f(x)$ and $g(x)$ were before we simplified the answer.

a. $y = |3x - 2|$

b. $y = \dfrac{1}{x^3 - 5x + 3}$