NATIONAL OPEN UNIVERSITY OF NIGERIA

MATHEMATICS FOR ECONOMISTS 1

ECO 255 COURSE GUIDE

Course Developer:

ODISHIKA Vivian Anietem

Economics Unit,
School of Arts and Social Sciences,
National Open University of Nigeria.
14/16 Ahmadu Bello Way,
Victoria Island, Lagos.

MODULE 1 NUMBER SYSTEM, INEQUALITITES, EXPONENT AND ROOTS

Unit 1 Number System
Unit 2 Inequalities
Unit 3 Exponent and Roots

UNIT 1 NUMBER SYSTEM

1.0 Introduction
2.0 Objectives
3.0 Main Content
 3.1 Introduction to Number System
 3.2 Properties of Number System
 3.3 Real Number
 3.4 Binary Numbers
 3.5 Imaginary Numbers
 3.6 Complex Numbers
4.0 Conclusion
5.0 Summary
6.0 Tutor-Marked Assignment
7.0 References/Further Readings

1.0 INTRODUCTION

Column weight	1000	100	10	1	0.1	0.01	0.001
Column Value	3000	400	50	6	0.7	0.08	0.009

We can deduce the following from Table 1 above:

- As we move left through the columns the columns increase in weight, i.e., each symbol gets "heavier".
- As we move right through the columns the numbers decrease in weight.
- Columns are numbered. Starting from the left of the decimal point and moving left, the numbers increase positively. At the right of the decimal point the column numbers increase negatively.
- You can see in the third row that the number base (10) is raised to the power of the column or index. 10^3 means 10 raised to the power 3, or 10 * 10 * 10. 10 is the base, 3 is the index or column.

We could write the numbers as:

3000 + 400 + 50 + 6 + 0.7 + 0.08 + 0.009

Or as:

$3 * 10^3 + 4 * 10^2 + 5 * 10^1 + 6 * 10^0 + 7 * 10^{-1} + 8 * 10^{-2} + 9 * 10^{-3}$.

SELF ASSESSMENT EXERCISE

Given the column of index value of 5, 4, 3, 2, 1, 0, -3, -4, -5., and digit or coefficient value of 7, 8, 9, 10, 11, 12, 13, 14 and 15. Write out the column value using base 10.

3.2 PROPERTIES OF NUMBER SYSTEM

The sum, difference, product and quotient (provided the denominator $\neq 0$) of real numbers is a real number. Thus, addition, subtraction and multiplication as well as division of real numbers are feasible in the field of real numbers.

Real number is a set of numbers that includes the integers or counting numbers (number that are written without a fractional component) and all the rational numbers (numbers that cannot be represented as a ratio of two whole numbers, such as π (3.142) and e (2.7183).

The figure above proves that a complex number can be visually represented as a pair of numbers (a, b) forming a vector on a diagram called an Argand diagram, representing the complex plane. "Re" is the real axis, "Im" is the imaginary axis, and i is the imaginary unit which satisfies the equation $i^2 = -1$. In the above figure, a and b represents the real numbers, while i represents the imaginary number, thus their combination formed the complex number.

Complex numbers allow for solutions to certain equations that have no real solutions: the equation $(x + 1)^2 = -9$ has no real solution, since the square of a real number is either 0 or positive. Complex numbers provide a solution to this problem. The idea is to extend the real numbers with the imaginary unit i where $i^2 = -1$, so that solutions to equations like the preceding one can be found. In this case the solutions are $-1 + 3i$ and $-1 - 3i$, as can be verified using the fact that $i^2 = -1$:

$$((-2 + 4i) + 2)^2$$

$$(4i)^2 = (4^2)(i^2)$$

$$16(-1) = -16$$

Also,

$$((-2 - 4i) + 2)^2$$

$$(-4i)^2 = (-4^2)(i^2)$$

$$16(-1) = -16$$

In fact not only quadratic equations (an equation in which the highest power of an unknown quantity is a square), but all polynomial equations (an expression consisting of variables and coefficients that involves only the operations of addition, subtraction, multiplication and non-negative integer exponents) with real or complex coefficients in a single variable can be solved using complex numbers.

Put differently, if C denotes the set of complex numbers, then $C = \{a + bi \mid a \in R, b \in R$ and $i = \sqrt{1}\}$. If $a = 0$, Z becomes an imaginary number and if $b = 0$, Z becomes a real number. The number $\hat{Z} = a - bi$ is said to be the conjunction (changing of the middle sign) of $Z = a + bi$.

For example, $3 - 2i$ and $3 + 2i$ are conjugate complex number.

If $a < 0$, then $-a > 0$

For example, 4 > 0 and -4 < 0.Similarly, -2 < 0 and 2 > 0.Whenever we multiply an inequality by -1, the inequality sign flips. This is also true when both numbers are non-zero: 4 > 2 and -4 < - 2 ; 6 < 7 and -6 > - 7 ; -2 < 5 and 2 > - 5.

In fact, when we multiply or divide both sides of an inequality by any negative number, the sign always flips. For instance, 4 > 2, so 4(- 3) < 2(- 3): -12 < - 6. -2 < 6, so -2/-2>6/-2 : 1 > -3. This leads to the multiplication and division properties of inequalities for negative numbers.

(3a) Multiplication and Division Properties of Inequalities for positive numbers:

If $a<b$ and $c>0$, then $ac<bc$ and $<a/c <b/c$

If $a >b$ and $c>0$, then $ac>bc$and $>a/c >b/c$

(3b) Multiplication and Division Properties of Inequalities for negative numbers:

If $a<b$ and $c<0$, then $ac >bc$and $>a/c >b/c$

If $a >b$ and $c < 0$, then $ac<bc$ and $<a/c <b/c$

Note: All the above properties apply to ≤ and ≥.

4. PROPERTIES OF RECIPROCAL

Note the following properties:

If $a>0$, then $1/a>0$

If $a<0$, then$1/a<0$

When we take the reciprocal of both sides of an equation, something interesting happens. If the numbers on both sides have the same sign, the inequality sign flips. For example, 2 < 3, but

1/2>1/3. Similarly,-1/3>-2/3, but -3 <-3/2.

We can write this as a formal property:

If a > 0 and b >0 , or a < 0 and b < 0 , and a < b , then 1/a>1/b.

If a > 0 and b >0 , or a < 0 and b < 0 , and a > b , then 1/a<1/b.

Note: All the above properties apply to ≤ and ≥.

6.0 TUTOR-MARKED ASSIGNMENT

- If angle F measures x(x – 3) + 90 degree, for which of the following values of x is F right? {0, 1, 2, 3}.
- Solve for x: -2x – 5 ≠ -3 – x.

7.0 REFERENCES/FURTHER READINGS

Carter .M (2001). Foundation of Mathematical Economics, The MIT Press, Cambridge, Massachusetts

Chiang A.C and Wainwright .K (2005). Fundamental Methods of Mathematical Economics. 4th edition-McGraw-hill

Ekanem, O.T (2004). Essential Mathematics for Economics and Business, Mareh: Benin City

UNIT 3 EXPONENT AND ROOTS

10^2x6 = 600.

Example 3: Simplify √810

√810 = √2x3x3x3x3x5

√2x3x3x3x3x5 = 3x3x√2x5

3x3x√2x5 = 9x√10

9^2x10 = 810.

Similarly, to simplify a cube root, factor the number inside the "(3√)" sign. If a factor appears three times, cross out all three and write the factor one time outside the cube root sign.

Example4: Find the cube root of 8.

3√8

3√2*2*2

Since 2 appear three times, we cross it out and write 2 as our answer. So the cube root of 8 is 2.

Example 5: Find the cube root of 216.

3√216

3√6*6*6

Thus giving us an answer of 6.

3.6.2 APPROXIMATING SQUARE ROOTS

It is very difficult to know the square root of a number (other than a perfect square) just by looking at it. And one cannot simply divide by some given number every time to find a square root. Thus, is it helpful to have a method for approximating square roots? To employ this method, it is useful to first memorize the square roots of the perfect squares. Here are the steps to approximate a square root:

1. Pick a perfect square that is close to the given number. Take its square root.
2. Divide the original number by this result.
3. Take the arithmetic mean of the result of I and the result of II by adding the two numbers and dividing by 2 (this is also called "taking an average").
4. Divide the original number by the result of III.
5. Take the arithmetic mean of the result of III and the result of IV.
6. Repeat steps IV-VI using this new result, until the approximation is sufficiently close.

2.0 INTRODUCTION

Equation is the process of equating one number to another. It is also a statement which shows that the values of two mathematical expressions are equal.

There so many types of equations such as the Linear Equation, Quadratic Equation, Polynomial Equation, Trigonometric Equation, Radical Equation and Exponential Equation. Solving equations means finding the value (or set of values)or unknown variables contained in the equation.

Working with single equations: $7x - 2 = 8 + 2x$.

We combine like terms to reduce the equation to:

$$7x - 2x = 8 + 2$$
$$5x = 10$$
$$x = 2.$$

We have seen equations with one variable, which generally have a finite number of solutions. In this unit, we will begin to deal with systems of equations; that is, with a set of two or more equations with the same variables. We limit our discussion to systems of linear equations, since our techniques for solving even a single equation of higher degree are quite limited.

Systems of linear equations can have zero, one, or an infinite number of solutions, depending on whether they are consistent or inconsistent, and whether they are dependent or independent. The first sub-unit will cover the introductory aspect of systems of linear equations, while the second and third sub-units will focus on the different methods of solving systems of linear equations (substitution, addition and subtraction). Substitution is useful when one variable in an equation of the system has a coefficient of 1 or a coefficient that easily divides the equation. If one of the variables has a coefficient of 1, substitution is very useful and easy to do. However, many systems of linear equations are not quite so neat (not easy to calculate) and substitution can be difficult, thus an alternative method for solving systems of linear equations (the Addition/Subtraction method) is introduced.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- Explain broadly, the equation system
- Solve problems of linear equation using substitution method.
- Solve problems of linear equation using the addition/subtraction method.
- Graph systems of equations solution.

3.0 MAIN CONTENT

From equation (2), $2x + 2y = 6$

$$2y = -2x + 6$$
$$y = -x + 3$$

In the above solution, the slope for the first and second equation is 2/3 and -1 respectively, while the y-intercept is -2 and 3 respectively.

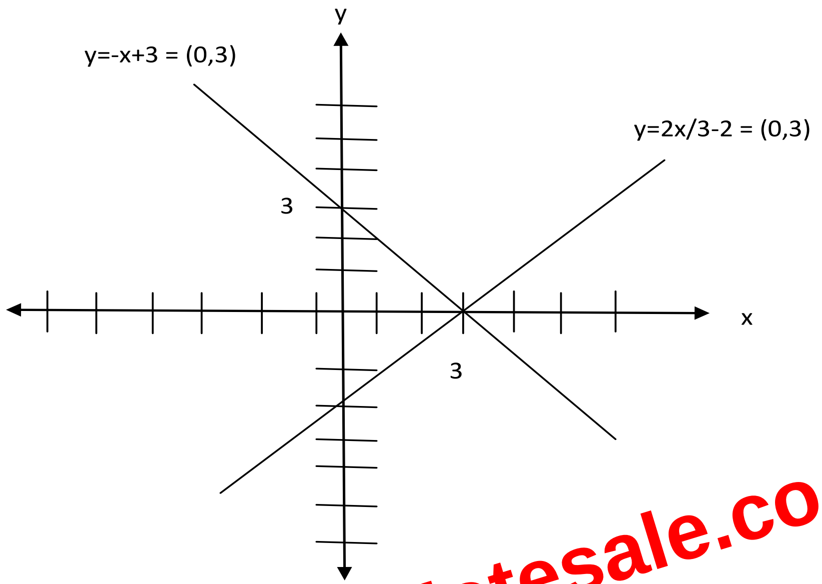


Figure1. Systems of Equation Graph

Since the two lines intersect at the point (0, 3), this point is a solution to the system. Thus, the solution set to the system of equations is {(0, 3)}

To check, plug (0, 3) with y = 0, and x = 3 into the equationy = 2x/3 – 2 and y = -x + 3.

y = -x + 3 = -3 + 3? Yes.

y = 2x/3 – 2 = 2(3)/3 – 2 = 6/3 – 2 = 2 – 2=0

2. CLASSIFICATION OF SYSTEMS

There are three possibilities for the manner in which the graphs of two linear equations could meet; the lines could intersect once, not intersect at all (be parallel), or intersect an infinite number of times (in which case the two lines are actually the same).

1. If the two equations describe the same line, and thus lines that intersect an infinite number of times, the system is dependent and consistent.
2. If the two equations describe lines that intersect once, the system is independent and consistent.
3. If the two equations describe parallel lines, and thus lines that do not intersect, the system is independent and inconsistent.

A system is consistent if it has one or more solutions. A system of two equations is dependent if all solutions to one equation are also solutions to the other equation.

The following chart will help determine if an equation is consistent and if an equation is dependent:

SELF ASSESSMENT EXERCISE

Notice that y in both equations have opposite signs, thus adding the two equations together eliminates y, thus resulting in $6x = 8$.
We can solve the above equation completely:

$$6x = 8$$
$$x = 1.33 \qquad (3)$$

To find y, we need to impute $x = 1.33$ into equation (1)

$$4(1.33) + 2y = 14$$
$$5.32 + 2y = 14$$
$$2y = 14 - 5.32$$
$$2y = 8.68$$
$$y = 4.34$$

The solution to the above problem is thus $x = 1.33$, $y = 4.34$.

Example: $4x - 4y = 12 \qquad (1)$
$3x + 4y = 8 \qquad (2)$

Substituting equation (2) from (1) gives:

$$x = 4 \qquad (3)$$

We impute $x = 4$ into equation (1) in order to find y.

$$4(4) - 4y = 12$$
$$16 - 4y = 12$$
$$-4y = 12 - 16$$
$$-4y = -4$$

Dividing through by -4:

$$y = 1.$$

We impute $y = 1$ into equation 1 to confirm the value for x.

$$4x - 4y = 12$$
$$4x - 4(1) = 12$$
$$4x - 4 = 12$$
$$4x = 12 + 4$$
$$4x = 16$$
$$x = 4.$$

We can see that the value for our x corresponds to the initial value of 4, thus confirming that our result is in line.
Another style of solving using the addition/ subtraction method is presented below.
Example: $7x + 2y = 47 \qquad (1)$
$5x - 4y = 1 \qquad (2)$

Notice that the value of y in equation (1) and (2) is not the same, thus making it impossible to eliminate them through addition or subtraction. The technique used in this case is multiplying equation (1) with the value of y in equation (2), and multiplying equation (2) with the value of y in equation (1).
Based on the question above, we multiply equation (1) with 4 which belongs to y in equation (2), and we multiply equation (2) with 2 which belongs to y in equation (1) to get a uniform model.

$(7x + 2y = 47) \times 4$
$(5x - 4y = 1) \times 2$
$28x + 8y = 188 \qquad (3)$
$10x - 8y = 2 \qquad (4)$

Eliminating equation (3) and (4) by adding the two equations together yielded:

$38x = 190$
$x = 5.$

Now that we have a value for x, we can substitute this into equation (2) in order to find y.
Substituting:

$5x - 4y = 1$
$5(5) - 4y = 1$
$25 - 4y = 1$
$-4y = -24$

Dividing both side by -4

$y = 6.$

SELF ASSESSMENT EXERCISE

Solve this pair of simultaneous equations using the substitution method.

$$3x + 7b = 27$$
$$5x + 2y = 16.$$

4.0 CONCLUSION

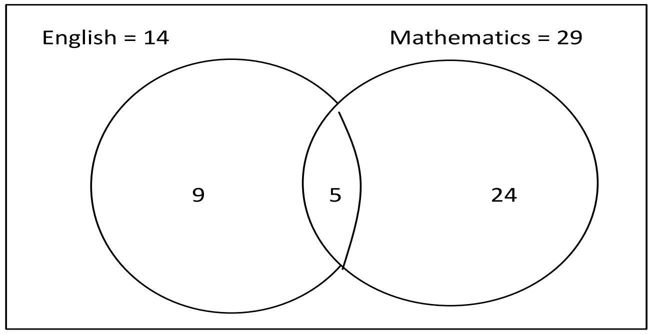
- Simultaneous equation refers to a condition where two or more unknown variables are related to each other through an equal number of equations.
- To solve a simultaneous equation, first eliminate one of the letter terms and find the value of the remaining letter.
- Substitution method involves the transformation of one of the equations such that one variable is defined in terms of the other.
- The elimination method (Addition/subtraction) is done by adding or subtracting the equations from one another for the purpose of canceling variable terms.
- When the coefficient of y or x in each equations are the same, and the signs of y and x are opposite, then adding or subtracting each side of the equation will eliminate y or x.

5.0 SUMMARY

This unit focused on simultaneous equation, which is a condition where two or more unknown variables are related to each other through an equal number of equations. In order to explain this topic, the different methods of solving simultaneous equation problems were utilized. The substitution method defines one variable in terms of the other, while the elimination method (addition/subtraction) involves adding or subtracting the equations from one another in order to form a unified equation.

6.0 TUTOR-MARKED ASSIGNMENT

Solve the following equations using the substitution and the elimination method:



This tells us that the total of 9 + 5 + 24 = 38 students are in either English or Mathematics or both. This leaves two students unaccounted for, so they must be the ones taking neither class.

From the above Venn diagram, we can deduce the following:

- 2 students are taking neither of the subjects.
- There are 38 students in at least one of the classes.
- There is a 24/40 (60%) probability that a randomly-chosen students in this group is taking Mathematics but not English.

SELF ASSESSMENT EXERCISE

Using Venn diagram, show the relationship between A = {1,2,3,4,5} and B = {3,4,5,6,7}.

4.0 CONCLUSION

- Set theory is the branch of mathematics that studies sets, which are collections of objects.
- A derived binary relation between two set is the subset relation, which is also called set inclusion.
- A is called proper subset of B if and only if A is a subset of B, but B is not a subset of A.
- Venn diagram shows the relationship between sets in picture form.
- The combination of set A and B is called union (U) of A and B.
- The overlapping of set A and B is called the intersection of A and B.
- The interior of a set circle symbolically represents the elements of the set, while the exterior represents elements that are not members of the set.

5.0 SUMMARY

This unit focused on the set theory. Set theory is a branch of mathematics which deals with the formal properties of sets as units (without regard to the nature of their individual constituents) and the expression of other branches of mathematics in terms of sets. Sub-unit 3.1 introduced us to set theory, the next sub-unit, focused on the symbols and properties of set operations, explaining the different properties such as the identity law, the involution laws, idempotent laws, e.t.c., while the last sub-unit introduced the concept of Venn diagram which uses diagram to show the relationship between sets.

7.0 References/Further Readings

3.0 INTRODUCTION

In its simplest form, a logarithm answers the question of how many of one number is to be multiplied to get another number; i.e., how many 2's do we multiply together to get 8?, the answer to this question is 2 *x* 2 *x* 2 = 8, so we needed to multiply 2 three (3) times in order to get 8; so in this case, the logarithm is 3, and we write it as $\log_2 8 = 3$.Like many types of functions, the exponential (a number representing the power to which a number is to be raised, i.e., 2^3; 3 is the exponent) function is the inverse of logarithm function, and that is the focus of this unit.

In order to do justice to this unit, the first sub-unit introduces us broadly to the meaning of the logarithmic function. This sub-unit also addresses the domain and range of a logarithmic function, which are inverses of those of its corresponding exponential function. The second sub-unit presents the two special logarithmic functions (the common logarithmic function and the natural logarithmic function). The common logarithm is $\log_{10}x$, while the natural logarithm is $\log_e x$. Sub-unit three deals with the properties of logarithms. The eight properties discussed in this section are helpful in evaluating logarithmic expressions by hand or using a calculator. They are also useful in simplifying and solving equations containing logarithms or exponents. The last sub-unit focuses on solving exponential and logarithm functions.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- Explain the concept Logarithm
- Know the difference between an exponent and logarithm
- Differentiate between the common and the natural logarithm function
- Understand the properties of logarithm.

3.0 MAIN CONTENT

3.1 INTRODUCTION TO LOGARITHM

Logarithms are the opposite of exponents, just as subtraction is the opposite of addition and division is the opposite if multiplication. Given, for example two variables *x* and *y*, such that $x = a^y$, this is an exponential function, which can be written in log form $\log_a x = y$. In general, if $a^y = x$, then $\log_a x = y$.

From the above specification, $x = a^y$ is the exponential function, while $\log_a x = y$ is the equivalent logarithm function which is pronounced as log-base-*a* of *x* equals *y*.

The value of "*a*" is the base of the logarithms, just as "*a*" is the base in the exponential expression "$_a x$". And, just as the *"a"* in an exponential is always positive (greater than

$2\log_3 6 - \log_3 4 = 2.$

$\log_3 6^2 - \log_3 4 = 2.$

$\log_3 36 - \log_3 4 = 2.$

$\text{Log}_3 36/4 = \log_3 9 = 2.$

Thus 1 is the solution to the above equation.

SELF ASSESSMENT EXERCISE

Solve for *x*: 2log3x + log3

4.0 CONCLUSION

Logarithm answers the question of how many of one variable is to be multiplied together to get another number.

Logarithm function is the inverse of exponential function, i.e., if $a^y = x$, then $\log_a x = y$.

Simplifying the logs makes it easy for us to solve.

The two special logarithm functions comprises of the common logarithm function and the natural logarithm function.

The common logarithm function is any logarithm with base 10, while the base of a natural logarithm function is denoted by "*e*", which is mostly written as "*lnx*".

In order to solve an equation containing an exponent variable, exponential quantity of the model must be isolated, then taking logarithm to the base of the exponent of both sides.

In order to solve an equation containing a logarithm, we need to consider the properties of logarithm which compresses the expressions into one.

5.0 SUMMARY

This unit focused on the logarithm function. Logarithm implies a quantity representing the power to which a fixed number (the base) must be raised to produce a given number. It is also the inverse of an exponent. The sub-topics focuses on the introduction, properties and the two special logarithms function respectively, while the last sub-unit presented detailed information on how to solve logarithm and exponential problem.

6.0 TUTOR-MARKED ASSIGNMENT

- Solve for *x*: $\log_8 x + \log_8 7x - 9$
- Evaluate log36 and *ln*62

7.0 REFERENCES/FURTHER READINGS

Brian .B and Tamblyn .I (2012). Understanding Math, Introduction to Logarithms (Kindle Edition). Solid State Press: California.

Chiang A.C and Wainwright .K (2005). Fundamental Methods of Mathematical Economics, 4th edition-McGraw-hill

Ekanem O.T (2004). Essential Mathematics for Economics and Business. Mareh: Benin City

UNIT 3 PARTIAL DERIVATIVES

The chain rule is a rule for differentiating compositions of functions. We've been using the standard chain rule for functions of one variable throughout the last two sub-units; it is now time to extend the chain rule out to more complicated situations. In order for us to use the chain rule extensively, we need to first review the notation for the chain rule for functions of one variable.
The chain rule states formally that:

$$D\{f(g(x))\} = f^{t}(g(x))\, g^{t}(x).$$

However, we rarely use this formal approach when applying the chain rule to specific problems. Instead, we invoke an intuitive approach. For example, it is sometimes easier to think of the functions f and g as "layers" of a problem. Function f is the "outer layer'" and function g is the "inner layer". Thus, the chain rule tells us to first differentiate the outer layer, leaving the inner layer unchanged (the term $f^{t}(g(x))$), then differentiate the inner layer (the term $g^{t}(x)$). This process will become clearer as we apply them to some problems.

Example 1: Differentiate y = $(3x + 1)^2$.
The outer layer is "the square" and the inner layer is $(3x+1)$. Differentiate the square first, leaving $(3x+1)$ unchanged. Then differentiate $(3x+1)$.) Thus

$$Đ(3x + 1)^2$$
$$= 2(3x + 1)^{2-1}Đ(3x + 1)$$

By differentiating further, we get:

$$= 2(3x + 1)^*(3)$$
$$= 6(3x + 1).$$

Example 2: Differentiate $y = (2 – 4x + 5x^4)^{10}$.
The outer layer is the 10th power, and the inner layer is $(2 – 4x + 5x^4)$. Differentiate the 10th power first, leaving $(2 – 4x + 5x^4)$ unchanged. Then differentiate $(2 – 4x + 5x^4)$.

$$Đ(2 – 4x + 5x^4)^{10} = 10(2 – 4x + 5x^4)^{10-1} * Đ(2 – 4x + 5x^4)$$
$$= 10(2 – 4x + 5x^4)^{10-1} * (-4 + 20x^3)$$
$$= 10(2 – 4x + 5x^4)^9 * (20x^3 - 4).$$

Example 3: Differentiate y = $\sin(5x)$
The outer layer is the "sin function" and the inner layer is $(5x)$. Differentiate the "sin function" first, leaving $(5x)$ unchanged. Then differentiate $(5x)$.

$$Đ\{\sin(5x)\} = \cos(5x) * Đ(5x)$$
$$= \cos(5x) * (5)$$
$$= 5\cos(5x).$$

SELF ASSESSMENT EXERCISE
Using the chain rule, differentiate y = $\cos 2(x^3)$.

In the case of minimization, we can assume, for example, that firms choose input bundles to minimize the cost of producing any given output; an analysis of the problem of minimizing the cost of achieving a certain payoff greatly facilitates the study of a payoff-maximizing consumer.

SELF ASSESSMENT EXERCISE
Explain briefly, the concept of optimization.

3.2 SOLVING OPTIMIZATION USING LAGRANGIAN MULLTIPLIER

We are going to consider the constraint on consumption before introducing income.
First, we want to look at the optimization of a single variable without constraint.
For example, given TU = $25x + 5x^2 - 2/3x^3 = 0$, calculate the saturation rate of consumption at the point of diminishing marginal utility.
In order to solve this problem, we differentiate the total utility equation, as the differentiation of total utility gives the marginal utility.

$$MU = dTU/dx = 25 + 10x - 6/3x^2 = 0$$
$$MU = 25 + 10x - 2x^2 = 0.$$

Since the marginal utility defines the slope of the total utility, and the slope of a function is zero at its maximum or minimum point, we set MU = 0.
Similarly, the marginal utility of TU = 10x4 + 17x2 + 16 = 0 is the differentiation of the total utility function.

MU = dTU/dx = $40x^3$ + 28[illegible]x = 0.

Now, let us consider multiple commodities. Here, we consider not only x_1, but $x_2 \ldots x_n$. i.e., TU = $f(x_1, x_2, \ldots x_n)$.
In order for us to find the marginal utility of a function similar to the one above, we need to differentiate partially and hold other x constant.

Example: Determine the maximum or the minimum level of satisfaction from the following two commodities. TU = $10x_1 + 1.5x_1x_2^2 + 2x_1^2x_2 + 5x_2$.
For us to solve this problem, we need to solve for the marginal utility of each x while holding others constant.

$$MUx_1 = dTU/dx_1 = 10 + 1.5x_2^2 + 4x_1x_2 = 0.$$
$$MUx_2 = dTU/dx_2 = 3x_1x_2 + 2x_1^2 + 5 = 0.$$

Now, let us determine the utility maximizing combination subject to income constraint.
Here, price will be introduced, thus our equation looks like this:

$$P_1x_1 + p_2x_2 + p_3x_3 \ldots + p_nx_n = Y.$$

Where P_1 represents the price for good x_1, and Y represents the income.
For us to solve this kind of problem, we will introduce the Joseph Lagrange multiplier (λ).

Given $Z = f(x_1, x_2, x_3 \ldots x_n) + \lambda(1 - p_1x_1 - p_2x_2 - p_3x_3 \ldots - p_nx_n)$, we proceed to differentiate the function partially.

$$\partial Z/\partial x_1 = \partial TU/\partial x_1 - \lambda p_1 = 0 \qquad (1)$$
$$\partial Z/\partial x_2 = \partial TU/\partial x_2 - \lambda p_2 = 0 \qquad (2)$$
$$\partial Z/\partial x_3 = \partial TU/\partial x_3 - \lambda p_3 = 0 \qquad (3)$$
$$\partial Z/\partial x_n = \partial TU/\partial x_n - \lambda p_n = 0 \qquad (4)$$
$$\partial Z/\partial \lambda = 1 - p_1x_1 - p_2x_2 - p_3x_3 \ldots - p_nx_n = 0 (5).$$

We have successfully solved equation 1 to 5 simultaneously in order to determine the consumption level of the commodities that would maximize the total utility (TU) function.

From equation (1), $\partial TU/\partial x_1 - \lambda p_1 = 0$

$$\partial TU/\partial x_1 = \lambda p_1$$
$$\lambda = \partial TU/\partial x_1 / p_1.$$
$$\lambda = MUx_1 / p_1$$

Note, $\partial TU/\partial x_1 = MUx_1$, while P_1 is the price for x_1 good.
Similar condition applies to all other equations simultaneously.

For equation (2), $\partial TU/\partial x_2 - \lambda p_2 = 0$

$$\partial TU/\partial x_2 = \lambda p_2$$
$$\lambda = \partial TU/\partial x_2 / p_2.$$
$$\lambda = MUx_2 / p_2$$

For equation (3), $\partial TU/\partial x_3 - \lambda p_3 = 0$

$$\partial TU/\partial x_3 = \lambda p_3$$
$$\lambda = \partial TU/\partial x_3 / p_3$$
$$\lambda = MUx_3 / p_3.$$

•

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For equation (4) $\partial TU/\partial x_n - \lambda p_n = 0$

$$\partial TU/\partial x_n = \lambda p_n$$
$$\lambda = \partial TU/\partial x_n / p_n$$
$$\lambda = MUx_n / p_n.$$

In mathematics, the three dots mean we solve until we get to the last equation which is equation (4) in our case.

In summary, $\lambda = \partial TU/\partial x_1 / p_1 = \partial TU/\partial x_2 / p_2 = \partial TU/\partial x_3 / p_3 = \ldots = \partial TU/\partial x_n / p_n$.

Now, since we have the value for our x_2, we can continue by imputing this value which is our equation (5) into equation (4) in order to get the actual value for x_1.

From equation (4): $x_1 = 4.8 + 0.4x_2$

$$x_1 = 4.8 + 0.4(8.69)$$
$$x_1 = 4.8 + 3.5$$
$$x_1 = 4.8 + 3.5$$
$$x_1 = 8.3.$$

This means that for the consumer to maximize his utility giving the price of good $x_1 = 2$ and good $x_2 = 5$ and income = 60, he must consume 8.3 quantity of good x_1 and 8.7 unit of good x_2.

SELF ASSESSMENT EXERCISE

Maximize $U = 10x_1^2 + 15x_2^4 - 21x_1^2x_2$

Subject to: income = 100, $px_1 = 4$, $px_2 = 3$.

3.4 SOLVING OPTIMIZATION USING MATRIX

Matrix can also be used to solve optimization problem.

Let us consider the same problem of maximizing $TU = 12x_1 + 18x_2 - 0.5x_1^2 - 0.5x_2^2$

Subject to the income constraint (Y) = 60, $px_1 = 2$, and $px_2 = 5$.

Solution:

For us to solve this equation, we will also introduce the Lagrange multiplier.

$$TU = 12x_1 + 18x_2 - 0.5x_1^2 - 0.5x_2^2 + \lambda(60 - 2x_1 - 5x_2)$$
$$TU = 12x_1 + 18x_2 - 0.5x_1^2 - 0.5x_2^2 + \lambda 60 - \lambda 2x_1 - \lambda 5x_2$$

Here, after differentiating the total utility equation partially with respect to the particular commodity, what we need to do next is simply to rearrange it in the following way.

$$\partial TU/\partial x_1 = 12 - x_1 - 2\lambda = 0$$
$$x_1 + 2\lambda = 12 \qquad (1)$$

$$\partial TU/\partial x_2 = 18 - x_2 - 5\lambda = 0$$
$$x_2 + 5\lambda = 18 \qquad (2)$$

$$\partial TU/\partial \lambda = 60 - 2x_1 - 5x_2 = 0$$
$$2x_1 + 5x_2 = 60 \qquad (3)$$

We can arrange the above equations into the matrix box using the Crammer's rule.

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 5 \\ 2 & 5 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \lambda \end{pmatrix} = \begin{pmatrix} 12 \\ 18 \\ 60 \end{pmatrix}$$

1.0 INTRODUCTION

The term linear programming consists of two words, linear and programming. Linear programming considers only linear relationship between two or more variables. By linear relationship, we mean that relations between the variables can be represented by straight lines. Programming means planning or decision-making in a systematic way. Linear programming is the technique for maximizing or minimizing a linear function of several variables such as output or cost. Linear programming can also be referred to as optimization of an outcome based on some set of constraints using a linear mathematical model. LP involves linear function of two or more variables which are to be optimized subject to a set of linear constraints at least one of which must be expressed as inequality. The sub-unit one focuses on the introduction to LP, while sub-unit two andthreefocuses on the assumptions, merits and demerits as well as the LP calculation using simplex algorithm.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- Understand the concept of linear programming
- State the assumptions, merits and demerits of linear programming
- Solve linear programming problem using simplex algorithm.

3.0 MAIN CONTENT

3.1 INTRODUCTION TO LINEAR PROGRAMMING

The linear programming problem is that of choosing non-negative values of certain variables so as to maximize or minimize a given linear function subject to a given set of linear inequality constraints. It can also be referred to as the use of linear mathematical relations to plan production activities.

Linear programming is a resource allocation tool in production economics. Put differently, linear programming is a tool of analysis which yields the optimum solution for the linear objective function subject to the constraints in the form of linear inequalities. This is to say that linear programming aims at the maximization or minimization of an objective, subject to a constraint. For instance, a company may want to determine the quantity of good x and y to be produced in order to minimize cost

	Zj	19,200	**-160.8**	240	0	-7.2	
	Pj - Zj	-	**360.8**	0	0	7.2	
Stage 3							
200	X1	60.2	1	0	0.05	-0.02	
240	X2	39.7	0	1	-0.03	0.04	
	Zj	21568	200	240	2.8	5.6	
	Pj - Zj	-	0	0	-2.8	-5.6	

Source: Authors Computation.
Note: stage three (3) maximizes the equation.

The **bolded** row and column are both our pivot row and column, while the pivot element is acquired by observing the highest value in row Pj – Zj and the lowest values in column ϴ. The pivot value is the value in block bracket (**[30]**) in stage 1.
Since the pivot element belong to column X_2 and S_2 row, we replace S_2 with X_2 in stage 2, and the same principle applies to stage 3.

Stage 2: Here, we make x_2 the subject of the formula in equation 2

$$20x_1 + 30x_2 + 0s_1 + s_2 = 2400$$

$$30x_2 = 2400 - 20x_1 - 0s_1 - s_2$$

Dividing through by 30 gives:

$$x_2 = 80 - 0.67x_1 - 0s_1 - 0.03s_2 \qquad (3)$$

Equation (3) above is imputed in the X_2 row in stage 2.
In order to get our S_1 row value, we impute equation (3) into (1) which gives:

$$30x_1 + 15(80 - 0.67x_1 - 0s_1 - 0.03s_2) + s_1 + 0s_2 = 2400$$

$$30x_1 + 1200 – 10.05x_1 – 0s_1 – 0.45s_2 + s_1 + 0s_2 = 2400$$

$$19.95x_1 + s_1 – 0.45s_2 = 1200 \qquad (4)$$

Making s_1 the subject of the formula:

$$s_1 = 1200 – 19.95x_1 + 0.45s_2 \qquad (5)$$

Stage 3: Here, the pivot element is identified, and it belongs to the tow S1, thus we will replace S_1 with X_1 in stage3. Here, we will make x_1 the subject of the formula in (4):

$$19.95x_1 + s_1 – 0.45s_2 = 1200$$

$$x_1 = 60.2 – 0.05s_1 + 0.02s_2 \qquad (6)$$

Equation (6) above is imputed in the X_1 row in stage 3.
Imputing (6) into (3) gives:

$$x_2 = 80 - 0.67x_1 - 0s_1 - 0.03s_2$$

By rearranging the above equation gives:

$$0.67x_1 + x_2 + 0s_1 + 0.03s_2 = 80$$

Thus our model gives:

$$0.67(60.2 – 0.05s_1 + 0.02s_2) + x_2 + 0.03s_2 = 80$$