

MATH 21

College Algebra 1

Lecture Notes

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Rules for Multiplying and Dividing Rational Expressions 1. $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} = \frac{ac}{bd}$. 2. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$.

Perform the indicated operations and simplify.

1. $\frac{2}{3} \cdot \frac{4}{5}$

2. $\frac{12}{9} \cdot \frac{4}{18}$

3. $\frac{x^2}{y} \cdot \frac{y^2}{x}$

4. $\frac{x+2}{x-2} \cdot \frac{x^2-6x+9}{x^2-2x}$

5. $\frac{5a^2+20a}{a^3-2a^2} \cdot \frac{a^2-a-12}{a^2-16}$

6. $\frac{2}{3} \div \frac{4}{5}$

7. $\frac{x-6}{x^2-2x+1} \div \frac{x^2-3x-18}{x-1}$

8. $\frac{9n^2-12n+4}{n^2-4n-32} \div \frac{3n^3-2n^2}{n^2+4n}$

9. $\frac{x^2-3xy+2y^2}{x^2y-xy^2} \div \frac{x-2y}{x^3-xy^2}$

10. $\frac{x^4-16}{x+2} \div \frac{x^2+4}{x+5}$

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Steps for Solving Word Problems**Step 1.** Carefully read the problem.**Step 2.** Define variables.**Step 3.** Set up equations.**Step 4.** Solve the equation.**Step 5.** Check your answer and write the answer as a sentence.

$$\text{Distance} = \text{Rate} \cdot \text{Time}$$

7. Solve $\frac{1}{A} = \frac{1}{B} + \frac{1}{C}$ for B .

8. Solve $\frac{x+3}{z-2} = \frac{2}{y}$ for x .

9. Harry flies his broom 96 miles in the same time it takes Luna to fly her thestral 56 miles. If the Harry flew 20 miles per hour faster than Luna, find their rates.

10. Luke can mow a lawn in 3 hours. Duke can mow the lawn in 2 hours. How long will it take them to mow the lawn if they are working together?

11. My wife (Jenny) can clean our cluttered living room in 30 minutes. My son (Andrew) can clutter up the living room in 40 minutes. If Jenny starts cleaning the cluttered living room while Andrew is busy cluttering it back up, how long will it take for the room to get cleaned?

12. It takes Nemo three times as long to clean a sea cucumber as it takes Marlin. If it takes them 1 hour to clean a sea cucumber when working together, then how long will it take Nemo working alone?

13. It takes Lester 1 hour longer than it takes Chester to paint a picture. If they paint 5 pictures in 6 hours, how many were painted by Lester?

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Properties of Exponents

1. $b^m \cdot b^n = b^{m+n}$

2. $(b^m)^n = b^{mn}$

3. $(ab)^n = a^n b^n$

4. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

5. $\frac{b^n}{b^m} = b^{n-m}$

6. $b^0 = 1$

7. $b^{-n} = \frac{1}{b^n}$

Simplify each of the following expressions.

1. $6^4 \cdot 6^{-2}$

10. $\frac{x^{-3}}{x^{-5}}$

2. $(3^{-2})^{-3}$

11. $(-4x^{-1}y^2)(6x^3y^{-4})$

3. $(5 \cdot 3^{-2})^2$

12. $\frac{108a^{-5}b^{-4}}{9a^{-2}b}$

4. $\left(\frac{6^{-1}}{4^{-2}}\right)^{-3}$

5. $\frac{2^{-3}}{2^{-5}}$

13. $\frac{4ab^2}{6a^3b^5}$

6. $x^{-1} \cdot x^3$

7. $(x^4)^5$

14. $2^{-1} - 3^{-1}$

8. $(a^3 \cdot b^{-1})^4$

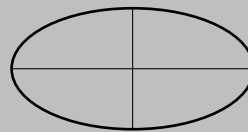
15. $a^{-2} + a^{-1}b^{-2}$

9. $\left(\frac{x^2}{y^3}\right)^2$

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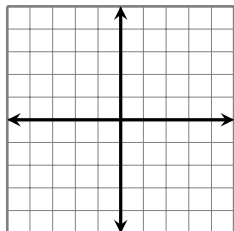
Ellipses

An **ellipse** that is centered at (h, k) is defined by an equation of the form $A(x - h)^2 + B(y - k)^2 = C$ ($A, B, C > 0$). The longer line segment is called the **major axis**, and the shorter is called the **minor axis**.

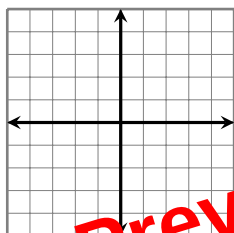


Graph each ellipse.

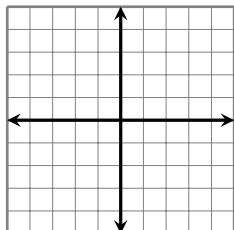
1. $9x^2 + 4y^2 = 36$



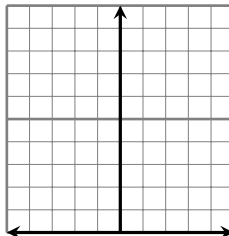
2. $x^2 + 4y^2 = 16$



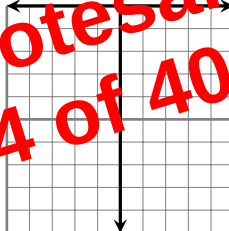
3. $9x^2 + 2y^2 = 36$



4. $9x^2 - 36x + 4y^2 - 24y + 36 = 0$



5. $4x^2 - 16x + y^2 + 8x + 16 = 0$



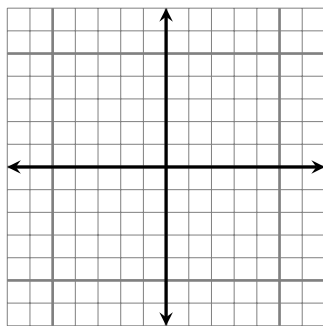
6. How are the graphs of $x^2 + 4y^2 = 36$ and $4x^2 + y^2 = 36$ related?

Solutions of Linear Systems

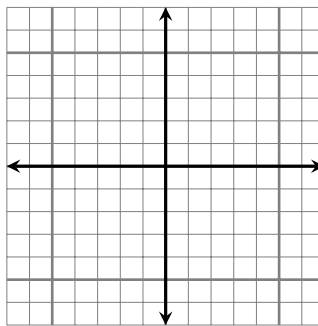
A **linear equation** in two variables has the form $Ax + By = C$. When we solve a system of equations, we find all ordered pairs that satisfy each of the equations in the system. This is equivalent to saying that the solution of a system of equations is the set of all points that are shared by the graphs of the equations.

Solve the systems of equations by graphing.

1.
$$\begin{aligned} 2x + y &= 4 \\ x - y &= -1 \end{aligned}$$



2.
$$\begin{aligned} y &= 2x - 3 \\ y &= 3x - 4 \end{aligned}$$

**The Substitution Method**

1. Solve one equation for one variable in terms of the other variable.
2. Substitute in place of that variable in the other equation.
3. Solve that equation.
4. Plug that solution into the other equation to get the value of the other variable.

Solve each system of equations.

3.
$$\begin{aligned} x - y &= -4 \\ 3x + 2y &= 8 \end{aligned}$$

4.
$$\begin{aligned} 9x - 2y &= -38 \\ 5x + y &= 0 \end{aligned}$$

5.
$$\begin{aligned} 5x - y &= 9 \\ 10x - 2y &= -6 \end{aligned}$$

6.
$$\begin{aligned} -x + 4y &= -22 \\ -4x + 7y &= -41 \end{aligned}$$