Contents

1.3 Basic Identities

Real numbers: a, b, c

34. Additive Identity
$a + 0 = a$

35. Additive Inverse
$a + (-a) = 0$

36. Commutative of Addition
$a + b = b + a$

37. Associative of Addition
$(a + b) + c = a + (b + c)$

38. Definition of Subtraction
$a - b = a + (-b)$

39. Multiplicative Identity
$a \cdot 1 = a$

40. Multiplicative Inverse
$a \cdot \frac{1}{a} = 1,\ a \neq 0$

41. Multiplication Times 0
$a \cdot 0 = 0$

42. Commutative of Multiplication
$a \cdot b = b \cdot a$

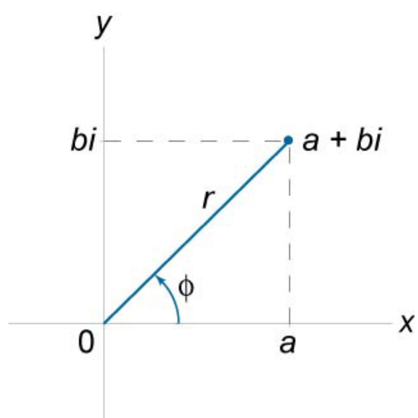

Figure 7.

55. Polar Presentation of Complex Numbers

$$a + bi = r(\cos\varphi + i\sin\varphi)$$

56. Modulus and Argument of a Complex Number

If $a + bi$ is a complex number, then

$r = \sqrt{a^2 + b^2}$ (modulus),

$\varphi = \arctan\frac{b}{a}$ (argument).

57. Product in Polar Representation

$$z_1 \cdot z_2 = r_1(\cos\varphi_1 + i\sin\varphi_1) \cdot r_2(\cos\varphi_2 + i\sin\varphi_2)$$
$$= r_1 r_2[\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2)]$$

58. Conjugate Numbers in Polar Representation

$$\overline{r(\cos\varphi + i\sin\varphi)} = r[\cos(-\varphi) + i\sin(-\varphi)]$$

59. Inverse of a Complex Number in Polar Representation

$$\frac{1}{r(\cos\varphi + i\sin\varphi)} = \frac{1}{r}[\cos(-\varphi) + i\sin(-\varphi)]$$

Chapter 2

Algebra

2.1 Factoring Formulas

Real numbers: a, b, c
Natural number: n

65. $a^2 - b^2 = (a+b)(a-b)$

66. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

67. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

68. $a^4 - b^4 = (a^2 - b^2)(a^2 + b^2) = (a-b)(a+b)(a^2 + b^2)$

69. $a^5 - b^5 = (a-b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$

70. $a^5 + b^5 = (a+b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)$

71. If n is odd, then
$a^n + b^n = (a+b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 - \ldots - ab^{n-2} + b^{n-1})$.

72. If n is even, then
$a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots + ab^{n-2} + b^{n-1})$,

$$a^n + b^n = (a+b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 - \ldots + ab^{n-2} - b^{n-1}).$$

2.2 Product Formulas

Real numbers: a, b, c
Whole numbers: n, k

73. $(a-b)^2 = a^2 - 2ab + b^2$

74. $(a+b)^2 = a^2 + 2ab + b^2$

75. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

76. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

77. $(a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$

78. $(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

79. Binomial Formula

$$(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n,$$

where ${}^nC_k = \dfrac{n!}{k!(n-k)!}$ are the binomial coefficients.

80. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$

81. $(a+b+c+\ldots+u+v)^2 = a^2 + b^2 + c^2 + \ldots + u^2 + v^2 +$
$+ 2(ab + ac + \ldots + au + av + bc + \ldots + bu + bv + \ldots + uv)$

2.6 Equations

Real numbers: a, b, c, p, q, u, v
Solutions: x_1, x_2, y_1, y_2, y_3

119. Linear Equation in One Variable

$$ax + b = 0, \; x = -\frac{b}{a}.$$

120. Quadratic Equation

$$ax^2 + bx + c = 0, \; x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

121. Discriminant

$$D = b^2 - 4ac$$

122. Viete's Formulas

If $x^2 + px + q = 0$, then

$$\begin{cases} x_1 + x_2 = -p \\ x_1 x_2 = q \end{cases}.$$

123. $ax^2 + bx = 0, \; x_1 = 0, \; x_2 = -\frac{b}{a}.$

124. $ax^2 + c = 0, \; x_{1,2} = \pm\sqrt{-\frac{c}{a}}.$

125. Cubic Equation. Cardano's Formula.

$$y^3 + py + q = 0,$$

185. Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos\alpha$,

$b^2 = a^2 + c^2 - 2ac\cos\beta$,

$c^2 = a^2 + b^2 - 2ab\cos\gamma$.

186. Law of Sines

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2R,$$

where R is the radius of the circumscribed circle.

187. $$R = \frac{a}{2\sin\alpha} = \frac{b}{2\sin\beta} = \frac{c}{2\sin\gamma} = \frac{bc}{2h_a} = \frac{ac}{2h_b} = \frac{ab}{2h_c} = \frac{abc}{4S}$$

188. $$r^2 = \frac{(p-a)(p-b)(p-c)}{p},$$

$$\frac{1}{r} = \frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c}$$

189. $$\sin\frac{\alpha}{2} = \sqrt{\frac{(p-b)(p-c)}{bc}},$$

$$\cos\frac{\alpha}{2} = \sqrt{\frac{p(p-a)}{bc}},$$

$$\tan\frac{\alpha}{2} = \sqrt{\frac{(p-b)(p-c)}{p(p-a)}}.$$

190. $$h_a = \frac{2}{a}\sqrt{p(p-a)(p-b)(p-c)},$$

$$h_b = \frac{2}{b}\sqrt{p(p-a)(p-b)(p-c)},$$

$$h_c = \frac{2}{c}\sqrt{p(p-a)(p-b)(p-c)}.$$

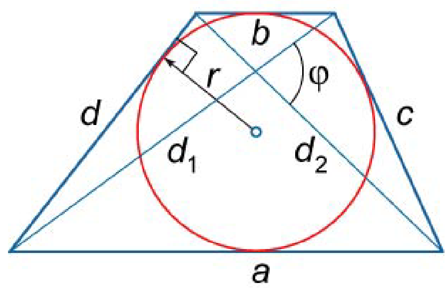

Figure 23.

230. $a + b = c + d$

231. $q = \frac{a+b}{2} = \frac{c+d}{2}$

232. $L = 2(a+b) = 2(c+d)$

233. $S = \frac{a+b}{2} \cdot h = \frac{c+d}{2} \cdot h = qh$

$S = \frac{1}{2} d_1 d_2 \sin\varphi$.

3.13 Kite

Sides of a kite: a, b
Diagonals: d_1, d_2
Angles: α, β, γ
Perimeter: L
Area: S

3.15 Tangential Quadrilateral

Sides of a quadrilateral: a, b, c, d
Diagonals: d_1, d_2
Angle between the diagonals: φ
Radius of inscribed circle: r
Perimeter: L
Semiperimeter: p
Area: S

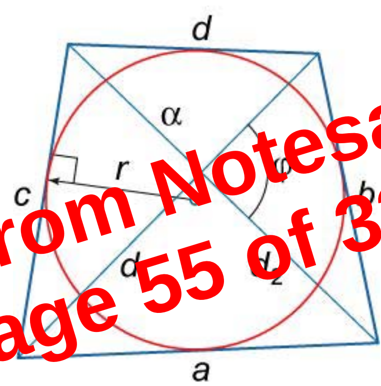

Figure 26.

242. $a + c = b + d$

243. $L = a + b + c + d = 2(a + c) = 2(b + d)$

244. $r = \dfrac{\sqrt{d_1^2 d_2^2 - (a - b)^2 (a + b - p)^2}}{2p}$,

where $p = \dfrac{L}{2}$.

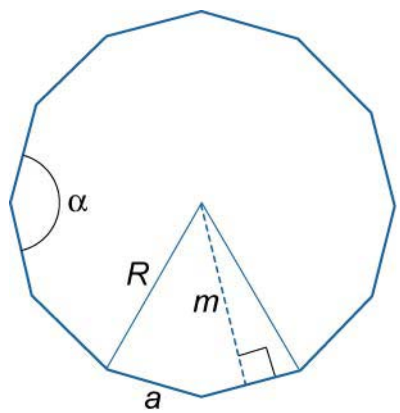

Figure 29.

254. $\alpha = \frac{n-2}{2} \cdot 180^\circ$

255. $\alpha = \frac{n-2}{2} \cdot 180^\circ$

256. $R = \frac{a}{2\sin\frac{\pi}{n}}$

257. $r = m = \frac{a}{2\tan\frac{\pi}{n}} = \sqrt{R^2 - \frac{a^2}{4}}$

258. $L = na$

259. $S = \frac{nR^2}{2}\sin\frac{2\pi}{n}$,

$S = pr = p\sqrt{R^2 - \frac{a^2}{4}}$,

274. $S = \frac{1}{2}\left[sR - a(R-h)\right] = \frac{R^2}{2}\left(\frac{\alpha\pi}{180°} - \sin\alpha\right) = \frac{R^2}{2}(x - \sin x)$,

$S \approx \frac{2}{3}ha$.

3.22 Cube

Edge: a
Diagonal: d
Radius of inscribed sphere: r
Radius of circumscribed sphere: r
Surface area: S
Volume: V

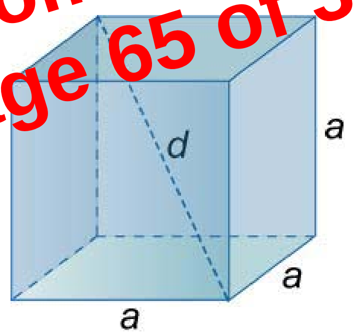

Figure 37.

275. $d = a\sqrt{3}$

276. $r = \frac{a}{2}$

286. $V = S_B h$

287. Cavalieri's Principle
Given two solids included between parallel planes. If every plane cross section parallel to the given planes has the same area in both solids, then the volumes of the solids are equal.

3.25 Regular Tetrahedron

Triangle side length: a
Height: h
Area of base: S_B
Surface area: S
Volume: V

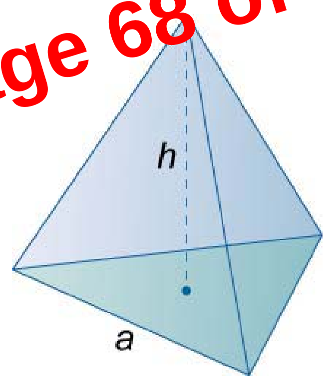

Figure 40.

288. $h = \sqrt{\frac{2}{3}}\, a$

3.27 Frustum of a Regular Pyramid

Base and top side lengths: $\begin{cases} a_1, a_2, a_3, \ldots, a_n \\ b_1, b_2, b_3, \ldots, b_n \end{cases}$

Height: h
Slant height: m
Area of bases: S_1, S_2
Lateral surface area: S_L
Perimeter of bases: P_1, P_2
Scale factor: k
Total surface area: S
Volume: V

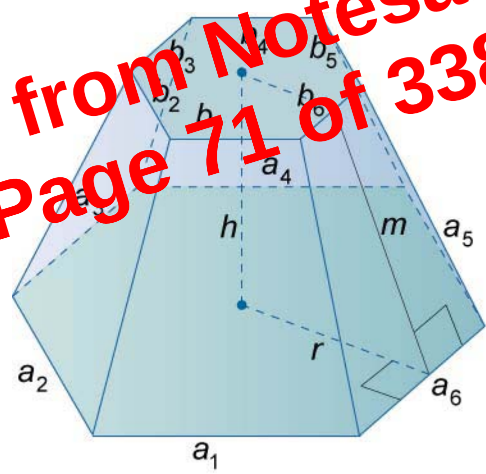

Figure 42.

298. $\frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3} = \ldots = \frac{b_n}{a_n} = \frac{b}{a} = k$

299. $\frac{S_2}{S_1} = k^2$

300. $S_L = \frac{m(P_1 + P_2)}{2}$

301. $S = S_L + S_1 + S_2$

302. $V = \frac{h}{3}\left(S_1 + \sqrt{S_1 S_2} + S_2\right)$

303. $V = \frac{hS_1}{3}\left[1 + \frac{b}{a} + \left(\frac{b}{a}\right)^2\right] = \frac{hS_1}{3}\left[1 + k + k^2\right]$

3.28 Rectangular Right Wedge

Sides of base: a, b
Top edge: c
Height: h
Lateral surface area: S_L
Area of base: S_B
Total surface area: S
Volume: V

326. $S = S_L + S_B = \pi R\left[h_1 + h_2 + R + \sqrt{R^2 + \left(\frac{h_1 - h_2}{2}\right)^2}\right]$

327. $V = \frac{\pi R^2}{2}(h_1 + h_2)$

3.32 Right Circular Cone

Radius of base: R
Diameter of base: d
Height: H
Slant height: m
Lateral surface area: S_L
Area of base: S_B
Total surface area: S
Volume: V

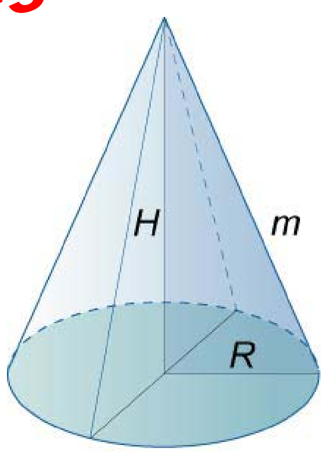

Figure 49.

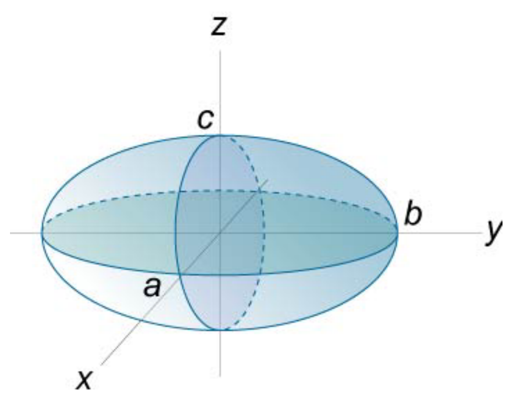

Figure 56.

355. $V = \frac{4}{3}\pi abc$

Prolate Spheroid

Semi-axes: a, b, b $(a > b)$
Surface area: S
Volume: V

356. $S = 2\pi b\left(b + \frac{a \arcsin e}{e}\right)$,

where $e = \frac{\sqrt{a^2 - b^2}}{a}$.

357. $V = \frac{4}{3}\pi b^2 a$

471. **Inverse Cosecant Function**

$$y = \operatorname{arccsc} x,\ \ x \in (-\infty, -1] \cup [1, \infty),\ \ \operatorname{arccsc} x \in \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right].$$

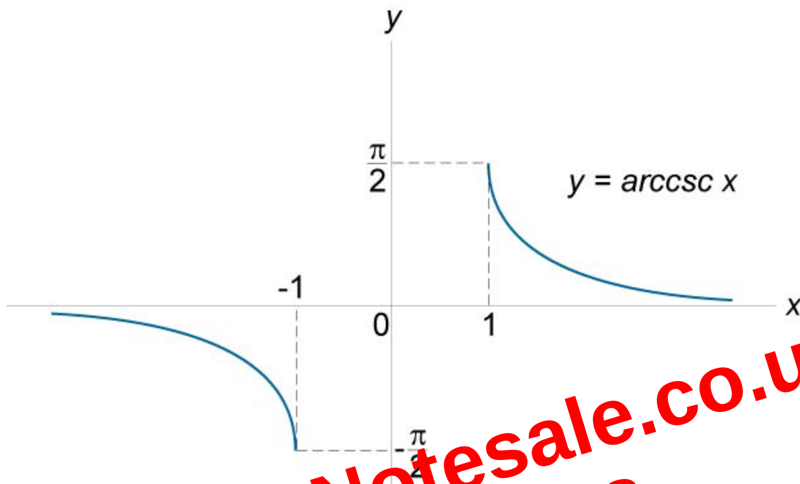

Figure 71.

4.18 Principal Values of Inverse Trigonometric Functions

472.

x	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
arcsin x	$0°$	$30°$	$45°$	$60°$	$90°$
arccos x	$90°$	$60°$	$45°$	$30°$	$0°$
x	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	
arcsin x	$-30°$	$-45°$	$-60°$	$-90°$	
arccos x	$120°$	$135°$	$150°$	$180°$	

482. $\arccos x = \frac{\pi}{2} - \arcsin x$

483. $\arccos x = \arcsin \sqrt{1 - x^2}$, $0 \le x \le 1$.

484. $\arccos x = \pi - \arcsin \sqrt{1 - x^2}$, $-1 \le x \le 0$.

485. $\arccos x = \arctan \frac{\sqrt{1 - x^2}}{x}$, $0 < x \le 1$.

486. $\arccos x = \pi + \arctan \frac{\sqrt{1 - x^2}}{x}$, $-1 \le x < 0$.

487. $\arccos x = \operatorname{arccot} \frac{x}{\sqrt{1 - x^2}}$, $-1 \le x \le 1$.

488. $\arctan(-x) = -\arctan x$

489. $\arctan x = \frac{\pi}{2} - \operatorname{arccot} x$

490. $\arctan x = \arcsin \frac{x}{\sqrt{1 + x^2}}$

491. $\arctan x = \arccos \frac{1}{\sqrt{1 + x^2}}$, $x \ge 0$.

492. $\arctan x = -\arccos \frac{1}{\sqrt{1 + x^2}}$, $x \le 0$.

529. Diagonal matrix is a square matrix with all elements zero except those on the leading diagonal.

530. Unit matrix is a diagonal matrix in which the elements on the leading diagonal are all unity. The unit matrix is denoted by I.

531. A null matrix is one whose elements are all zero.

5.4 Operations with Matrices

532. Two matrices A and B are equal if, and only if, they are both of the same shape $m \times n$ and corresponding elements are equal.

533. Two matrices A and B can be added (or subtracted) of, and only if, they have the same shape $m \times n$. If

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix},$$

$$B = [b_{ij}] = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \dots & b_{mn} \end{bmatrix},$$

then

$$AB = C = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1k} \\ c_{21} & c_{22} & \dots & c_{2k} \\ \vdots & \vdots & & \vdots \\ b_{m1} & c_{m2} & \dots & c_{mk} \end{bmatrix},$$

where

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \dots + a_{in}b_{nj} = \sum_{\lambda=1}^{n} a_{i\lambda}b_{\lambda j}$$

$(i = 1, 2, \dots, m; j = 1, 2, \dots, k)$.

Thus if

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}, \; B = [b_i] = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix},$$

then

$$AB = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} a_{11}b_1 & a_{12}b_2 & a_{13}b_3 \\ a_{21}b_1 & a_{22}b_2 & a_{23}b_3 \end{bmatrix}.$$

536. Transpose of a Matrix
If the rows and columns of a matrix are interchanged, then the new matrix is called the transpose of the original matrix. If A is the original matrix, its transpose is denoted A^T or $\tilde{A}$.

537. The matrix A is orthogonal if $AA^T = I$.

538. If the matrix product AB is defined, then
$(AB)^T = B^T A^T$.

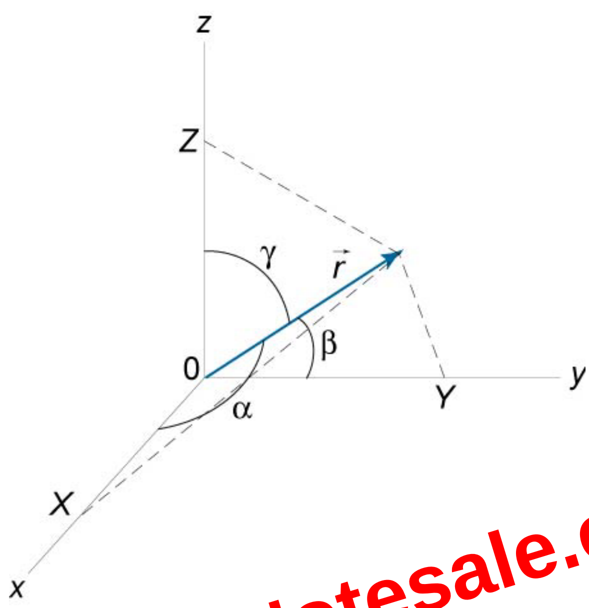

Figure 75.

555. If $\vec{r}(X, Y, Z) = \vec{r}_1(X_1, Y_1, Z_1)$, then
$X = X_1,\ Y = Y_1,\ Z = Z_1$.

6.2 Vector Addition

556. $\vec{w} = \vec{u} + \vec{v}$

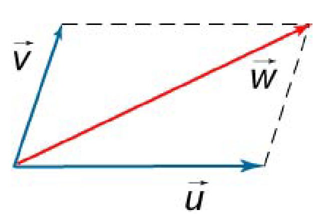

Figure 76.

609. Midpoint of a Line Segment

$$x_0 = \frac{x_1 + x_2}{2}, \ \lambda = 1.$$

7.2 Two-Dimensional Coordinate System

Point coordinates: x_0, x_1, x_2, y_0, y_1, y_2
Polar coordinates: r, φ
Real number: λ
Positive real numbers: a, b, c,
Distance between two points: d
Area: S

610. Distance Between Two Points

$$d = AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

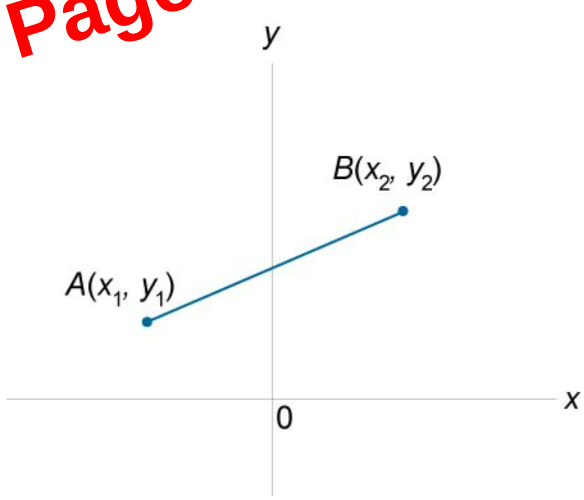

Figure 88.

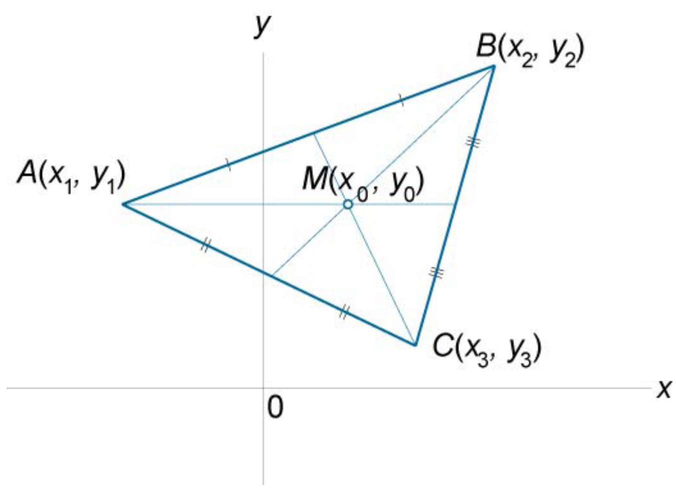

Figure 91.

614. Incenter (Intersection of Angle Bisectors) of a Triangle

$$x_0 = \frac{ax_1 + bx_2 + cx_3}{a + b + c}, \; y_0 = \frac{ay_1 + by_2 + cy_3}{a + b + c},$$

where $a = BC$, $b = CA$, $c = AB$.

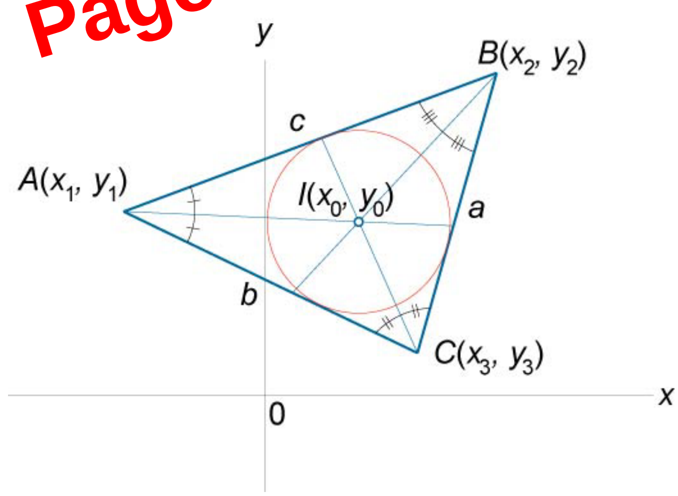

Figure 92.

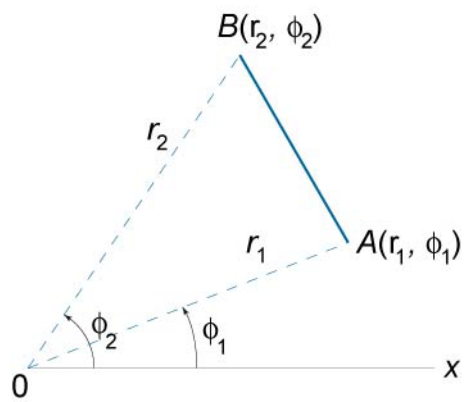

Figure 96.

620. Converting Rectangular Coordinates to Polar Coordinates

$x = r\cos\varphi,\ \ y = r\sin\varphi.$

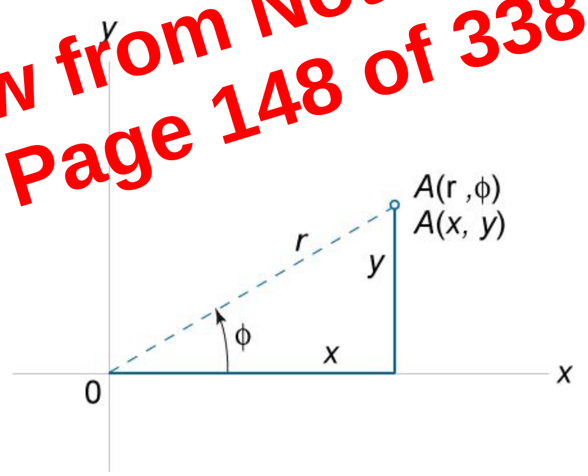

Figure 97.

621. Converting Polar Coordinates to Rectangular Coordinates

$r = \sqrt{x^2 + y^2},\ \ \tan\varphi = \frac{y}{x}.$

626. Equation of a Line Given a Point and the Gradient

$y = y_0 + k(x - x_0)$,

where k is the gradient, $P(x_0, y_0)$ is a point on the line.

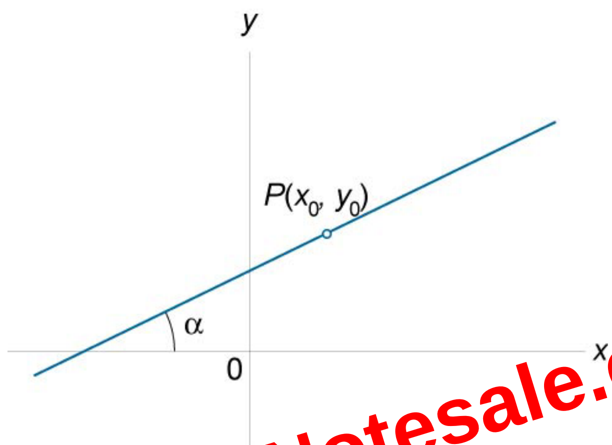

Figure 101.

627. Equation of a Line That Passes Through Two Points

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

or

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0.$$

651. General Form
$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$,
where $B^2 - 4AC < 0$.

652. General Form with Axes Parallel to the Coordinate Axes
$Ax^2 + Cy^2 + Dx + Ey + F = 0$,
where $AC > 0$.

653. Circumference
$L = 4aE(e)$,
where the function E is the complete elliptic integral of the second kind.

654. Approximate Formulas of the Circumference
$L = \pi\left(1.5(a+b) - \sqrt{ab}\right)$,
$L = \pi\sqrt{2(a^2 + b^2)}$.

655. $S = \pi ab$

7.6 Hyperbola

Transverse axis: a
Conjugate axis: b
Foci: $F_1(-c, 0)$, $F_2(c, 0)$
Distance between the foci: 2c
Eccentricity: e
Asymptotes: s, t
Real numbers: A, B, C, D, E, F, t, k

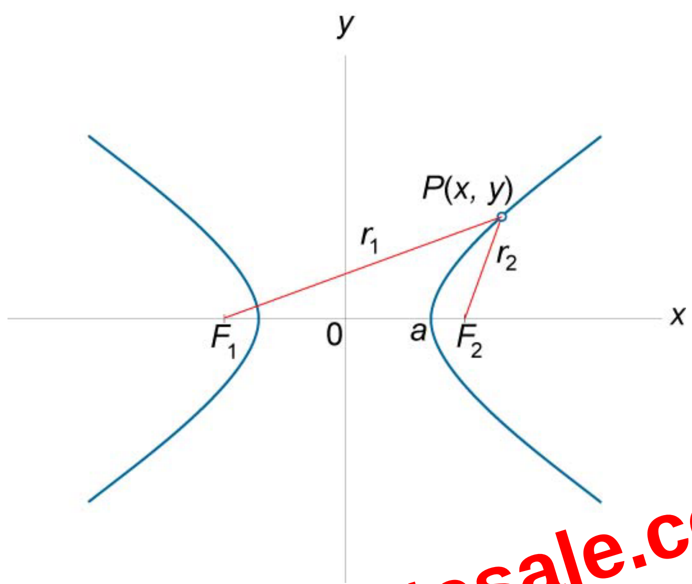

Figure 113.

658. Equations of Asymptotes

$$y = \pm \frac{b}{a} x$$

659. $c^2 = a^2 + b^2$

660. Eccentricity

$$e = \frac{c}{a} > 1$$

661. Equations of Directrices

$$x = \pm \frac{a}{e} = \pm \frac{a^2}{c}$$

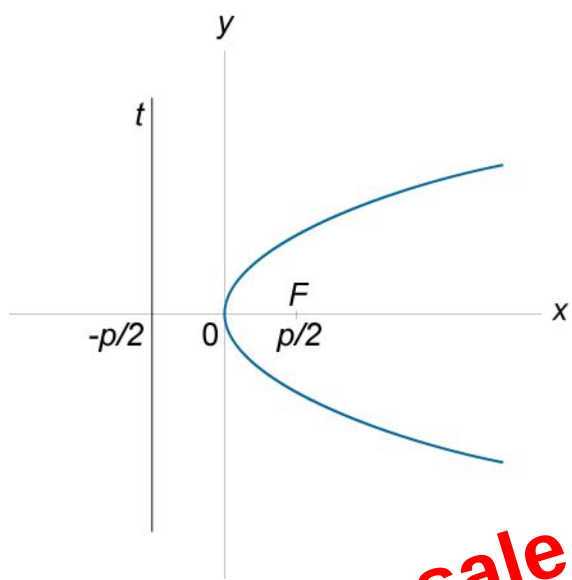

Figure 120.

Equation of the directrix

$x = -\frac{p}{2}$,

Coordinates of the focus

$F\left(\frac{p}{2}, 0\right)$,

Coordinates of the vertex

$M(0, 0)$.

667. General Form

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$,

where $B^2 - 4AC = 0$.

668. $y = ax^2$, $p = \frac{1}{2a}$.

Equation of the directrix

$F\left(x_0, y_0 + \frac{p}{2}\right),$

Coordinates of the vertex

$x_0 = -\frac{b}{2a},\ y_0 = ax_0^2 + bx_0 + c = \frac{4ac - b^2}{4a}.$

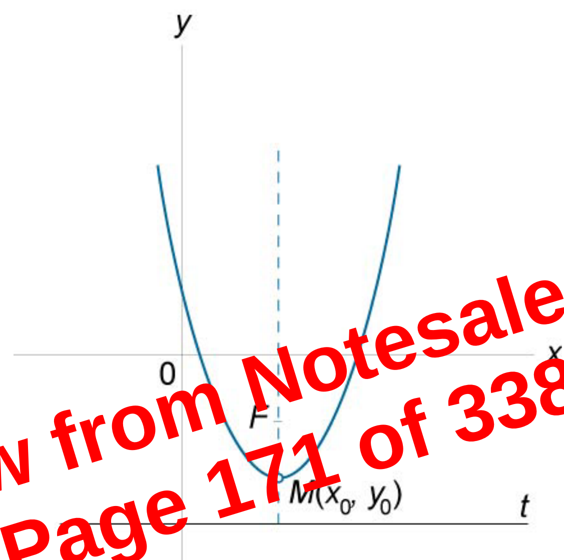

Figure 122.

7.8 Three-Dimensional Coordinate System

Point coordinates: $x_0, y_0, z_0, x_1, y_1, z_1, \ldots$

Real number: λ

Distance between two points: d

Area: S

Volume: V

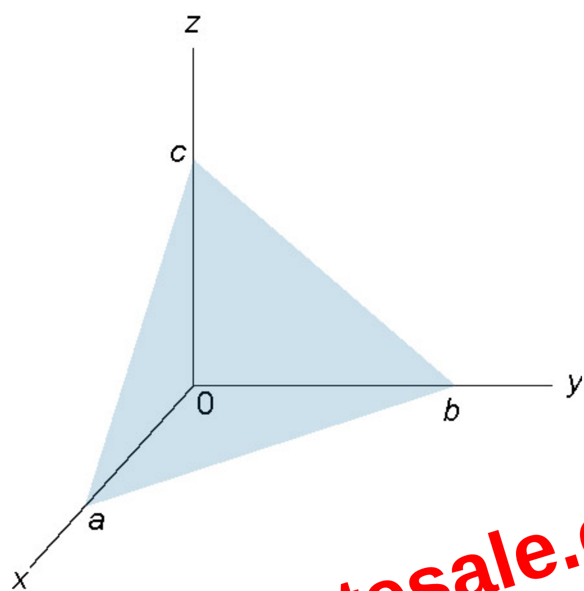

Figure 129.

680. Three Point Form

$$\begin{vmatrix} x - x_3 & y - y_3 & z - z_3 \\ x_1 - x_3 & y_1 - y_3 & z_1 - z_3 \\ x_2 - x_3 & y_2 - y_3 & z_2 - z_3 \end{vmatrix} = 0,$$

or

$$\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0.$$

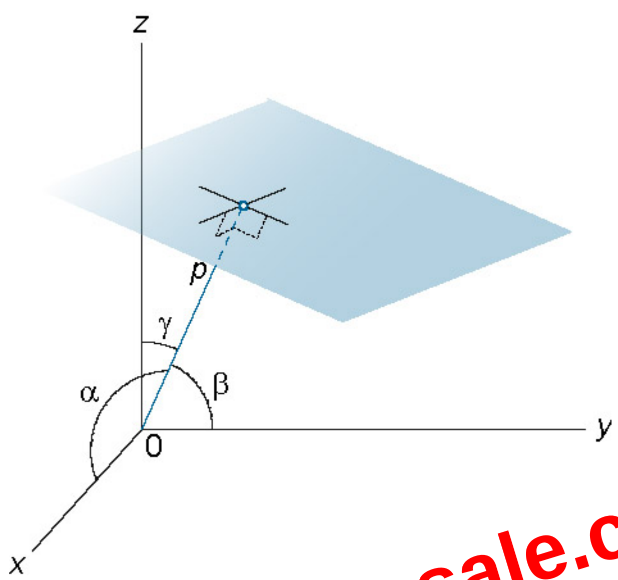

Figure 121.

682. Parametric Form

$$\begin{cases} x = x_1 + a_1 s + a_2 t, \\ y = y_1 + b_1 s + b_2 t, \\ z = z_1 + c_1 s + c_2 t \end{cases}$$

where (x, y, z) are the coordinates of any unknown point on the line , the point $P(x_1, y_1, z_1)$ lies in the plane, the vectors (a_1, b_1, c_1) and (a_2, b_2, c_2) are parallel to the plane.

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

687. Equation of a Plane Through $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$, and Parallel To the Vector (a, b, c)

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a & b & c \end{vmatrix} = 0$$

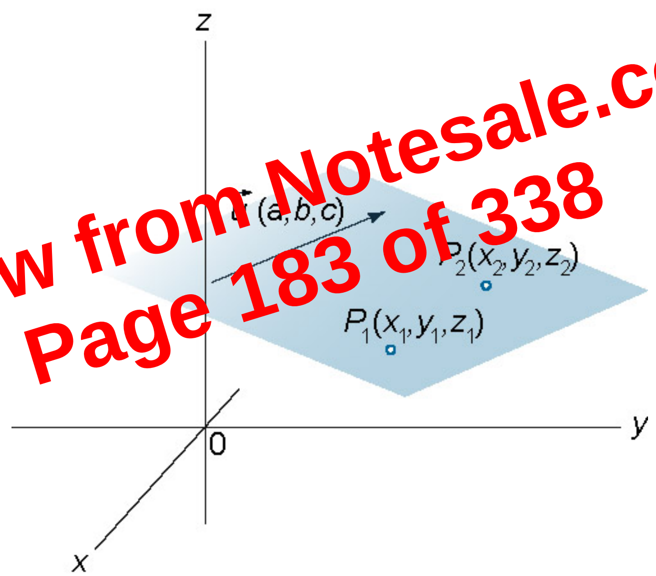

Figure 134.

688. Distance From a Point To a Plane
The distance from the point $P_1(x_1, y_1, z_1)$ to the plane $Ax + By + Cz + D = 0$ is

734. Square Root Function
$y = \sqrt{x}$, $x \in [0, \infty)$.

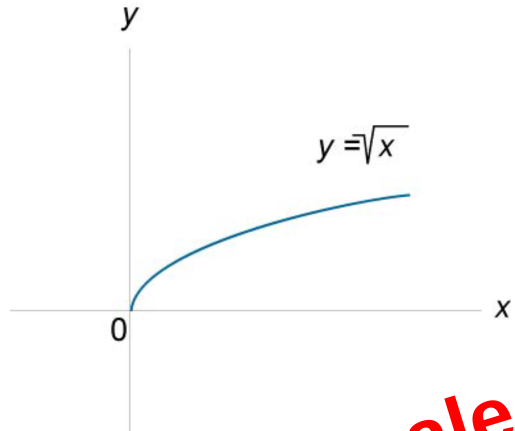

Figure 160.

735. Exponential Functions
$y = a^x$, $a > 0$, $a \neq 1$,
$y = e^x$ if $a = e$, $e = 2.71828182846\ldots$

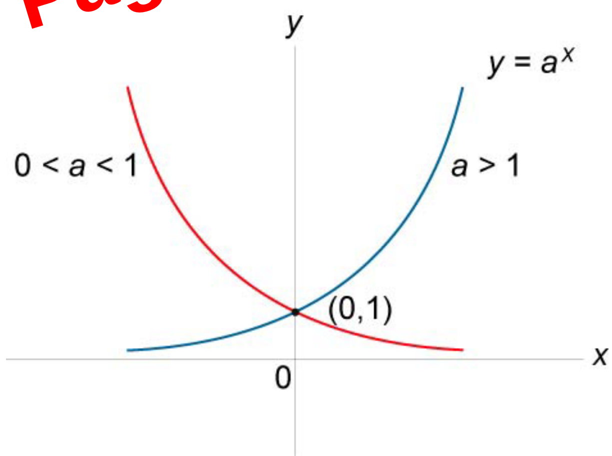

Figure 161.

742. Hyperbolic Cosecant Function

$y = \operatorname{csch} x$, $y = \operatorname{csch} x = \dfrac{1}{\sinh x} = \dfrac{2}{e^x - e^{-x}}$, $x \in R$, $x \neq 0$.

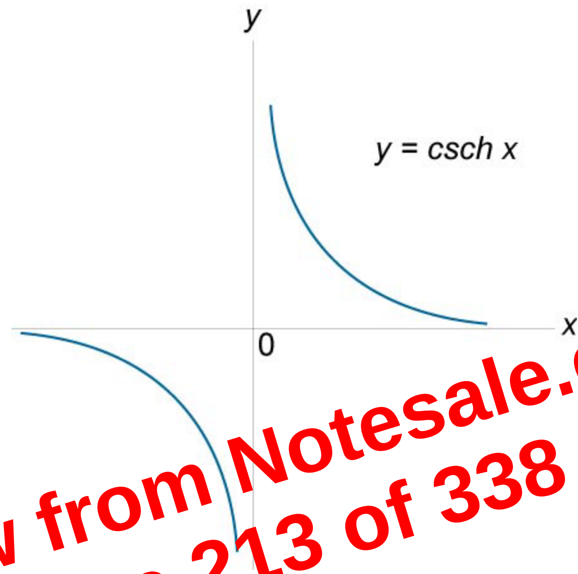

Figure 168.

743. Inverse Hyperbolic Sine Function

$y = \operatorname{arcsinh} x$, $x \in R$.

759. $\lim\limits_{x\to 0}\frac{\tan^{-1}x}{x}=1$

760. $\lim\limits_{x\to 0}\frac{\ln(1+x)}{x}=1$

761. $\lim\limits_{x\to \infty}\left(1+\frac{1}{x}\right)^{x}=e$

762. $\lim\limits_{x\to \infty}\left(1+\frac{k}{x}\right)^{x}=e^{k}$

763. $\lim\limits_{x\to 0}a^{x}=1$

8.3 Definition and Properties of the Derivative

Functions: f, g, y, u, v
Independent variable: x
Real constant: k
Angle: α

764. $y'(x)=\lim\limits_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}=\lim\limits_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\frac{dy}{dx}$

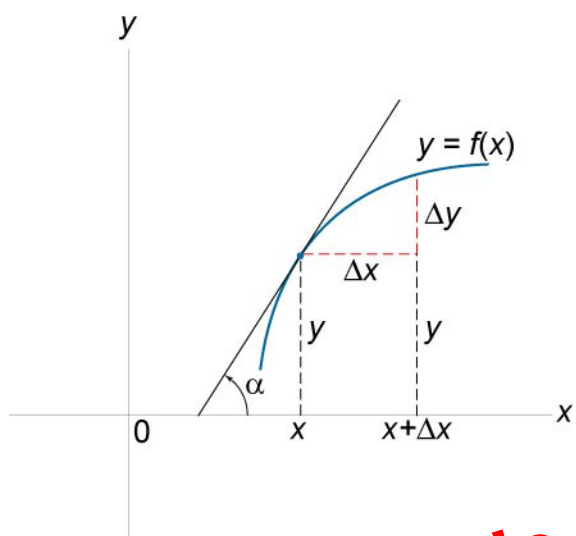

Figure 175

765. $\frac{dy}{dx} = \tan\alpha$

766. $\frac{d(u+v)}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

767. $\frac{d(u-v)}{dx} = \frac{du}{dx} - \frac{dv}{dx}$

768. $\frac{d(ku)}{dx} = k\frac{du}{dx}$

769. Product Rule

$$\frac{d(u \cdot v)}{dx} = \frac{du}{dx} \cdot v + u \cdot \frac{dv}{dx}$$

880. $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C,\ n \neq -1.$

881. $\int \frac{dx}{x} = \ln|x| + C$

882. $\int \frac{dx}{ax+b} = \frac{1}{a}\ln|ax+b| + C$

883. $\int \frac{ax+b}{cx+d}dx = \frac{a}{c}x + \frac{bc-ad}{c^2}\ln|cx+d| + C$

884. $\int \frac{dx}{(x+a)(x+b)} = \frac{1}{a-b}\ln\left|\frac{x+b}{x+a}\right| + C,\ a \neq b.$

885. $\int \frac{xdx}{a+bx} = \frac{1}{b^2}\left(a+bx - a\ln|a+bx|\right) + C$

886. $\int \frac{x^2dx}{a+bx} = \frac{1}{b^3}\left[\frac{1}{2}(a+bx)^2 - 2a(a+bx) + a^2\ln|a+bx|\right] + C$

887. $\int \frac{dx}{x(a+bx)} = \frac{1}{a}\ln\left|\frac{a+bx}{x}\right| + C$

888. $\int \frac{dx}{x^2(a+bx)} = -\frac{1}{ax} + \frac{b}{a^2}\ln\left|\frac{a+bx}{x}\right| + C$

889. $\int \frac{xdx}{(a+bx)^2} = \frac{1}{b^2}\left(\ln|a+bx| + \frac{a}{a+bx}\right) + C$

890. $\int \frac{x^2 dx}{(a+bx)^2} = \frac{1}{b^3}\left(a + bx - 2a\ln|a+bx| - \frac{a^2}{a+bx} \right) + C$

891. $\int \frac{dx}{x(a+bx)^2} = \frac{1}{a(a+bx)} + \frac{1}{a^2}\ln\left|\frac{a+bx}{x}\right| + C$

892. $\int \frac{dx}{x^2-1} = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C$

893. $\int \frac{dx}{1-x^2} = \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| + C$

894. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C$

895. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$

896. $\int \frac{dx}{1+x^2} = \tan^{-1} x + C$

897. $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

898. $\int \frac{x\,dx}{x^2+a^2} = \frac{1}{2}\ln\left(x^2+a^2\right) + C$

899. $\int \frac{dx}{a+bx^2} = \frac{1}{\sqrt{ab}}\arctan\left(x\sqrt{\frac{b}{a}} \right) + C,\ ab > 0.$

925. $\int x\sqrt{x^2+a^2}\,dx = \frac{1}{3}\left(x^2+a^2\right)^{3/2} + C$

926. $\int x^2\sqrt{x^2+a^2}\,dx = \frac{x}{8}\left(2x^2+a^2\right)\sqrt{x^2+a^2} -$

$-\frac{a^4}{8}\ln\left|x+\sqrt{x^2+a^2}\right| + C$

927. $\int \frac{\sqrt{x^2+a^2}}{x^2}\,dx = -\frac{\sqrt{x^2+a^2}}{x} + \ln\left|x+\sqrt{x^2+a^2}\right| + C$

928. $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left|x+\sqrt{x^2+a^2}\right| + C$

929. $\int \frac{\sqrt{x^2+a^2}}{x}\,dx = \sqrt{x^2+a^2} + a\ln\left|\frac{x}{a+\sqrt{x^2+a^2}}\right| + C$

930. $\int \frac{x\,dx}{\sqrt{x^2+a^2}} = \sqrt{x^2+a^2} + C$

931. $\int \frac{x^2\,dx}{\sqrt{x^2+a^2}} = \frac{x}{2}\sqrt{x^2+a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2+a^2}\right| + C$

932. $\int \frac{dx}{x\sqrt{x^2+a^2}} = \frac{1}{a}\ln\left|\frac{x}{a+\sqrt{x^2+a^2}}\right| + C$

933. $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + C$

934. $\int x\sqrt{x^2-a^2}\,dx = \frac{1}{3}\left(x^2-a^2\right)^{3/2} + C$

1090. Area of a Region

$$A = \int_a^b \int_{g(x)}^{f(x)} dydx \quad \text{(for a type I region)}.$$

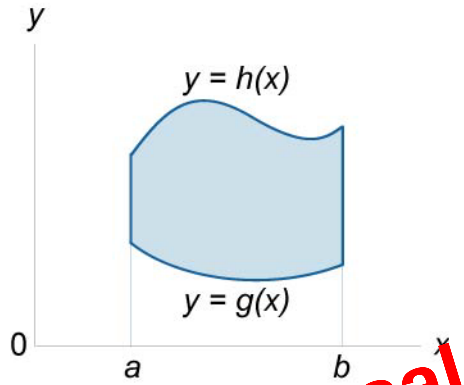

Figure 198.

$$A = \int_c^d \int_{p(y)}^{q(y)} dxdy \quad \text{(for a type II region)}.$$

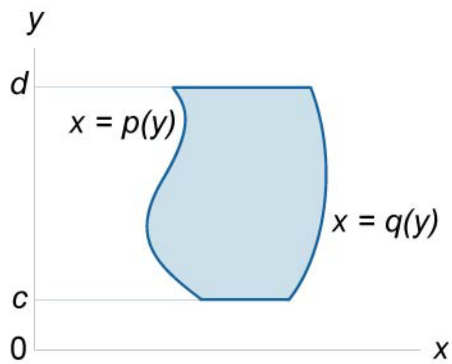

Figure 199.

1091. Volume of a Solid

$$V = \iint_R f(x,y)\,dA.$$

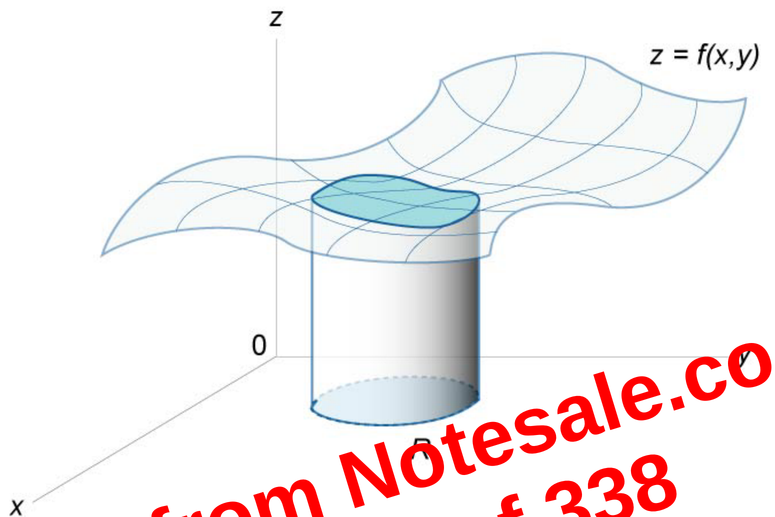

Figure 200.

If R is a type I region bounded by $x = a$, $x = b$, $y = h(x)$, $y = g(x)$, then

$$V = \iint_R f(x,y)\,dA = \int_a^b \int_{h(x)}^{g(x)} f(x,y)\,dy\,dx.$$

If R is a type II region bounded by $y = c$, $y = d$, $x = q(y)$, $x = p(y)$, then

$$V = \iint_R f(x,y)\,dA = \int_c^d \int_{p(y)}^{q(y)} f(x,y)\,dx\,dy.$$

$$\bar{y} = \frac{M_x}{m} = \frac{1}{m}\iint_R y\rho(x,y)dA = \frac{\iint_R y\rho(x,y)dA}{\iint_R \rho(x,y)dA}.$$

1097. Charge of a Plate

$$Q = \iint_R \sigma(x,y)dA,$$

where electrical charge is distributed over a region R and its charge density at a point (x,y) is $\sigma(x,y)$.

1098. Average of a Function

$$\mu = \frac{1}{S}\iint_R f(x,y)dA,$$

where $S = \iint_R dA$.

9.11 Triple Integral

Functions of three variables: $f(x,y,z)$, $g(x,y,z)$, ...

Triple integrals: $\iiint_G f(x,y,z)dV$, $\iiint_G g(x,y,z)dV$, ...

Riemann sum: $\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} f(u_i,v_j,w_k)\Delta x_i \Delta y_j \Delta z_k$

Small changes: Δx_i, Δy_j, Δz_k

Limits of integration: a, b, c, d, r, s

Regions of integration: G, T, S

Cylindrical coordinates: r, θ, z

Spherical coordinates: r, θ, φ

Volume of a solid: V

1123. Properties of Line Integrals of Vector Fields

$$\int_{-C}\left(\vec{F}\cdot d\vec{r}\right)=-\int_{C}\left(\vec{F}\cdot d\vec{r}\right),$$

where -C denote the curve with the opposite orientation.

$$\int_{C}\left(\vec{F}\cdot d\vec{r}\right)=\int_{C_1\cup C_2}\left(\vec{F}\cdot d\vec{r}\right)=\int_{C_1}\left(\vec{F}\cdot d\vec{r}\right)+\int_{C_2}\left(\vec{F}\cdot d\vec{r}\right),$$

where C is the union of the curves C_1 and C_2.

1124. If the curve C is parameterized by $\vec{r}(t)=\langle x(t),y(t),z(t)\rangle$, $\alpha\le t\le\beta$, then

$$\int_C Pdx+Qdy+Rdz=$$

$$=\int_{\alpha}^{\beta}\left(P(x(t),y(t),z(t))\frac{dx}{dt}+Q(x(t),y(t),z(t))\frac{dy}{dt}+R(x(t),y(t),z(t))\frac{dz}{dt}\right)dt$$

1125. If C lies in the xy-plane and given by the equation $y=f(x)$, then

$$\int_C Pdx+Qdy=\int_a^b\left(P(x,f(x))+Q(x,f(x))\frac{df}{dx}\right)dx.$$

1126. Green's Theorem

$$\iint_R\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dxdy=\oint_C Pdx+Qdy,$$

where $\vec{F}=P(x,y)\vec{i}+Q(x,y)\vec{j}$ is a continuous vector function with continuous first partial derivatives $\frac{\partial P}{\partial y}$, $\frac{\partial Q}{\partial x}$ in a some domain R, which is bounded by a closed, piecewise smooth curve C.

1127. Area of a Region R Bounded by the Curve C

$$S = \iint_R dxdy = \frac{1}{2}\oint_C xdy - ydx$$

1128. Path Independence of Line Integrals

The line integral of a vector function $\vec{F} = P\vec{i} + Q\vec{j} + R\vec{k}$ is said to be path independent, if and only if P, Q, and R are continuous in a domain D, and if there exists some scalar function $u = u(x, y, z)$ (a scalar potential) in D such that

$$\vec{F} = \text{grad}\, u\text{, or } \frac{\partial u}{\partial x} = P\text{, } \frac{\partial u}{\partial y} = Q\text{, } \frac{\partial u}{\partial z} = R\text{.}$$

Then

$$\int_C \vec{F}(\vec{r}) \cdot d\vec{r} = \int_C Pdx + Qdy + Rdz = u(B) - u(A).$$

1129. Test for a Conservative Field

A vector field of the form $\vec{F} = \text{grad}\, u$ is called a conservative field. The line integral of a vector function $\vec{F} = P\vec{i} + Q\vec{j} + R\vec{k}$ is path independent if and only if

$$\text{curl}\, \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \vec{0}.$$

If the line integral is taken in xy-plane so that

$$\int_C Pdx + Qdy = u(B) - u(A),$$

then the test for determining if a vector field is conservative can be written in the form

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}.$$

$$\frac{\partial \vec{r}}{\partial u} = \frac{\partial x}{\partial u}(u,v)\vec{i} + \frac{\partial y}{\partial u}(u,v)\vec{j} + \frac{\partial z}{\partial u}(u,v)\vec{k},$$

$$\frac{\partial \vec{r}}{\partial v} = \frac{\partial x}{\partial v}(u,v)\vec{i} + \frac{\partial y}{\partial v}(u,v)\vec{j} + \frac{\partial z}{\partial v}(u,v)\vec{k}$$

and $\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v}$ is the cross product.

1141. If the surface S is given by the equation $z = z(x,y)$ where $z(x,y)$ is a differentiable function in the domain $D(x,y)$, then

$$\iint_S f(x,y,z)\,dS = \iint_{D(x,y)} f(x,y,z(x,y))\sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\,dxdy.$$

1142. Surface Integral of the Vector Field $\vec{F}$ over the Surface S

- If S is oriented outward, then

$$\iint_S \vec{F}(x,y,z)\cdot d\vec{S} = \iint_S \vec{F}(x,y,z)\cdot \vec{n}\,dS$$

$$= \iint_{D(u,v)} \vec{F}(x(u,v),y(u,v),z(u,v))\cdot\left[\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v}\right]dudv.$$

- If S is oriented inward, then

$$\iint_S \vec{F}(x,y,z)\cdot d\vec{S} = \iint_S \vec{F}(x,y,z)\cdot \vec{n}\,dS$$

$$= \iint_{D(u,v)} \vec{F}(x(u,v),y(u,v),z(u,v))\cdot\left[\frac{\partial \vec{r}}{\partial v} \times \frac{\partial \vec{r}}{\partial u}\right]dudv.$$

$d\vec{S} = \vec{n}\,dS$ is called the vector element of the surface. Dot means the scalar product of the appropriate vectors. The partial derivatives $\frac{\partial \vec{r}}{\partial u}$ and $\frac{\partial \vec{r}}{\partial v}$ are given by

$$I_{xz} = \iint_S y^2 \mu(x,y,z)dS.$$

1156. Moments of Inertia about the x-axis, y-axis, and z-axis

$$I_x = \iint_S (y^2 + z^2)\mu(x,y,z)dS,$$

$$I_y = \iint_S (x^2 + z^2)\mu(x,y,z)dS,$$

$$I_z = \iint_S (x^2 + y^2)\mu(x,y,z)dS.$$

1157. Volume of a Solid Bounded by a Closed Surface

$$V = \frac{1}{3}\left|\oiint_S xdydz + ydxdz + zdxdy\right|$$

1158. Gravitational Force

$$\vec{F} = Gm\iint_S \mu(x,y,z)\frac{\vec{r}}{r^3}dS,$$

where m is a mass at a point $\langle x_0, y_0, z_0 \rangle$ outside the surface,

$\vec{r} = \langle x - x_0, y - y_0, z - z_0 \rangle$,

$\mu(x,y,z)$ is the density function,

and G is gravitational constant.

1159. Pressure Force

$$\vec{F} = \iint_S p(\vec{r})d\vec{S},$$

where the pressure $p(\vec{r})$ acts on the surface S given by the position vector $\vec{r}$.

1160. Fluid Flux (across the surface S)

$$\Phi = \oiint_S \vec{v}(\vec{r})\cdot d\vec{S},$$

Acceleration of gravity: g
Current: I
Resistance: R
Inductance: L
Capacitance: C

10.1 First Order Ordinary Differential Equations

1164. Linear Equations

$$\frac{dy}{dx} + p(x)y = q(x).$$

The general solution is

$$y = \frac{\int u(x)q(x)dx + C}{u(x)},$$

where

$$u(x) = \exp\left(\int p(x)dx\right).$$

1165. Separable Equations

$$\frac{dy}{dx} = f(x,y) = g(x)h(y)$$

The general solution is given by

$$\int \frac{dy}{h(y)} = \int g(x)dx + C,$$

or

$$H(y) = G(x) + C.$$

1166. Homogeneous Equations

The differential equation $\frac{dy}{dx} = f(x, y)$ is homogeneous, if the function $f(x, y)$ is homogeneous, that is
$f(tx, ty) = f(x, y)$.

The substitution $z = \frac{y}{x}$ (then $y = zx$) leads to the separable equation
$x\frac{dz}{dx} + z = f(1, z)$.

1167. Bernoulli Equation
$\frac{dy}{dx} + p(x)y = q(x)y^n$.

The substitution $z = y^{1-n}$ leads to the linear equation
$\frac{dz}{dx} + (1-n)p(x)z = (1-n)q(x)$.

1168. Riccati Equation
$\frac{dy}{dx} = p(x) + q(x)y + r(x)y^2$

If a particular solution y_1 is known, then the general solution can be obtained with the help of substitution
$z = \frac{1}{y - y_1}$, which leads to the first order linear equation
$\frac{dz}{dx} = -[q(x) + 2y_1 r(x)]z - r(x)$.

1169. Exact and Nonexact Equations

The equation

$$M(x,y)dx + N(x,y)dy = 0$$

is called exact if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x},$$

and nonexact otherwise.

The general solution is

$$\int M(x,y)dx + \int N(x,y)dy = C.$$

1170. Radioactive Decay

$$\frac{dy}{dt} = -ky,$$

where $y(t)$ is the amount of radioactive element at time t, k is the rate of decay.

The solution is

$y(t) = y_0 e^{-kt}$, where $y_0 = y(0)$ is the initial amount.

1171. Newton's Law of Cooling

$$\frac{dT}{dt} = -k(T - S),$$

where $T(t)$ is the temperature of an object at time t, S is the temperature of the surrounding environment, k is a positive constant.

The solution is

$$T(t) = S + (T_0 - S)e^{-kt},$$

where $T_0 = T(0)$ is the initial temperature of the object at time $t = 0$.

1176. Differential Equations with x Missing

$y'' = f(y, y')$.

Set $u = y'$. Since

$$y'' = \frac{du}{dx} = \frac{du}{dy}\frac{dy}{dx} = u\frac{du}{dy},$$

we have

$$u\frac{du}{dy} = f(y, u),$$

which is a first order differential equation.

1177. Free Undamped Vibrations

The motion of a Mass on a Spring is described by the equation

$$m\ddot{y} + ky = 0,$$

where

m is the mass of the object,
k is the stiffness of the spring,
y is displacement of the mass from equilibrium.

The general solution is

$$y = A\cos(\omega_0 t - \delta),$$

where

A is the amplitude of the displacement,
ω_0 is the fundamental frequency, the period is $T = \frac{2\pi}{\omega_0}$,
δ is phase angle of the displacement.

This is an example of simple harmonic motion.

1178. Free Damped Vibrations

$m\ddot{y} + \gamma\dot{y} + ky = 0$, where

γ is the damping coefficient.

There are 3 cases for the general solution:

where I is the current in an RLC circuit with an ac voltage source $V(t) = E_0 \sin(\omega t)$.

The general solution is

$I(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t} + A \sin(\omega t - \varphi)$,

where

$$r_{1,2} = \frac{-R \pm \sqrt{R^2 - \frac{4L}{C}}}{2L},$$

$$A = \frac{\omega E_0}{\sqrt{\left(L\omega^2 - \frac{1}{C}\right)^2 + R^2\omega^2}},$$

$$\varphi = \arctan\left(\frac{L\omega}{R} - \frac{1}{RC\omega}\right),$$

C_1, C_2 are constants depending on initial conditions.

10.3. Some Partial Differential Equations

1181. The Laplace Equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

applies to potential energy function $u(x, y)$ for a conservative force field in the xy-plane. Partial differential equations of this type are called elliptic.

1182. The Heat Equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial t}$$

1193. $1+2+3+\ldots+n=\dfrac{n(n+1)}{2}$

1194. $2+4+6+\ldots+2n=n(n+1)$

1195. $1+3+5+\ldots+(2n-1)=n^2$

1196. $k+(k+1)+(k+2)+\ldots+(k+n-1)=\dfrac{n(2k+n-1)}{2}$

1197. $1^2+2^2+3^2+\ldots+n^2=\dfrac{n(n+1)(2n+1)}{6}$

1198. $1^3+2^3+3^3+\ldots+n^3=\left[\dfrac{n(n+1)}{2}\right]^2$

1199. $1^2+3^2+5^2+\ldots+(2n-1)^2=\dfrac{n(4n^2-1)}{3}$

1200. $1^3+3^3+5^3+\ldots+(2n-1)^3=n^2(2n^2-1)$

1201. $1+\dfrac{1}{2}+\dfrac{1}{4}+\dfrac{1}{8}+\ldots+\dfrac{1}{2^n}+\ldots=2$

1202. $\dfrac{1}{1\cdot 2}+\dfrac{1}{2\cdot 3}+\dfrac{1}{3\cdot 4}+\ldots+\dfrac{1}{n(n+1)}+\ldots=1$

1203. $1+\dfrac{1}{1!}+\dfrac{1}{2!}+\dfrac{1}{3!}+\ldots+\dfrac{1}{(n-1)!}+\ldots=e$

1224. Integration of Power Series

Let $f(x)=\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \ldots$ for $|x|<R$.

Then, for $|x|<R$, the indefinite integral $\int f(x)dx$ exists and

$$\int f(x)dx = \int a_0 dx + \int a_1 x dx + \int a_2 x^2 dx + \ldots$$

$$= a_0 x + a_1 \frac{x^2}{2} + a_2 \frac{x^3}{3} + \ldots = \sum_{n=0}^{\infty} a_n \frac{x^{n+1}}{n+1} + C.$$

11.10 Taylor and Maclaurin Series

Whole number: n
Differentiable function: $f(x)$
Remainder term: R_n

1225. Taylor Series

$$f(x)=\sum_{n=0}^{\infty} f^{(n)}(a)\frac{(x-a)^n}{n!} = f(a)+f'(a)(x-a)+\frac{f''(a)(x-a)^2}{2!}+\ldots$$

$$+\frac{f^{(n)}(a)(x-a)^n}{n!}+R_n.$$

1226. The Remainder After n+1 Terms is given by

$$R_n = \frac{f^{(n+1)}(\xi)(x-a)^{n+1}}{(n+1)!}, \quad a<\xi<x.$$

1227. Maclaurin Series

1234. $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} \pm \ldots$

1235. $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} + \ldots, \; |x| < \frac{\pi}{2}.$

1236. $\cot x = \frac{1}{x} - \left(\frac{x}{3} + \frac{x^3}{45} + \frac{2x^5}{945} + \frac{2x^7}{4725} + \ldots \right), \; |x| < \pi.$

1237. $\arcsin x = x + \frac{x^3}{2 \cdot 3} + \frac{1 \cdot 3x^5}{2 \cdot 4 \cdot 5} + \ldots + \frac{1 \cdot 3 \cdot 5 \ldots (2n-1)x^{2n+1}}{2 \cdot 4 \cdot 6 \ldots (2n)(2n+1)} + \ldots,$

$|x| < 1.$

1238. $\arccos x = \frac{\pi}{2} - \left(x + \frac{x^3}{2 \cdot 3} + \frac{1 \cdot 3x^5}{2 \cdot 4 \cdot 5} + \ldots + \frac{1 \cdot 3 \cdot 5 \ldots (2n-1)x^{2n+1}}{2 \cdot 4 \cdot 6 \ldots (2n)(2n+1)} + \ldots \right),$

$|x| < 1.$

1239. $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots + \frac{(-1)^n x^{2n+1}}{2n+1} \pm \ldots, \; |x| \le 1.$

1240. $\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots + \frac{x^{2n}}{(2n)!} + \ldots$

1241. $\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \ldots + \frac{x^{2n+1}}{(2n+1)!} + \ldots$

where
x is a particular outcome,
t is a variable of integration.

1279. $P(\alpha < X < \beta) = F\left(\frac{\alpha - \mu}{\sigma}\right) - F\left(\frac{\beta - \mu}{\sigma}\right)$,

where
X is normally distributed random variable,
F is cumulative normal distribution function,
$P(\alpha < X < \beta)$ is interval probability.

1280. $P(|X - \mu| < \varepsilon) = 2F\left(\frac{\varepsilon}{\sigma}\right)$,

where
X is normally distributed random variable,
F is cumulative normal distribution function.

1281. Cumulative Distribution Function

$F(x) = P(X < x) = \int_{-\infty}^{x} f(t)dt$,

where t is a variable of integration.

1282. Bernoulli Trials Process

$\mu = np \quad , \sigma^2 = npq$,

where
n is a sequence of experiments,
p is the probability of success of each experiments,
q is the probability of failure, $q = 1 - p$.

1283. Binomial Distribution Function

$b(n,p,q) = \binom{n}{k} p^k q^{n-k}$,

1294. Markov Inequality

$$P(X > k) \le \frac{E(X)}{k},$$

where k is some constant.

1295. Variance of Discrete Random Variables

$$\sigma^2 = V(X) = E\left[(X-\mu)^2\right] = \sum_{i=1}^{n} (x_i - \mu)^2 p_i \,,$$

where

x_i is a particular outcome,

p_i is its probability.

1296. Variance of Continuous Random Variables

$$\sigma^2 = V(X) = E\left[(X-\mu)^2\right] = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$$

1297. Properties of Variance

$V(X+Y) = V(X) + V(Y)$,

$V(X-Y) = V(X) + V(Y)$,

$V(X+c) = V(X)$,

$V(cX) = c^2 V(X)$,

where c is a constant.

1298. Standard Deviation

$$D(X) = \sqrt{V(X)} = \sqrt{E\left[(X-\mu)^2\right]}$$

1299. Covariance

$\operatorname{cov}(X,Y) = E\left[(X-\mu(X))(Y-\mu(Y))\right] = E(XY) - \mu(X)\mu(Y)$,

where

X is random variable,

$V(X)$ is the variance of X,

μ is the expected value of X or Y.