Stock Market Trading Volume

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Abstract

If price and quantity are the fundamental building blocks of any theory of market interactions, the importance of trading volume in understanding the behavior of financial markets is clear. However, while many economic models of financial markets have been developed to explain the behavior of prices—predictability, variability, and information content—far less attention has been devoted to explaining the behavior of trading volume. In this article, we hope to expand our understanding of trading volume by developing well-articulated economic models of asset prices and volume and empirically estimating them using recently available daily volume data for individual securities from the University of Chicago's Center for Research in Securities Prices. Our theoretical contributions include: (1) an economic definition of volume that is most consistent with theoretical models of trading activity; (2) the derivation of volume implications of basic portfolio theory; and (3) the development of an intertemporal equilibrium model of asset market in which the trading process is determined endogenously by liquidity needs and risk-sharing motives. Our empirical contributions include: (1) the construction of a volume/returns database extract of the CRSP volume data; (2) comprehensive exploratory data analysis of both the time-series and cross-sectional properties of trading volume; (3) estimation and inference for price/volume relations implied by asset-pricing models; and (4) a new approach for empirically identifying factors to be included in a linear-factor model of asset returns using volume data.

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construct the hedging portfolio and its returns. We find that the hedging-portfolio returns consistently outperforms other factors in predicting future returns to the market portfolio, an implication of the intertemporal equilibrium model. We then use the returns to the hedging and market portfolios as two risk factors in a cross-sectional test along the lines of Fama and MacBeth (1973), and find that the hedging portfolio is comparable to other factors in explaining the cross-sectional variation of expected returns.

We conclude with suggestions for future research in Section 8.

2 Measuring Trading Activity

Any empirical analysis of trading activity in the market must start with a proper measure of volume. The literature on trading activity in financial markets is extensive and a number of measures of volume have been proposed and studied.[3] Some studies of aggregate trading activity use the total number of shares traded as a measure of volume (see Epps and Epps (1976), Gallant, Rossi, and Tauchen (1992), Hiemstra and Jones (1994), and Ying (1966)). Other studies use aggregate *turnover*—the total number of shares traded divided by the total number of shares outstanding—as a measure of volume (see Campbell, Grossman, Wang (1993), LeBaron (1992), Smidt (1990), and the *1996 NYSE Fact Book*). Individual share volume is often used in the analysis of price/volume and volatility/volume relations (see Andersen (1996), Epps and Epps (1976), and Lamoureux and Lastrapes (1990, 1994)). Studies focusing on the impact of information events on trading activity use individual turnover as a measure of volume (see Bamber (1986, 1987), Lakonishok and Smidt (1986), Morse (1980), Richardson, Sefcik, Thompson (1986), Stickel and Verrecchia (1994)). Alternatively, Tkac (1996) considers individual dollar volume normalized by aggregate market dollar-volume. And even the total number of trades (Conrad, Hameed, and Niden (1994)) and the number of trading days per year (James and Edmister (1983)) have been used as measures of trading activity. Table 1 provides a summary of the various measures used in a representative sample of the recent volume literature. These differences suggest that different applications call for different volume measures.

In order to proceed with our analysis, we need to first settle on a measure of volume. After developing some basic notation in Section 2.1, we review several volume measures in Section 2.2 and provide some economic motivation for turnover as a canonical measure of

[3]See Karpoff (1987) for an excellent introduction to and survey of this burgeoning literature.

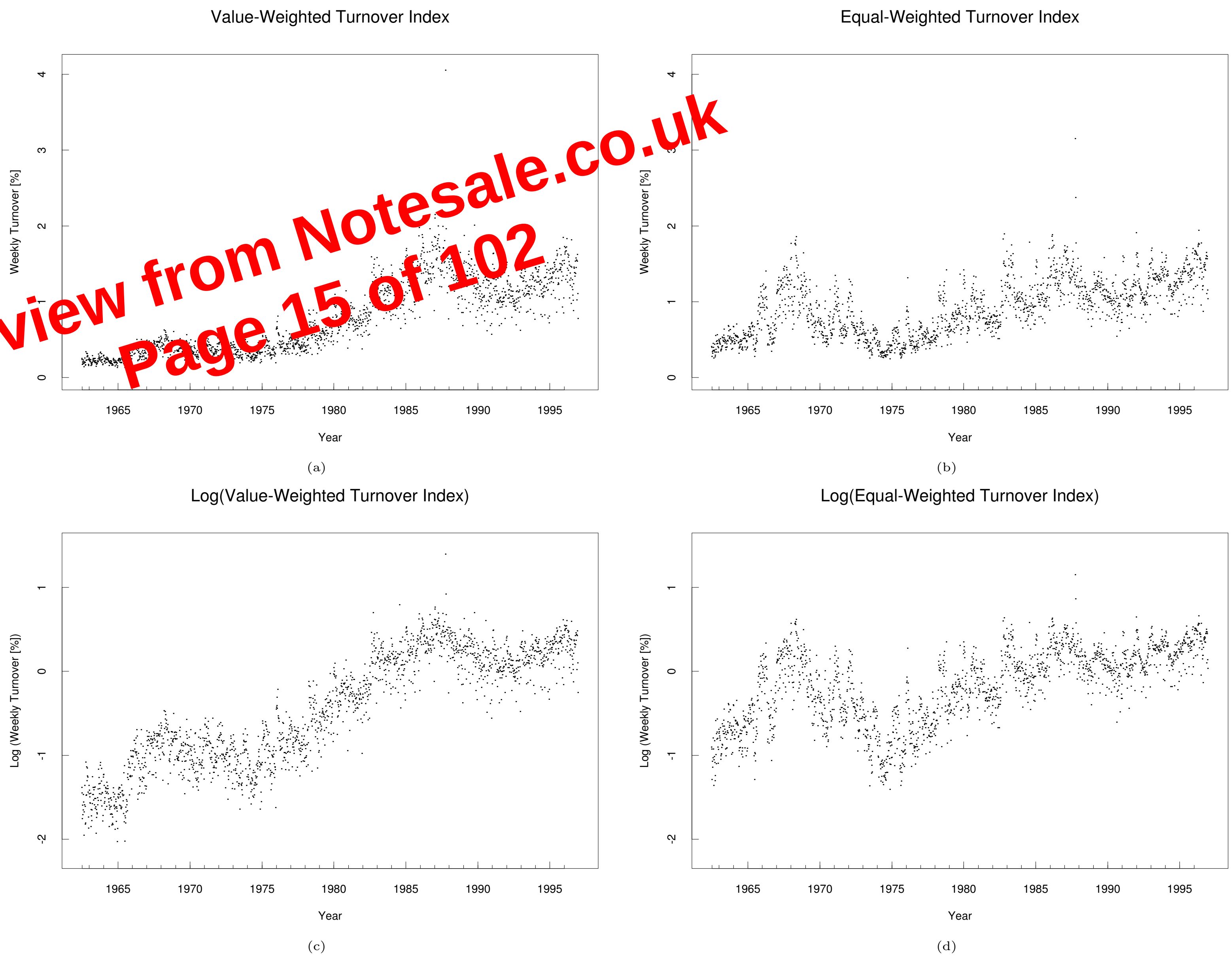

Figure 1: Weekly Value-Weighted and Equal-Weighted Turnover Indexes, 1962 to 1996.

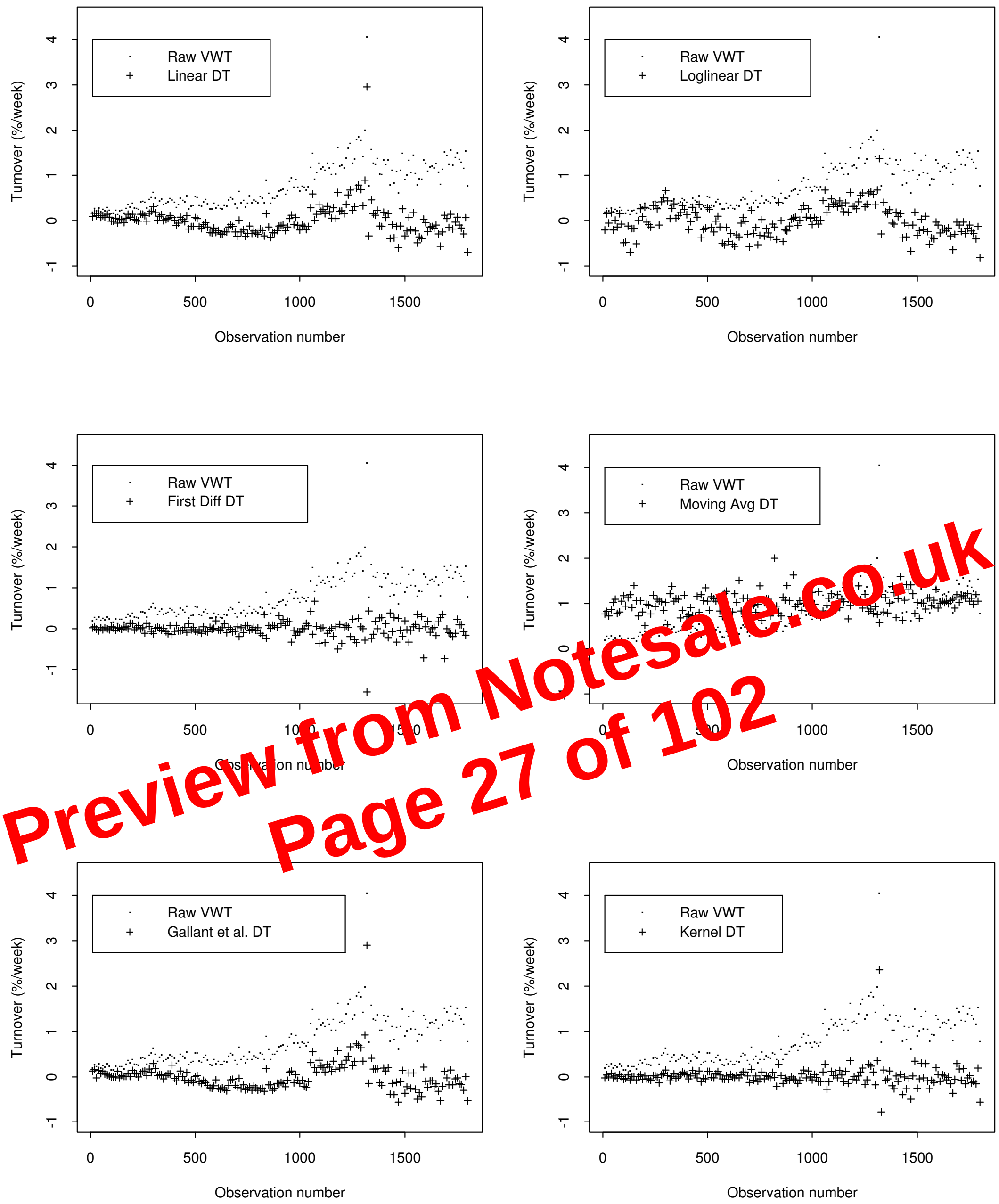

Figure 2: Raw and Detrended Weekly Value-Weighted Turnover Indexes, 1962 to 1996.

	$\overline{\tau}_j$	$\tilde{\tau}_j$	$\hat{\alpha}_{\tau,j}$	$\hat{\beta}_{\tau,j}$	$\hat{\sigma}_{\epsilon,\tau,j}$	$\hat{\alpha}_{r,j}$	$\hat{\beta}_{r,j}$	$\hat{\sigma}_{\epsilon,r,j}$	v_j	p_j	d_j	SP_j^{500}	$\hat{\gamma}_{r,j}(1)$
							1962 to 1966 (234 weeks)						
μ	0.576	0.374	0.009	2.230	0.646	0.080	1.046	4.562	17.404	1.249	0.059	0.175	−2.706
m	0.397	0.272	0.092	0.725	0.391	0.064	1.002	3.893	17.263	1.445	0.058	0.000	−0.851
s	0.641	0.372	1.065	5.062	0.889	0.339	0.529	2.406	1.737	0.965	0.081	0.380	8.463
							1967 to 1971 (261 weeks)						
μ	0.900	0.610	−0.361	3.134	0.910	0.086	1.272	5.367	17.930	1.442	0.049	0.178	−1.538
m	0.641	0.446	−0.128	1.948	0.612	0.081	1.225	5.104	17.791	1.522	0.042	0.000	−0.623
s	0.827	0.547	0.954	3.559	0.940	0.383	0.537	1.991	1.566	0.685	0.046	0.382	4.472
							1972 to 1976 (261 weeks)						
μ	0.521	0.359	−0.025	1.472	0.535	0.085	0.986	6.252	17.574	0.823	0.072	0.162	−3.084
m	0.420	0.291	0.005	1.040	0.403	0.086	0.955	5.825	17.346	0.883	0.063	0.000	−1.007
s	0.408	0.292	0.432	1.595	0.473	0.319	0.429	2.619	1.784	0.890	0.067	0.369	8.262
							1977 to 1981 (261 weeks)						
μ	0.780	0.553	0.043	1.199	0.749	0.254	0.950	5.081	18.155	1.074	0.099	0.176	−1.748
m	0.629	0.449	0.052	0.818	0.566	0.215	0.936	4.737	18.094	1.212	0.086	0.000	−0.622
s	0.561	0.405	0.638	1.348	0.643	0.356	0.428	2.097	1.769	0.805	0.097	0.381	5.100
							1982 to 1986 (261 weeks)						
μ	1.160	0.833	0.005	0.957	1.135	0.113	0.873	5.419	18.629	1.143	0.090	[illegible]	−1.627
m	0.998	0.704	0.031	0.713	0.902	0.146	0.863	4.813	[illegible]	[illegible]	[illegible]	0.000	−0.573
s	0.788	0.605	0.880	1.018	0.871	0.455	0.437	2.581	[illegible]	[illegible]	0.126	0.385	8.405
							1987 to 1991 (261 weeks)						
μ	1.255	0.888	0.333	0.715	[illegible]	−0.007	0.977	[illegible]	[illegible]	[illegible]	0.095	0.191	−5.096
m	0.995	0.708	0.171	[illegible]	[illegible]	0.014	0.998	[illegible]	[illegible]	1.108	0.062	0.000	−0.386
s	1.039	0.773	[illegible]	1.229	1.272	[illegible]	[illegible]	5.417	2.013	1.097	0.134	0.393	44.246
							1992 to 1996 (261 weeks)						
μ	1.419	1.032	0.379	[illegible]	[illegible]	0.147	0.851	5.722	19.407	1.081	0.063	0.182	−3.600
m	1.114	0.834	0.239	0.511	0.997	0.113	0.831	4.674	19.450	1.297	0.042	0.000	−1.136
s	1.208	0.910	1.637	1.572	1.480	0.482	0.520	3.901	2.007	1.032	0.095	0.386	21.550

Summary statistics of variables for cross-sectional analysis of weekly turnover of NYSE or AMEX ordinary common shares (CRSP share codes 10 and 11, excluding 37 stocks containing Z-errors in reported volume) for subperiods of the sample period from July 1962 to December 1996. The variables are: $\overline{\tau}_j$ (average turnover); $\tilde{\tau}_j$ (median turnover); $\hat{\alpha}_{\tau,j}$, $\hat{\beta}_{\tau,j}$, and $\hat{\sigma}_{\epsilon,\tau,j}$ (the intercept, slope, and residual, respectively, from the time-series regression of an individual security's turnover on market turnover); $\hat{\alpha}_{r,j}$, $\hat{\beta}_{r,j}$, and $\hat{\sigma}_{\epsilon,r,j}$ (the intercept, slope, and residual, respectively, from the time-series regression of an individual security's return on the market return); v_j (natural logarithm of market capitalization); p_j (natural logarithm of price); d_j (dividend yield); SP_j^{500} (S&P 500 indicator variable); and $\hat{\gamma}_{r,j}(1)$ (first-order return autocovariance). The statistics are: μ (mean); m (median); and s (standard deviation).

Table 10: Summary Statistics for Cross-Sectional Analysis of Weekly Turnover

	$\overline{\tau}_j$	$\tilde{\tau}_j$	$\hat{\alpha}_{\tau,j}$	$\hat{\beta}_{\tau,j}$	$\hat{\sigma}_{\epsilon,\tau,j}$	$\hat{\alpha}_{r,j}$	$\hat{\beta}_{r,j}$	$\hat{\sigma}_{\epsilon,r,j}$	v_j	p_j	d_j	SP_j^{500}
					1962 to 1966 (2,073 stocks)							
$\tilde{\tau}_j$	93.1											
$\hat{\alpha}_{\tau,j}$	−8.6	1.9										
$\hat{\beta}_{\tau,j}$	56.6	43.9	−86.9									
$\hat{\sigma}_{\epsilon,\tau,j}$	88.8	70.3	−11.8	54.1								
$\hat{\alpha}_{r,j}$	14.9	10.7	−12.0	16.9	14.8							
$\hat{\beta}_{r,j}$	56.3	59.3	−15.8	40.8	43.2	1.5						
$\hat{\sigma}_{\epsilon,r,j}$	36.1	25.4	−19.5	34.0	45.8	16.3	29.2					
v_j	−19.2	−11.4	9.6	−17.5	−28.9	−3.0	1.9	−62.7				
p_j	−7.6	1.7	14.6	−16.0	−20.1	1.6	3.2	−77.1	78.7			
d_j	−11.4	−9.3	9.3	−13.2	−12.2	0.4	−17.0	−27.9	13.1	20.7		
SP_j^{500}	−5.0	−0.6	4.8	−6.4	−10.2	−6.6	2.4	−24.2	43.1	32.0	4.8	
$\hat{\gamma}_{r,j}(1)$	−0.6	3.0	5.7	−5.1	−7.6	−14.4	1.9	−63.2	31.1	52.7	12.9	10.7
					1967 to 1971 (2,292 stocks)							
$\tilde{\tau}_j$	96.8											
$\hat{\alpha}_{\tau,j}$	−30.9	−23.0										
$\hat{\beta}_{\tau,j}$	77.6	70.6	−83.8									
$\hat{\sigma}_{\epsilon,\tau,j}$	92.2	80.7	−38.2	77.9								
$\hat{\alpha}_{r,j}$	10.3	8.7	4.2	1.9	12.5							
$\hat{\beta}_{r,j}$	59.2	60.4	−31.2	55.4	50.0	−12.6						
$\hat{\sigma}_{\epsilon,r,j}$	56.3	49.5	−36.7	57.0	60.7	−1.5	61.3					
v_j	−32.5	−25.3	32.7	−40.5	−41.1	1.1	−23.7	−67.6				
p_j	−19.8	−11.9	35.6	−35.3	−30.1	16.7	−22.1	−68.9	77.0			
d_j	−38.2	−37.2	19.8	−35.3	−35.2	3.0	−51.9	−57.1	28.0	28.3		
SP_j^{500}	−14.0	−10.6	11.9	−16.1	−18.2	2.2	−11.5	−30.9	47.9	35.2	13.3	
$\hat{\gamma}_{r,j}(1)$	−8.7	−6.8	11.7	−12.8	−11.4	8.8	−14.9	−40.7	30.7	43.8	[illegible]	12.3
					1972 to 1976 (2,084 stocks)							
$\tilde{\tau}_j$	96.5											
$\hat{\alpha}_{\tau,j}$	2.5	8.9										
$\hat{\beta}_{\tau,j}$	67.4	60.2	−72.0									
$\hat{\sigma}_{\epsilon,\tau,j}$	83.9	69.4	−5.9	62.6								
$\hat{\alpha}_{r,j}$	8.5	7.2	−7.7	11.1	7.5							
$\hat{\beta}_{r,j}$	54.3	54.3	[illegible]	49.4	39.7	−14.8						
$\hat{\sigma}_{\epsilon,r,j}$	22.2	[illegible]	−2.9	17.9	[illegible]	−11.3	29.9					
v_j	0.6	12.0	3.8	−2.7	−21.7	−5.3	12.6	−65.2				
p_j	[illegible]	17.4	8.8	[illegible]	−11.7	14.6	1.8	−76.1	83.7			
d_j	−20.9	−18.3	[illegible]	[illegible]	−20.9	9.4	−34.2	−41.6	19.4	25.0		
SP_j^{500}	1.2	8.6	1.5	−0.4	−13.1	−2.2	9.1	−28.2	50.5	37.9	2.6	
$\hat{\gamma}_{r,j}(1)$	0.0	3.2	6.4	−5.2	−5.6	5.3	−8.3	−57.1	32.9	50.6	23.8	11.6
					1977 to 1981 (2,352 stocks)							
$\tilde{\tau}_j$	96.4											
$\hat{\alpha}_{\tau,j}$	6.7	11.0										
$\hat{\beta}_{\tau,j}$	61.9	55.1	−72.9									
$\hat{\sigma}_{\epsilon,\tau,j}$	83.0	67.4	3.5	54.9								
$\hat{\alpha}_{r,j}$	10.6	2.8	−8.2	16.9	22.7							
$\hat{\beta}_{r,j}$	59.8	63.8	−11.0	47.1	35.6	3.2						
$\hat{\sigma}_{\epsilon,r,j}$	28.5	18.3	−8.2	25.6	42.8	30.8	24.9					
v_j	5.3	15.7	6.7	−2.0	−16.5	−26.8	16.4	−63.4				
p_j	8.1	17.1	11.7	−3.6	−10.8	−9.0	12.2	−70.1	80.8			
d_j	−18.4	−18.2	3.8	−15.2	−14.7	1.4	−27.9	−27.3	9.9	13.0		
SP_j^{500}	2.5	8.4	−0.4	2.5	−8.9	−19.0	8.5	−28.5	51.6	35.1	2.8	
$\hat{\gamma}_{r,j}(1)$	0.2	3.0	1.8	−1.3	−5.3	−3.6	−2.3	−55.6	31.5	52.1	14.7	10.5

Table 11a: Correlation Matrix for Weekly Turnover Regressors

	$\overline{\tau}_j$	$\tilde{\tau}_j$	$\hat{\alpha}_{\tau,j}$	$\hat{\beta}_{\tau,j}$	$\hat{\sigma}_{\epsilon,\tau,j}$	$\hat{\alpha}_{r,j}$	$\hat{\beta}_{r,j}$	$\hat{\sigma}_{\epsilon,r,j}$	v_j	p_j	d_j	SP_j^{500}
						1982 to 1986 (2,644 stocks)						
$\tilde{\tau}_j$	96.2											
$\hat{\alpha}_{\tau,j}$	−12.0	−5.6										
$\hat{\beta}_{\tau,j}$	71.3	64.3	−77.8									
$\hat{\sigma}_{\epsilon,\tau,j}$	80.0	62.8	−19.8	64.7								
$\hat{\alpha}_{r,j}$	−7.4	−10.9	−14.5	6.2	2.4							
$\hat{\beta}_{r,j}$	46.4	50.6	−12.6	38.3	24.8	−32.5						
$\hat{\sigma}_{\epsilon,r,j}$	15.4	7.3	12.3	0.7	25.2	−17.7	15.6					
v_j	19.0	29.7	−8.3	18.8	−5.0	−3.1	27.6	−55.7				
p_j	9.0	16.5	−12.4	15.3	−5.9	22.3	10.3	−76.1	75.3			
d_j	−6.7	−7.6	−4.1	−0.5	−2.5	15.5	−12.6	−21.4	16.6	20.5		
SP_j^{500}	15.5	22.7	−2.0	12.1	−1.6	−3.8	18.2	−24.7	57.3	37.5	8.0	
$\hat{\gamma}_{r,j}(1)$	5.2	5.6	−8.9	9.5	4.1	18.9	−0.4	−39.2	15.7	32.6	7.1	5.2
						1987 to 1991 (2,471 stocks)						
$\tilde{\tau}_j$	94.1											
$\hat{\alpha}_{\tau,j}$	17.1	25.8										
$\hat{\beta}_{\tau,j}$	50.8	39.2	−76.0									
$\hat{\sigma}_{\epsilon,\tau,j}$	79.1	56.6	−1.0	53.0								
$\hat{\alpha}_{r,j}$	7.1	5.1	16.8	−9.7	9.2							
$\hat{\beta}_{r,j}$	45.4	49.4	5.0	25.5	22.3	−15.0						
$\hat{\sigma}_{\epsilon,r,j}$	3.1	−3.6	−0.7	2.5	12.7	24.4	−2.6					
v_j	20.3	31.7	3.3	10.4	−2.0	5.6	22.4	−48.1				
p_j	12.3	22.0	6.4	2.5	−5.7	10.8	11.2	−62.0	80.4			
d_j	−1.2	−1.9	−1.8	0.8	1.6	2.9	−4.7	−10.9	12.9	15.7		
SP_j^{500}	16.1	25.4	−1.4	11.6	−3.8	−2.4	19.1	−20.7	58.7	[illegible]	[illegible]	
$\hat{\gamma}_{r,j}(1)$	4.2	5.5	2.7	0.5	0.4	−39.5	11.7	[illegible]	[illegible]	23.0	2.9	4.4
						1992 to 1996 ([illegible] stocks)						
$\tilde{\tau}_j$	94.8											
$\hat{\alpha}_{\tau,j}$	6.8	10.8										
$\hat{\beta}_{\tau,j}$	55.8	49.1	[illegible]									
$\hat{\sigma}_{\epsilon,\tau,j}$	79.1	58.6	[illegible]	[illegible]								
$\hat{\alpha}_{r,j}$	−2.8	[illegible]	−13.5	9.6	[illegible]							
$\hat{\beta}_{r,j}$	46.6	[illegible]	0.0	28.7	[illegible]	−14.4						
$\hat{\sigma}_{\epsilon,r,j}$	[illegible]	6.4	[illegible]	[illegible]	36.3	24.2	4.2					
v_j	10.1	23.8	[illegible]	[illegible]	−18.8	−15.7	27.8	−61.5				
p_j	5.8	17.2	−3.3	6.1	−17.4	−8.4	16.2	−76.8	81.5			
d_j	−9.5	−8.3	−1.5	−4.5	−9.3	0.4	−6.4	−14.6	13.3	15.4		
SP_j^{500}	6.6	15.9	−8.8	11.5	−12.3	−9.1	17.5	−24.2	56.7	37.7	11.0	
$\hat{\gamma}_{r,j}(1)$	2.3	4.9	−2.3	3.2	−3.8	1.2	12.1	−23.2	19.1	29.3	5.0	4.5

Correlation matrix of variables for cross-sectional analysis of weekly turnover of NYSE or AMEX ordinary common shares (CRSP share codes 10 and 11, excluding 37 stocks containing Z-errors in reported volume) for subperiods of the sample period from July 1962 to December 1996. The variables are: $\overline{\tau}_j$ (average turnover); $\tilde{\tau}_j$ (median turnover); $\hat{\alpha}_{\tau,j}$, $\hat{\beta}_{\tau,j}$, and $\hat{\sigma}_{\epsilon,\tau,j}$ (the intercept, slope, and residual, respectively, from the time-series regression of an individual security's turnover on market turnover); $\hat{\alpha}_{r,j}$, $\hat{\beta}_{r,j}$, and $\hat{\sigma}_{\epsilon,r,j}$ (the intercept, slope, and residual, respectively, from the time-series regression of an individual security's return on the market return); v_j (natural logarithm of market capitalization), p_j (natural logarithm of price); d_j (dividend yield); SP_j^{500} (S&P 500 indicator variable); and $\hat{\gamma}_{r,j}(1)$ (first-order return autocovariance).

Table 11b: Correlation Matrix for Weekly Turnover Regressors (continued)

c	$\hat{\alpha}_{r,j}$	$\hat{\beta}_{r,j}$	$\hat{\sigma}_{\epsilon,r,j}$	v_j	p_j	d_j	SP500$_j$	$\hat{\gamma}_{r,j}(1)$	R^2 (%)
				1982 to 1986 (261 weeks, 2,644 stocks)					
−1.385	0.051	0.543	0.062	0.071	0.085	−0.223	0.091	0.006	31.6
(0.180)	(0.025)	(0.027)	(0.007)	(0.010)	(0.023)	(0.081)	(0.031)	(0.001)	
−0.193	0.018	0.583	0.057	—	0.170	−0.182	0.187	0.005	30.4
(0.051)	(0.024)	(0.027)	(0.007)		(0.020)	(0.081)	(0.028)	(0.001)	
−1.602	0.080	0.562	0.048	0.091	—	−0.217	0.085	0.006	31.3
(0.170)	(0.023)	(0.027)	(0.005)	(0.009)		(0.081)	(0.031)	(0.001)	
				1987 to 1991 (261 weeks, 2,471 stocks)					
−1.662	0.155	0.791	0.038	0.078	0.066	−0.138	0.131	0.003	31.9
(0.223)	(0.027)	(0.034)	(0.005)	(0.013)	(0.024)	(0.097)	(0.041)	(0.001)	
−0.313	0.153	0.831	0.035	—	0.158	−0.128	0.252	0.003	30.9
(0.052)	(0.027)	(0.033)	(0.005)		(0.019)	(0.098)	(0.036)	(0.001)	
−1.968	0.171	0.795	0.031	0.100	—	−0.122	0.119	0.003	[illegible]
(0.195)	(0.026)	(0.034)	(0.005)	(0.010)		(0.097)	(0.041)	(0.001)	
				1992 to 1996 (261 weeks, 2,520 stocks)					
−1.004	−0.087	0.689	0.077	0.040	0.262	−0.644	[illegible]	0.000	29.6
(0.278)	(0.034)	(0.033)	(0.007)	(0.016)	[illegible]	[illegible]	(0.049)	(0.001)	
−0.310	−0.095	0.708	0.076	—	[illegible]	−0.641	[illegible]	−0.001	29.4
(0.061)	(0.034)	(0.032)	[illegible]		(0.026)	[illegible]	[illegible]	(0.001)	
−2.025	−0.025	0.711	[illegible]	0.115	—	[illegible]	−0.005	0.000	27.8
(0.249)	[illegible]	[illegible]	(0.006)	[illegible]		(0.166)	(0.049)	(0.001)	

Table 12b: Cross-sectional regressions of median weekly turnover of NYSE and AMEX ordinary common shares (CRSP share codes 10 and 11, excluding 37 stocks containing Z-errors in reported volume) for five-year subperiods of the sample period from July 1982 to December 1996. The explanatory variables are: $\hat{\alpha}_{r,j}$, $\hat{\beta}_{r,j}$, and $\hat{\sigma}_{\epsilon,r,j}$ (the intercept, slope, and residual, respectively, from the time-series regression of an individual security's return on the market return); v_j (natural logarithm of market capitalization), p_j (natural logarithm of price); d_j (dividend yield); SP500$_j$ (S&P 500 indicator variable); and $\hat{\gamma}_{r,j}(1)$ (first-order return autocovariance).

responsible for another 5% to 20%. And in one case—in-sample sorting on betas relative to the equal-weighted index during 1987–1991—the third principal component accounts for an additional 10%. These figures suggest that the trend in turnover is unlikely to be the source of the dominant first principal component.

In summary, the results of Tables 13 and 14 indicate that a one-factor model for turnover is a reasonable approximation, at least in the case of turnover-beta-sorted portfolios, and that a two-factor model captures well over 90% of the time-series variation in turnover. This lends some support to the practice of estimating "abnormal" volume by using an event-study style "market model", e.g., Bamber (1986), Jain and Joh (1988), Lakonishok and Smidt (1986), Morse (1980), Richardson, Sefcik, Thompson (1986), Stickel and Verrecchia (1994), and Tkac (1996).

As compelling as these empirical results are, several qualifications should be kept in mind. First, we have provided little statistical inference for our principal components decomposition. In particular, the asymptotic standard errors reported in Tables 13 and 14 were computed under the assumption of IID Gaussian data, hardly appropriate for weekly US stock returns and even less convincing for turnover (see Muirhead (1982, Chapter 9) for further details). Perhaps nonparametric methods such as the moving-block bootstrap can provide better indications of the statistical significance of our estimated eigenvalues. Monte Carlo simulations should also be conducted to check the finite-sample properties of our estimators.

More importantly, the economic interpretation of the first two principal components or, alternatively, identifying the specific factors is a challenging issue that principal components cannot resolve. More structure must be imposed on the data—in particular, an intertemporal model of trading—to obtain a better understanding for the sources of turnover variation, and we present such structure in the next section.

7 Volume Implications of Intertemporal Asset-Pricing Models

In this section, we analyze the volume implications of intertemporal asset pricing models and how volume is related to returns. We first develop an intertemporal equilibrium model of stock trading and pricing with multiple assets and heterogeneous investors. We derive the behavior of volume and returns. We show that both volume and returns are driven by the

Period	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\theta}_4$	$\hat{\theta}_5$	$\hat{\theta}_6$	$\hat{\theta}_7$	$\hat{\theta}_8$	$\hat{\theta}_9$	$\hat{\theta}_{10}$
Out-of-Sample Turnover-Beta-Sorted Turnover-Differences Portfolios (τ^{VW})										
1967 to 1971	82.6	7.1	5.1	2.0	1.6	0.8	0.5	0.1	0.1	0.1
	(7.2)	(0.6)	(0.5)	(0.2)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)
1972 to 1976	81.2	6.8	4.7	2.8	2.0	1.0	0.9	0.4	0.2	0.1
	(7.1)	(0.6)	(0.4)	(0.2)	(0.2)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)
1977 to 1981	85.2	4.5	2.9	2.6	1.6	1.2	0.8	0.5	0.5	0.2
	(7.5)	(0.4)	(0.3)	(0.2)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)
1982 to 1986	81.3	5.1	3.5	2.7	2.2	1.7	1.3	0.9	0.7	0.6
	(7.1)	(0.4)	(0.3)	(0.2)	(0.2)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)
1987 to 1991	73.1	10.9	4.1	3.0	2.2	1.7	1.6	1.4	1.1	0.9
	(6.4)	(1.0)	(0.4)	(0.3)	(0.2)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)
1992 to 1996	78.4	8.6	4.0	2.8	2.1	1.2	1.0	0.9	0.6	0.4
	(6.9)	(0.8)	(0.4)	(0.2)	(0.2)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)
Out-of-Sample Turnover-Beta-Sorted Turnover-Differences Portfolios (τ^{EW})										
1967 to 1971	82.2	8.0	4.5	2.3	1.4	0.7	0.4	0.3	0.1	0.0
	(7.2)	(0.7)	(0.4)	(0.2)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)
1972 to 1976	79.3	7.5	4.8	4.0	1.9	1.3	0.6	0.4	0.2	0.1
	(7.0)	(0.7)	(0.4)	(0.4)	(0.2)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)
1977 to 1981	80.3	5.3	4.8	3.8	2.0	1.4	1.2	0.7	0.5	0.2
	(7.0)	(0.5)	(0.4)	[illegible]	(0.2)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)
1982 to 1986	82.6	5.0	3.0	2.6	2.0	1.7	1.1	0.9	0.7	0.4
	(7.3)	(0.4)	(0.3)	(0.2)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)	(0.0)
1987 to 1991	77.2	5.5	4.3	2.7	2.5	2.3	1.8	1.6	1.2	1.0
	(6.8)	(0.5)	(0.4)	(0.2)	(0.2)	(0.2)	(0.2)	(0.1)	(0.1)	(0.1)
1992 to 1996	80.4	6.4	4.6	2.6	1.7	1.4	1.1	0.7	0.5	0.4
	(7.1)	(0.6)	(0.4)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)

Table 14: Eigenvalues $\hat{\theta}_i$, $i = 1, \ldots, 10$ of the covariance matrix of the first-differences of the weekly turnover of ten out-of-sample-beta-sorted portfolios of NYSE and AMEX ordinary common shares (CRSP share codes 10 and 11, excluding 37 stocks containing Z-errors in reported volume)—in percentages (where the eigenvalues are normalized to sum to 100%)—for subperiods of the sample period from July 1962 to December 1996. Turnover betas are calculated in two ways: with respect to a value-weighted turnover index (τ^{VW}) and an equal-weighted turnover index (τ^{EW}). Standard errors for the normalized eigenvalues are given in parentheses and are calculated under the assumption of IID normality.

underlying risks of the economy. The results presented here are from Lo and Wang (2001b).

7.1 An Intertemporal Capital Asset-Pricing Model

Since our purpose is to draw qualitative implications on the joint behavior of return and volume, the model is kept as parsimonious as possible. Several generalizations of the model are discussed in Lo and Wang (2001b).

The Economy

We consider an economy defined on a set of discrete dates: $t = 0, 1, 2, \ldots$. There are J risky stocks, each pays a stream of dividends over time. As before, D_{jt} denote the dividend of stock j at date t, $j = 1, \cdots, J$, and $D_t \equiv [D_{1t} \cdots D_{Jt}]$ denote the column vector of dividends. Without loss of generality, in this section we assume that the total number of shares outstanding is one for each stock.

A stock portfolio can be expressed in terms of its shares of each stock, denoted by $S \equiv [S_1 \ldots S_J]$, where S_j is the number of stock j shares in the portfolio ($j = 1, \ldots, J$). A portfolio of particular importance is the market portfolio, denoted by S^M, which is given by

$$S^M = \iota \tag{20}$$

where ι is a vector of 1's with rank J. $D_{Mt} \equiv \iota^\top D_t$ gives the dividend of the market portfolio, which is the aggregate dividend.

In addition to the stocks, there is also a risk-free bond that yields a constant, positive interest r per time period.

There are I investors in the economy. Each investor is endowed with equal shares of the stocks and no bond. Every period, investor i, $i = 1, \ldots, I$, maximizes his expected utility of the following form:

$$\mathrm{E}_t \left[-e^{-W^i_{t+1} - (\lambda_X X_t + \lambda_Y Y^i_t) D_{Mt+1} - \lambda_Z (1+Z^i_t) X_{t+1}} \right] \tag{21}$$

where W^i_{t+1} is investor i's wealth next period, X_t, Y^i_t, Z^i_t are three one-dimensional state variables, and λ_X, λ_Y, λ_Z are non-negative constants. Apparently, the utility function in

(21) is state-dependent. We further assume

$$\sum_{i=1}^{I} Y_t^i = \sum_{i=1}^{I} Z_t^i = 0 \tag{22}$$

where $t = 0, 1, \ldots$.

For simplicity, we assume that all the exogenous shocks, D_t, X_t, $\{Y_t^i, Z_t^i, i = 1, \ldots, I\}$, are IID over time with zero means. For tractability, we further assume that D_{t+1} and X_{t+1} are jointly normally distributed:

$$u_{t+1} \equiv \begin{pmatrix} D_{t+1} \\ X_{t+1} \end{pmatrix} \overset{d}{\sim} N(\cdot, \sigma) \quad \text{where} \quad \sigma = \begin{pmatrix} \sigma_{DD} & \sigma_{DX} \\ \sigma_{XD} & \sigma_{XX} \end{pmatrix}. \tag{23}$$

Without loss of generality, σ_{DD} is assumed to be positive definite.

Our model has several features that might seem unusual. One feature of the model is that investors are assumed to have a myopic, but state-dependent utility function in (21). The purpose for using this utility function is to capture the dynamic nature of the investment problem without explicitly solving a dynamic optimization problem. The state dependence of the utility function is assumed to have the following properties. The marginal utility of wealth depends on the dividend of the market portfolio (the aggregate dividend), as reflected in the second term in the exponential of the utility function. When the aggregate dividend goes up, the marginal utility of wealth goes down. The marginal utility of wealth also depends on future state variables, in particular X_{t+1}, as reflected in the third term in the exponential of the utility function. This utility function can be interpreted as the equivalent of a value function from an appropriately specified dynamic optimization problem (see, for example, Wang (1994) and Lo and Wang (2001a)). More discussion is given in Lo and Wang (2001b) on this point.

Another feature of the model is the IID assumption for the state variables. This might leave the impression that the model is effectively static. This impression, however, is false since the state-dependence of investors' utility function introduces important dynamics over time. We can allow richer dynamics for the state variables without changing the main properties of the model.

The particular form of the utility function and the normality of distribution for the state variables are assumed for tractability. These assumptions are restrictive. But we hope with

p, the $\bar{R}^2$ should be less than that of (50).

It is impractical to compare (50) to all possible portfolios, and uninformative to compare it to random portfolios. Instead, we need only make comparisons to "optimal forecast portfolios", portfolios that are optimal forecasters of R_{Mt}, since by construction, no other portfolios can have higher levels of predictability than these. The following proposition shows how to construct optimal forecasting portfolios (OFPs) (see Lo and Wang, 2001 for details):

Proposition 7 *Let Γ_0 and Γ_1 denote the contemporaneous and first-order autocovariance matrix of the vector of all returns. For any arbitrary target portfolio q with weights $w_q = (w_{q1}; \ldots; w_{qN})$, define $A \equiv {\Gamma_0}^{-1}\Gamma_1 w_q {w_q}' {\Gamma_1}'$. The optimal forecast portfolio of w_q is given by the normalized eigenvector of A corresponding to its largest eigenvalue.*

Since Γ_0 and Γ_1 are unobservable, they must be estimated using historical data. Given the large number of stocks in our sample (over 2,000 in each subperiod) and the relatively short time series in each subperiod (261 weekly observations), the standard estimators for Γ_0 and Γ_1 are not viable. However, it is possible to construct OFPs from a much smaller number of "basis portfolios", and then compare the predictive power of these OFPs to the hedging portfolio. As long as the basis portfolios are not too specialized, the $\bar{R}^2$s are likely to be similar to those obtained from the entire universe of all stocks.

We form several sets of basis portfolios by sorting all the J stocks into K groups of equal numbers ($K \leq J$) according to: market capitalization, market beta, and SIC codes, and then construct value-weighted portfolios within each group.[38] This procedure yields K basis portfolios for which the corresponding Γ_0 and Γ_1 can be estimated using the portfolios' weekly returns within each subperiod. Based on the estimated autocovariance matrices, the OFP can be computed easily according to Proposition 7.

In selecting the number of basis portfolios K, we face the following trade-off: fewer portfolios yield better sampling properties for the covariance matrix estimators, but less desirable properties for the OFP since the predictive power of the OFP is obviously maximized when when $K = J$. As a compromise, for the OFPs based market capitalization and market betas, we choose K to be 10, 15, 20, and 25. For the OFP based on SIC codes, we choose 13 industry groupings, described in more detail below.

[38] It is important that we use value-weighted portfolios here so that the market portfolio, whose return we wish to predict, is a portfolio of these basic portfolios (recall that the target portfolio ω_q that we wish to forecast is a linear combination of the vector of returns for which Γ_k is the k-th order autocovariance matrix).

Parameter	Beta10	Beta15	Beta20	Beta25	Cap10	Cap15	Cap20	Cap25	SIC13	R_H	Q_H	$\log(\text{Cap}^{-1})$	VW	EW	TBill
							January 1967 to December 1971 (261 Weeks)								
Intercept	0.002	0.002	0.001	0.002	0.001	0.002	0.002	0.002	0.001	0.001	0.172	0.746	0.001	0.001	—
t-Stat	1.330	1.360	1.150	1.430	1.240	1.520	[illegible]	1.380	0.920	1.270	1.200	2.330	1.240	1.250	—
Slope	0.103	−0.034	−0.153	0.171	−0.262	0.173	−0.039	−0.176	−0.208	0.138	0.154	0.027	0.191	0.092	—
t-Stat	1.810	−0.550	−1.890	1.780	[illegible]	1.079	−0.240	−1.070	−2.860	3.460	3.900	2.330	3.130	2.080	—
$\overline{R}^2$	0.013	0.001	[illegible]	[illegible]	0.014	0.005	0.000	0.005	0.031	0.045	0.056	0.021	0.037	0.016	—
							January 1972 to December 1976 (261 Weeks)								
Intercept	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.103	0.389	0.001	0.001	—
t-Stat	0.650	0.640	0.560	0.670	0.830	0.640	0.730	0.630	0.630	0.820	0.760	1.410	0.700	0.640	—
Slope	0.023	0.204	−0.315	0.079	0.235	0.098	−0.169	0.069	0.040	−0.054	−0.023	0.014	−0.003	0.048	—
t-Stat	0.120	[illegible]	−2.630	0.580	1.660	0.660	−1.180	0.430	0.430	−1.430	−1.900	1.410	−0.060	0.910	—
$\overline{R}^2$	0.000	0.005	0.026	0.001	0.011	0.002	0.005	0.001	0.001	0.008	0.014	0.008	0.000	0.003	—
							January 1977 to December 1981 (261 Weeks)								
Intercept	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.223	0.151	0.002	0.002	—
t-Stat	1.750	1.600	1.800	1.640	1.770	1.760	1.800	1.530	1.749	1.500	1.370	0.720	1.570	1.380	—
Slope	0.007	0.071	0.065	0.033	0.075	0.003	−0.204	−0.186	0.150	0.049	0.013	0.005	0.069	0.080	—
t-Stat	0.040	0.870	0.460	0.510	0.230	0.010	−0.850	−0.990	1.130	1.810	1.760	0.710	1.110	1.370	—
$\overline{R}^2$	0.000	0.003	0.001	0.001	0.000	0.000	0.003	0.004	0.005	0.013	0.012	0.002	0.005	0.007	—

Table 18a: Forecast of weekly market-portfolio returns by lagged weekkly returns of the beta-sorted optimal forecast portfolios (OFPs), the market-capitalization-sorted OFP's, the SIC-sorted OFP, the return and dollar return on the hedging portfolio, minus log-market-capitalization, the lagged returns on the CRSP value- and equal-weighted portfolios, and lagged constant-maturity (three-month) Treasury bill rates from 1962 to 1981 in five-year subperiods. The value of ϕ is 1.25 for the return R_H and 1.5 for the dollar return Q_H on the hedging portfolio, respectively.

Model	Statistic	$\hat{\gamma}_{0t}$	$\hat{\gamma}_{1t}$	$\hat{\gamma}_{2t}$	$\overline{R}^2$ (%)
January 1972 to December 1976 (261 Weeks)					
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \epsilon_{pt}$	Mean:	0.002	0.000		10.0
	S.D.:	0.015	0.021		10.9
	t-Stat:	1.639	0.348		
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \gamma_{2t}\widehat{\beta}_p^{HR} + \epsilon_{pt}$	Mean:	0.004	−0.002	−0.002	14.3
$(\phi = 1.25)$	S.D.:	0.035	0.035	0.037	10.9
	t-Stat:	2.040	−1.047	−0.820	
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \gamma_{2t}\widehat{\beta}_p^{HQ} + \epsilon_{pt}$	Mean:	0.004	−0.002	−0.104	15.5
$(\phi = 1.50)$	S.D.:	0.032	0.034	3.797	10.9
	t-Stat:	2.162	−1.081	−0.442	
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \gamma_{2t}\widehat{\beta}_p^{SMB} + \epsilon_{pt}$	Mean:	0.001	0.000	0.063	12.1
	S.D.:	0.014	0.024	1.142	10.8
	t-Stat:	1.424	0.217	0.898	
January 1977 to December 1981 (261 Weeks)					
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \epsilon_{pt}$	Mean:	0.001	0.003		11.7
	S.D.:	0.011	0.022		[illegible]
	t-Stat:	1.166	2.566		
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \gamma_{2t}\widehat{\beta}_p^{HR} + \epsilon_{pt}$	Mean:	0.003	−0.001	−0.012	13.1
$(\phi = 4.75)$	S.D.:	[illegible]	0.020	0.051	12.4
	t-Stat:	3.748	−0.902	−3.712	
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \gamma_{2t}\widehat{\beta}_p^{HQ} + \epsilon_{pt}$	Mean:	0.003	−0.001	−1.564	12.5
$(\phi = 4.25)$	S.D.:	0.013	0.020	6.104	12.2
	t-Stat:	3.910	−0.754	−4.140	
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \gamma_{2t}\widehat{\beta}_p^{SMB} + \epsilon_{pt}$	Mean:	0.001	0.000	0.299	14.9
	S.D.:	0.011	0.017	1.088	13.4
	t-Stat:	2.251	−0.164	4.433	
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \gamma_{2t}\widehat{\beta}_p^{OFP} + \epsilon_{pt}$	Mean:	0.003	0.001	0.001	14.1
	S.D.:	0.018	0.023	0.036	11.6
	t-Stat:	2.735	0.843	0.632	

Table 20a: Cross-sectional regression tests of various linear factor models along the lines of Fama and MacBeth (1973) using weekly returns for NYSE and AMEX stocks from 1962 to 1996, five-year subperiods for the portfolio-formation, estimation, and testing periods, and 100 portfolios in the cross-sectional regressions each week. The five linear-factor models are: the standard CAPM ($\widehat{\beta}_p^M$), and four two-factor models in which the first factor is the market beta and the second factors are, respectively, the hedging portfolio return beta ($\widehat{\beta}_p^{HR}$), the hedging portfolio dollar-return beta ($\widehat{\beta}_p^{HQ}$), the beta of a small-minus-big cap portfolio return ($\widehat{\beta}_p^{SMB}$), and the beta of the optimal forecast portfolio based on a set of 25 market-beta-sorted basis portfolios ($\widehat{\beta}_p^{OFP}$).

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