

Q.P. Code : 4827

(3 Hours)

[Total Marks : 80

- N.B.: (1) Question No.1 is compulsory.
 (2) Attempt any **three** from the remaining **six** questions.
 (3) **Figures** to the right indicate **full marks**.

Q1a Find Laplace Transform of $\frac{\sin t}{t}$ [20]

b Prove that $f(z) = \sinh z$ is analytic and find its derivative

c Find Fourier Series for $f(x) = 9 - x^2$ over $(-3, 3)$

d Find $Z\{f(k) * g(k)\}$ if $f(k) = \frac{1}{3^k}$, $g(k) = \frac{1}{5^k}$

Q2 a Prove that $\vec{F} = ye^{xy} \cos z i + xe^{xy} \cos z j - e^{xy} \sin z k$ is Irrotational. Find Scalar Potential for $\vec{F}$

Hence evaluate $\int_C \vec{F} \cdot d\vec{r}$ along the curve C joining the points $(0, 0, 0)$ and $(-1, 2, \pi)$ [6]

b Find the Fourier series for $f(x) = \frac{\pi - x}{2}$; $0 \leq x \leq 2\pi$. [6]

c Find Inverse Laplace Transform of i) $\frac{s+29}{(s+4)(s^2+9)}$ ii) $\frac{e^{-2s}}{s^2+8s+25}$ [8]

Q3 a Find the Analytic function $f(z) = u + iv$ if $u + v = \frac{x}{x^2 + y^2}$ [6]

b Find Inverse Z transform of $\frac{1}{(z-1/2)(z-1/3)}$, $1/3 < |z| < 1/2$ [6]

c Solve the Differential Equation $\frac{d^2 y}{dt^2} + y = t$, $y(0) = 1$, $y'(0) = 0$, using Laplace Transform [8]

Q4 a Find the Orthogonal Trajectory of $3x^2 y - y^3 = k$ [6]

b Using Green's theorem evaluate $\int_C (xy + y^2) dx + x^2 dy$, C is closed path formed by $y = x$, $y = x^2$ [6]

c Find Fourier Integral of $f(x) = \begin{cases} \sin x & 0 \leq x \leq \pi \\ 0 & x > \pi \end{cases}$. Hence show that $\int_0^\infty \frac{\cos(\lambda\pi/2)}{1-\lambda^2} d\lambda = \frac{\pi}{2}$ [8]