(3) **Ionisation energy (I.E.) :** Minimum energy required to move an electron from ground state to n = ∞ is called ionisation energy of the atom or ion
Ionisation energy of H atom = 13.6 eV
Ionisation energy of He^+ Ion = 54.4 eV
Ionisation energy of Li^{++} Ion = 122.4 eV

(4) **Ionisation potential (I.P.) :** Potential difference through which a free electron must be accelerated from rest such that its kinetic energy becomes equal to ionisation energy of the atom is called ionisation potential of the atom.
I.P of H atom = 13.6 V
I.P. of He^+ Ion = 54.4 V

(5) **Excitation energy :** Energy required to move an electron from ground state of the atom to any other exited state of the atom is called excitation energy of that state.
Energy in ground state of H atom = –13.6 eV
Energy in first excited state of H-atom = –3.4 eV
I^{st} excitation energy = 10.2 eV.

(6) **Excitation Potential :** Potential difference through which an electron must be accelerated from rest so that its kinetic energy becomes equal to excitation energy of any state is called excitation potential of that state.
I^{st} excitation energy = 10.2 eV.
I^{st} excitation potential = 10.2 V.

(7) **Binding energy or Seperation energy :** Energy required to move an electron from any state to n = ∞ is called binding energy of that state. or energy released during formation of an H-like atom/ion from n = ∞ to some particular n is called binding energy of that state.
Binding energy of ground state of H-atom = 13.6 eV

Ex. 13 First excitation potential of a hypothetical hydrogen like atom is 15 volt. Find third excitation potential of the atom.

Sol. Let energy of ground state = E_0

$E_0 = -13.6\,Z^2$ eV and $E_n = \dfrac{E_0}{n^2}$

$n = 2,\ E_2 = \dfrac{E_0}{4}$

given $\dfrac{E_0}{4} - E_0 = 15$

$-\dfrac{3E_0}{4} = 15$

for $n = 4,\quad E_4 = \dfrac{E_0}{16}$

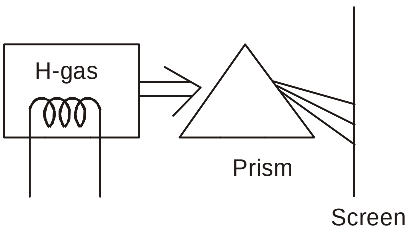
third excitation energy $= \dfrac{E_0}{16} - E_0$

$= -\dfrac{15}{16}E_0 = -\dfrac{15}{16}\cdot\left(\dfrac{-4\times15}{3}\right)$

$= \dfrac{75}{4}$ eV

∴ third excitation potential is $\dfrac{75}{4}$ V

10.3 **Emission spectrum of hydrogen atom :**



Under normal conditions the single electron in hydrogen atom stays in ground state (n = 1). It is excited to some higher energy state when it acquires some energy from external source. But it hardaly stays there for more than 10^{-8} second.

But $\left(-\frac{m_e e^4}{8\varepsilon_0^2 h^2}\right)$ is the ground state energy of hydrogen atom and hence is equal to – 13.6 eV.

From (iii), $E_n = -\frac{1872}{n^2} \times 13.6 \text{ eV} = \frac{-25459.2\,\text{eV}}{n^2}$

Thus, $E_1 = -25459.2$ eV and $E_3 = \frac{E_1}{9} = -2828.8$ eV. The energy difference is $E_3 - E_1 = 22630.4$eV.

The wavelength emitted is

$$\lambda = \frac{hc}{\Delta E} = \frac{1240\,\text{eV} - \text{nm}}{22630.4\,\text{eV}} = 55 \text{ pm}.$$

Ex. 24 A gas of hydrogen like atoms can absorb radiations of 68 eV. Consequently, the atoms emit radiations of only three different wavelength. All the wavelengths are equal or smaller than that of the absorbed photon.
(a) Determine the initial state of the gas atoms.
(b) Identify the gas atoms.
(c) Find the minimum wavelength of the emitted radiations.
(d) Find the ionization energy and the respective wavelength for the gas atoms.

Sol. (a) $\frac{n(n-1)}{2} = 3$

$\therefore$ $n = 3$

i.e., after excitation atom jumps to second excited state.

Hence $n_f = 3$. So n_i can be 1 or 2

If $n_i = 1$ then energy emitted is either equal to, greater than or less than the energy absorbed. Hence the emitted wavelength is either equal to, less than or greater than the absorbed wavelength.

Hence $n_i \neq 1$.

If $n_i = 2$, then $E_e \geq E_a$. Hence $\lambda_e \leq \lambda_a$

(b) $E_3 - E_2 = 68$ eV

$\therefore$ $(13.6)(Z^2)\left(\frac{1}{4} - \frac{1}{9}\right) = 68$

$\therefore$ $Z = 6$

(c) $\lambda_{min} = \frac{12400}{E_3 - E_1} = \frac{12400}{(13.6)(6)^2\left(1 - \frac{1}{9}\right)} = \frac{12400}{435.2} = 28.49$ **Ans.**

(d) Ionization energy = $(13.6)(6)^2 = 489.6$ eV **Ans.**

$$\lambda = \frac{12400}{489.6} = 25.33 \text{ Å}$$ **Ans.**

Ex. 25 An electron is orbiting in a circular orbit of radius r under the influence of a constant magnetic field of strength B. Assuming that Bohr's postulate regarding the quantisation of angular momentum holds good for this electron, find
(a) the allowed values of the radius 'r' of the orbit.
(b) the kinetic energy of the electron in orbit
(c) The potential energy of interaction between the magnetic moment of the orbital current due to the electron moving in its orbit and the magnetic field B.
(d) The total energy of the allowed energy levels.

Sol. (a) radius of circular path

$$r = \frac{mv}{Be} \quad \text{....(i)}$$

$$mvr = \frac{nh}{2\pi} \quad \text{....(ii)}$$

Solving these two equations, we get

$$r = \sqrt{\frac{nh}{2\pi Be}} \quad \text{and } v = \sqrt{\frac{nhBe}{2\pi m^2}}$$

Ex. 31 How many head-on, elastic collisions must a neutron have with deuterium nucleus to reduce its energy from 1 MeV to 0.025 eV.

Sol. Let mass of neutron = m and mass of deuterium = 2m

initial kinetic energy of neutron = K_0

Let after first collision kinetic energy of neutron and deuterium be K_1 and K_2.

Using C.O.L.M. along direction of motion

$$\sqrt{2mK_0} = \sqrt{2mK_1} + \sqrt{4mK_2}$$

velocity of seperation = velocity of approach

$$\frac{\sqrt{4mK_2}}{2m} - \frac{\sqrt{2mK_1}}{m} = \frac{\sqrt{2mK_0}}{m}$$

Solving equaiton (i) and (ii) we get

$$K_1 = \frac{K_0}{9}$$

Loss in kinetic eneryg after first collision

$$\Delta K_1 = K_0 - K_1$$

$$\Delta K_1 = \frac{8}{9} K_0 \qquad \text{....... (1)}$$

After second collision

$$\Delta K_2 = \frac{8}{9} K_1 = \frac{8}{9} \cdot \frac{K_0}{9}$$

$\therefore$ Total energy loss

$$\Delta K = \Delta K_1 + \Delta K_2 + \ldots.. + \Delta K_n$$

As,
$$\Delta K = \frac{8}{9} K_0 + \frac{8}{9^2} K_0 + \ldots\ldots\ldots + \frac{8}{9^n} K_0$$

$$\Delta K = \frac{8}{9} K_0 \left(1 + \frac{1}{9} + \ldots\ldots\ldots + \frac{1}{9^{n-1}}\right)$$

$$\frac{\Delta K}{K_0} = \frac{8}{9}\left[\frac{1 - \frac{1}{9^n}}{1 - \frac{1}{9}}\right] = 1 - \frac{1}{9^n}$$

Here $K_0 = 10^6$ eV, $\Delta K = (10^6 - 0.025)$ eV

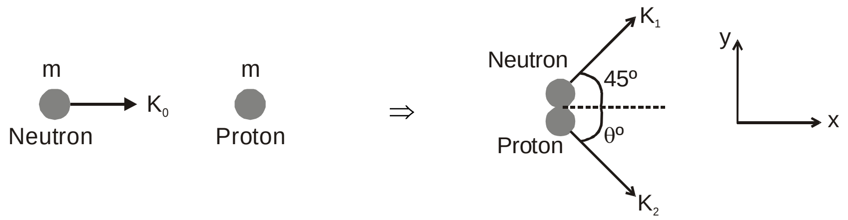
$$\therefore \quad \frac{1}{9^n} = \frac{K_0 - \Delta K}{K_0} = \frac{0.025}{10^6} \quad \text{or} \quad 9^n = 4 \times 10^7$$

Taking log both sides and solving, we get

$$n = 8$$

Ex. 32 A neutron with an energy of 4.6 MeV collides with protons and is retarded. Assuming that upon each collision neutron is deflected by 45º find the number of collisions which will reduce its energy to 0.23 eV.

Sol. Mass of neutron $\approx$ mass of proton = m



From conservation of momentum in y-direction

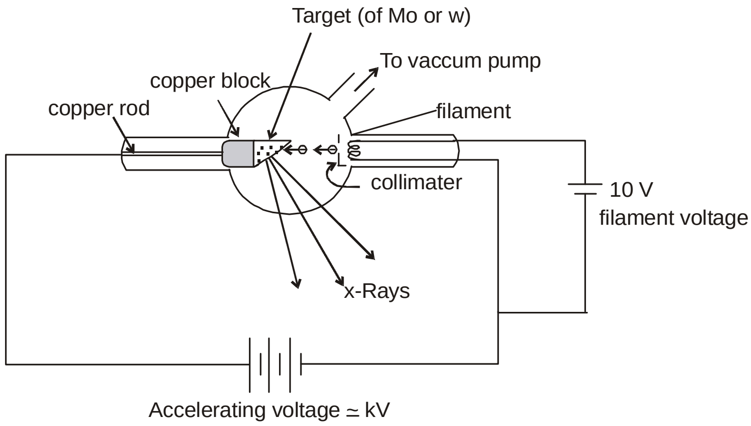
$$\sqrt{2mK_1} \sin 45^\circ = \sqrt{2mK_2} \sin \theta \qquad \text{....(i)}$$

In x-direction
$$\sqrt{2mK_0} - \sqrt{2mK_1} \cos 45^\circ = \sqrt{2mK_2} \cos \theta \qquad \text{....(ii)}$$

Squaring and adding equation (i) and (ii), we have

$$K_2 = K_1 + K_0 - \sqrt{2K_0K_1} \qquad \text{....(iii)}$$

13.1 Production of x-rays by coolidge tube :



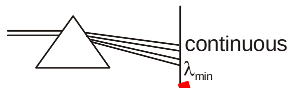
The melting point, specific heat capacity and atomic number of target should be high. When voltage is applied across the filament then filament on being heated emits electrons from it. Now for giving the beam shape of electrons, collimator is used. Now when electron strikes the target then x-rays are produced.

When electrons strike with the target, some part of energy is lost and converted into heat. Since, target should not melt or it can absorb heat so that the melting point, specific heat of target should be high.

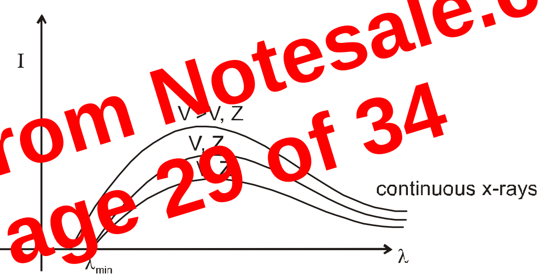
Here copper rod is attached so that heat produced can go behind and it can absorb heat and target does not get heated very high.

For more energetic electron, accelerating voltage is increased.
For more no. of photons voltage across filament is increased.

The x-ray were analysed by mostly taking their spectrum



13.2 Variation of Intensity of x-rays with λ is plotted as shown in figure :



1. The minimum wavelength corresponds to the maximum energy of the x-rays which in turn is equal to the maximum kinetic energy eV of the striking electrons thus

$$eV = h\nu_{max} = \frac{hc}{\lambda_{min}}$$

$$\lambda_{min} = \frac{hc}{eV} = \frac{12400}{V(\text{involts})} \text{ Å.}$$

We see that cutoff wavelength λ_{min} depends only on accelerating voltage applied between target and filament. It does not depend upon material of target, it is same for two different metals (Z and Z')

Ex. 33 An X-ray tube operates at 20 kV. A particular electron loses 5% of its kinetic energy to emit an X-ray photon at the first collision. Find the wavelength corresponding to this photon.

Sol. Kinetic energy acquired by the electron is

$$K = eV = 20 \times 10^3 \text{ eV}.$$

The energy of the photon

$$= 0.05 \times 20 = 10^3 \text{ eV} = 10^3 \text{ eV}.$$

Thus, $\frac{h\nu}{\lambda} = 10^3 \text{eV} = \frac{(4.14 \times 10^{-15} \text{eV} - \text{s}) \times (3 \times 10^8 \text{m/s})}{10^3 \text{eV}} = \frac{1242 \text{eV} - \text{nm}}{10^3 \text{eV}} = 1.24 \text{nm}$

$$\sqrt{\frac{\lambda_{c_0}}{\lambda_x}} = \frac{Z_x - 1}{Z_{c_0} - 1}$$

Substituting gives us

$$\sqrt{\frac{178.9\,\text{pm}}{143.5\,\text{pm}}} = \frac{Z_x - 1}{27 - 1}.$$

Solving for the unknown, we find $Z_x = 30.0$; the impurity is zinc.

Ex. 38 Find the constants a and b in Moseley's equation $\sqrt{v} = a(Z - b)$ from the following data.

Element	Z	Wavelength of K_α X-ray
Mo	42	71 pm
Co	27	178.5 pm

Sol. Moseley's equation is

$$\sqrt{v} = a(Z - b)$$

Thus,
$$\sqrt{\frac{c}{\lambda_1}} = a(Z_1 - b) \qquad \text{....(i)}$$

and
$$\sqrt{\frac{c}{\lambda_2}} = a(Z_2 - b) \qquad \text{....(ii)}$$

From (i) and (ii)
$$\sqrt{c}\left(\frac{1}{\sqrt{\lambda_1}} - \frac{1}{\sqrt{\lambda_2}}\right) = a(Z_1 - Z_2)$$

or,
$$a = \frac{\sqrt{c}}{(Z_1 - Z_2)}\left(\frac{1}{\sqrt{\lambda_1}} - \frac{1}{\sqrt{\lambda_2}}\right)$$

$$= \frac{(3 \times 10^8\,\text{m/s})^{1/2}}{42 - 27}\left[\frac{1}{(71 \times 10^{-12}\,\text{m})^{1/2}} - \frac{1}{(178.5 \times 10^{-12}\,\text{m})^{1/2}}\right]$$

$= 5.0 \times 10^7\ (\text{Hz})^{1/2}$

Dividing (i) by (ii),

$$\sqrt{\frac{\lambda_2}{\lambda_1}} = \frac{Z_1 - b}{Z_2 - b}$$

or,
$$\sqrt{\frac{178.5}{71}} = \frac{42 - b}{27 - b}$$

or, $b = 1.37$

Problem 1. Find the momentum of a 12.0 MeV photon.

Solution : $p = \frac{E}{c} = 12$ MeV/c.

Problem 2. Monochromatic light of wavelength 3000 Å is incident normally on a surface of area 4 cm². If the intensity of the light is 15×10^{-2} W/m², determine the rate at which photons strike the surface.

Solution : Rate at which photons strike the surface

$$= \frac{IA}{hc/\lambda} = \frac{6 \times 10^{-5}\,\text{J/s}}{6.63 \times 10^{-19}\,\text{J/photon}} = 9.05 \times 10^{13}\ \text{photon/s}.$$