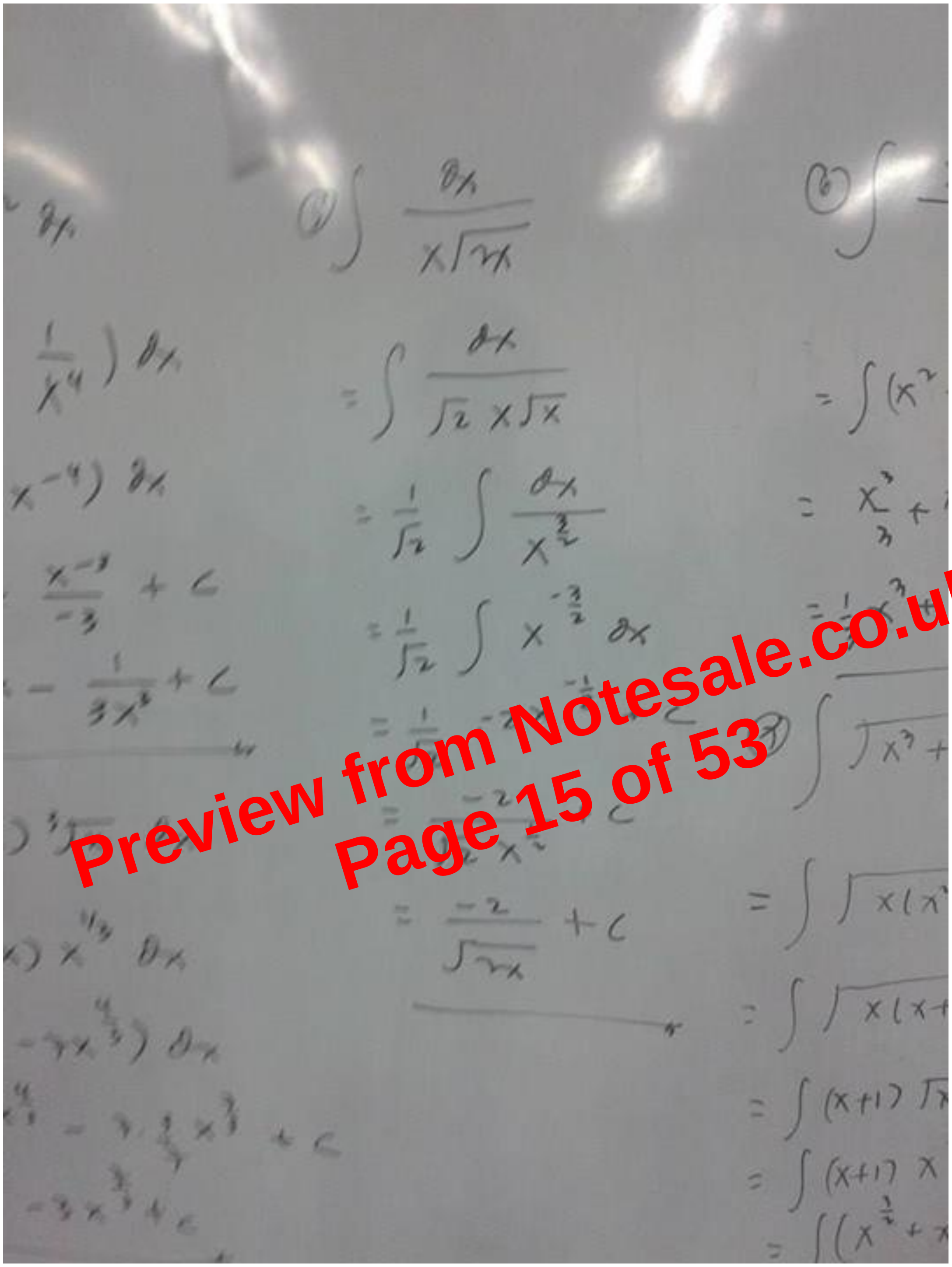


$$dx$$

$$\left(\frac{1}{x^4}\right) dx$$

$$x^{-4}) \, dx$$

$$\frac{x^{-3}}{-3} + C$$

$$- \frac{1}{3x^3} + C$$

$$) \sqrt[3]{x}$$

$$x) \, x^{1/3} \, dx$$

$$- 2x^{4/3}) \, dx$$

$$x^{4/3} - 3 \cdot \frac{3}{7} x^{7/3} + C$$

$$- 3x^{7/3} + c$$

$$⑥ \int \frac{dx}{x\sqrt{2x}}$$

$$= \int \frac{dx}{\sqrt{2}\, x\sqrt{x}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{x^{\frac{3}{2}}}$$

$$= \frac{1}{\sqrt{2}} \int x^{-\frac{3}{2}} \, dx$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + C$$

$$= \frac{-2}{\sqrt{2}\, x^{\frac{1}{2}}} + C$$

$$= \frac{-2}{\sqrt{2x}} + C$$

$$⑦ \int$$

$$= \int (x^2$$

$$= \frac{x^3}{3} +$$

$$= \frac{1}{3} x^3 +$$

$$⑧ \int \sqrt{x^3 +}$$

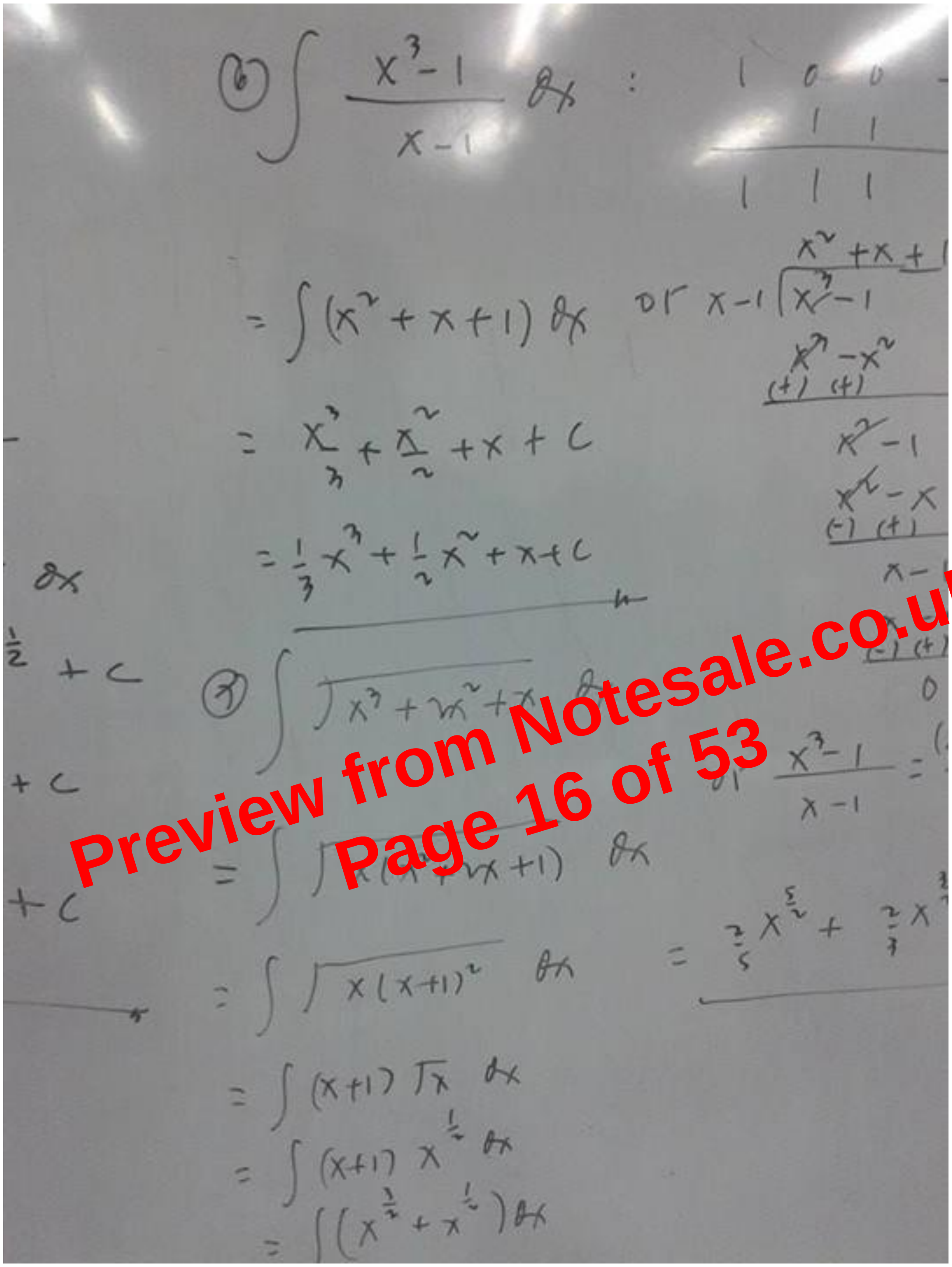
$$= \int \sqrt{x(x^2}$$

$$= \int \sqrt{x(x+}$$

$$= \int (x+1)\sqrt{x}$$

$$= \int (x+1)\, x^{\frac{1}{2}}$$

$$= \int (x^{\frac{3}{2}} + x$$



⑥ $\int \frac{x^3-1}{x-1}\,dx$:

1	0	0
	1	1
1	1	1

$= \int (x^2+x+1)\,dx$ or $x-1 \overline{\big)\,x^3-1}$ with quotient x^2+x+1

x^3-x^2

(+) (+)

x^2-1

x^2-x

(-) (+)

$x-1$

$x-1$

(-) (+)

0

$= \frac{x^3}{3}+\frac{x^2}{2}+x+C$

$= \frac{1}{3}x^3+\frac{1}{2}x^2+x+C$

dx

$\frac{1}{2}+C$

$+C$

$+C$

⑦ $\int \sqrt{x^3+2x^2+x}\,dx$

or $\frac{x^3-1}{x-1} =$ (

$= \int \sqrt{x(x^2+2x+1)}\,dx$

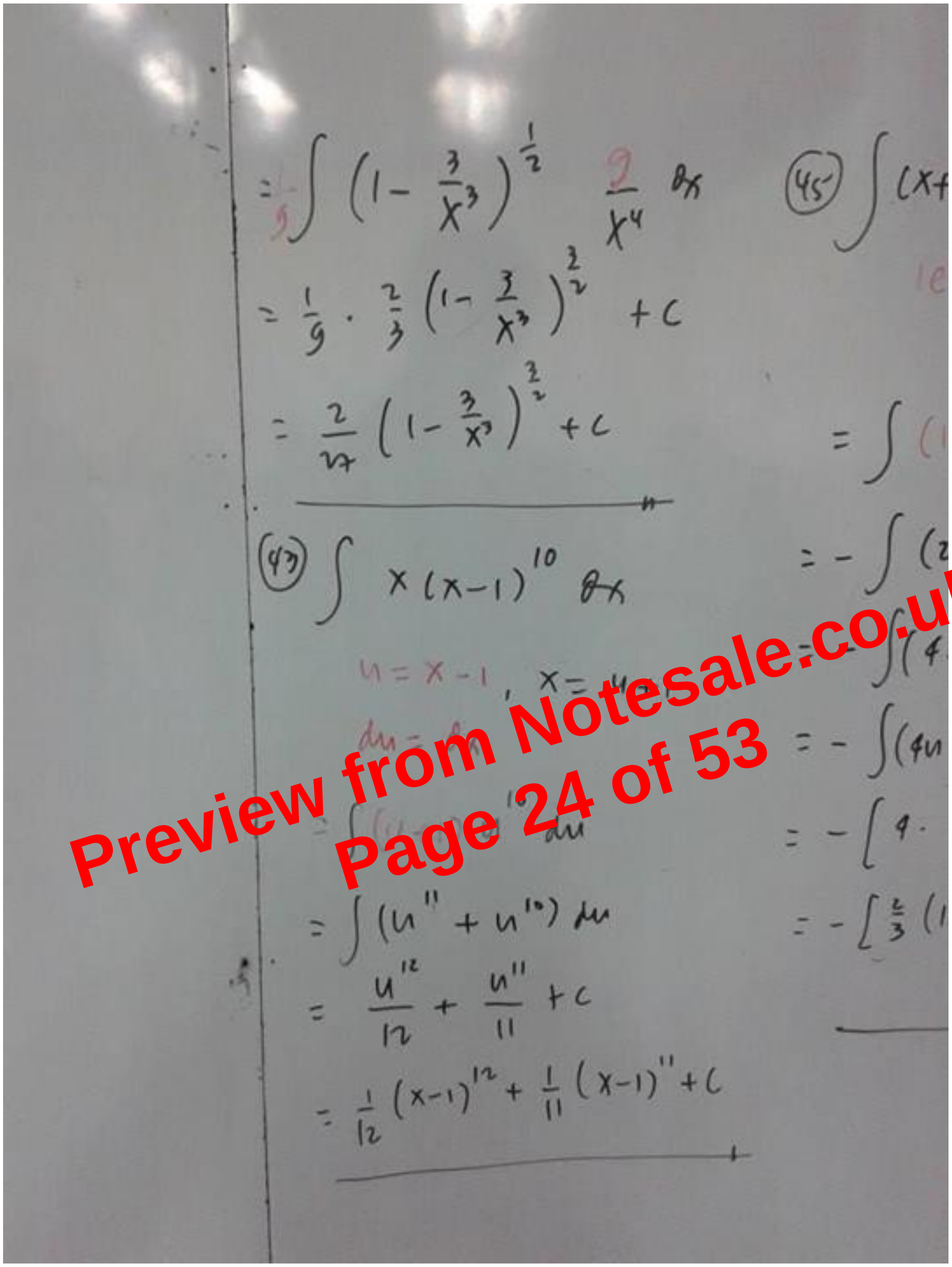
$= \int \sqrt{x(x+1)^2}\,dx$

$= \frac{2}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{3}$

$= \int (x+1)\sqrt{x}\,dx$

$= \int (x+1)\,x^{\frac{1}{2}}\,dx$

$= \int (x^{\frac{3}{2}}+x^{\frac{1}{2}})\,dx$



$= \frac{1}{9}\int \left(1-\frac{3}{x^3}\right)^{\frac{1}{2}} \frac{9}{x^4}\,dx$

$= \frac{1}{9}\cdot\frac{2}{3}\left(1-\frac{3}{x^3}\right)^{\frac{3}{2}} + C$

$= \frac{2}{27}\left(1-\frac{3}{x^3}\right)^{\frac{3}{2}} + C$

---

(47) $\int x(x-1)^{10}\,dx$

$u = x-1 \;,\; x = u+1$

$du = dx$

$= \int (u+1)u^{10}\,du$

$= \int (u^{11} + u^{10})\,du$

$= \frac{u^{12}}{12} + \frac{u^{11}}{11} + C$

$= \frac{1}{12}(x-1)^{12} + \frac{1}{11}(x-1)^{11} + C$

---

(45) $\int (x+$

le

$= \int ($

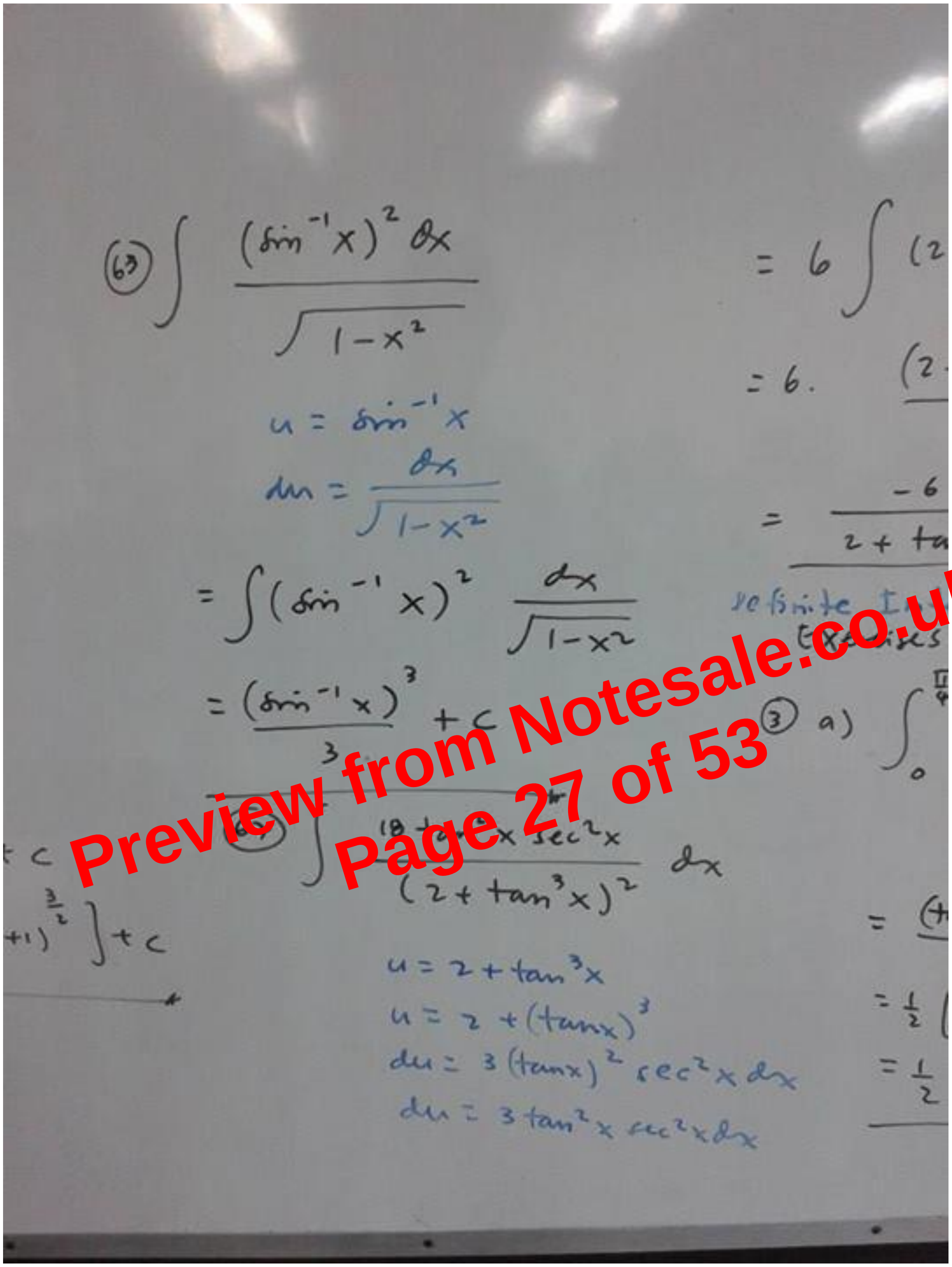
$= -\int (2$

$= \int (4$

$= -\int (4u$

$= -[4\cdot$

$= -[\frac{2}{3}(1$



(63) $\int \frac{(\sin^{-1}x)^2 \, dx}{\sqrt{1-x^2}}$

$u = \sin^{-1}x$

$du = \frac{dx}{\sqrt{1-x^2}}$

$= \int (\sin^{-1}x)^2 \frac{dx}{\sqrt{1-x^2}}$

$= \frac{(\sin^{-1}x)^3}{3} + c$

(64) $\int \frac{18\tan^{2}x\sec^2x}{(2+\tan^3x)^2}\, dx$

$u = 2 + \tan^3x$

$u = 2 + (\tan x)^3$

$du = 3(\tan x)^2 \sec^2x \, dx$

$du = 3\tan^2x \sec^2x \, dx$

$+ c$

$+1)^{\frac{3}{2}} \Big] + c$

$= 6 \int (2$

$= 6. \; (2$

$= \frac{-6}{2 + ta}$

Definite In

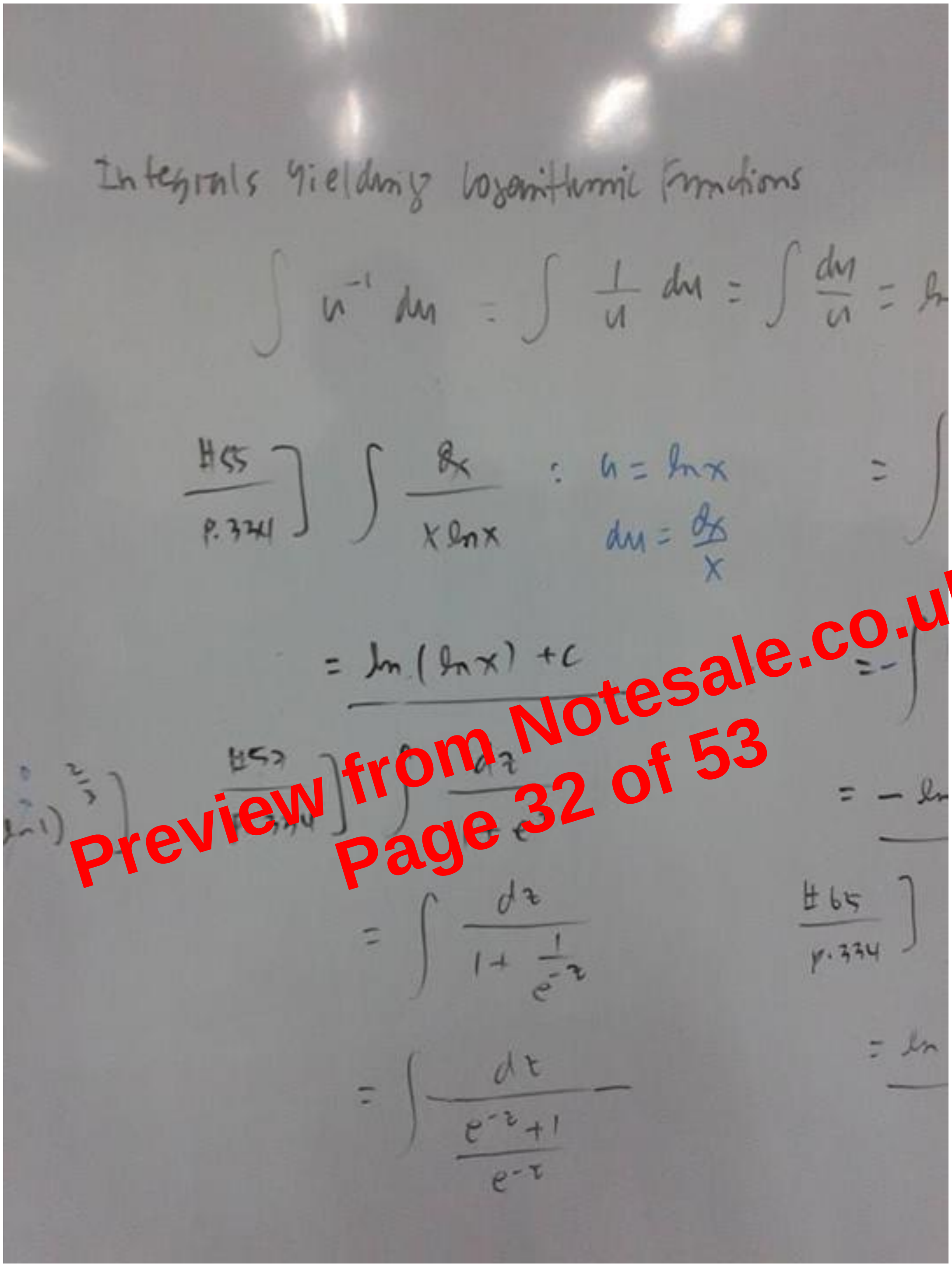
Exercises

③ a) $\int_0^{\frac{\pi}{4}}$

$= (+$

$= \frac{1}{2} ($

$= \frac{1}{2}$



Integrals Yielding Logarithmic Functions

$$\int u^{-1}\,du = \int \frac{1}{u}\,du = \int \frac{du}{u} = \ln$$

#55 / p.334] $\int \frac{dx}{x\ln x}$: $u = \ln x$, $du = \frac{dx}{x}$

$$= \ln(\ln x) + c$$

#57 / p.334] $\int \frac{dz}{1+e^{z}}$

$$= \int \frac{dz}{1+\frac{1}{e^{-z}}}$$

$$= \int \frac{dz}{\frac{e^{-z}+1}{e^{-z}}}$$

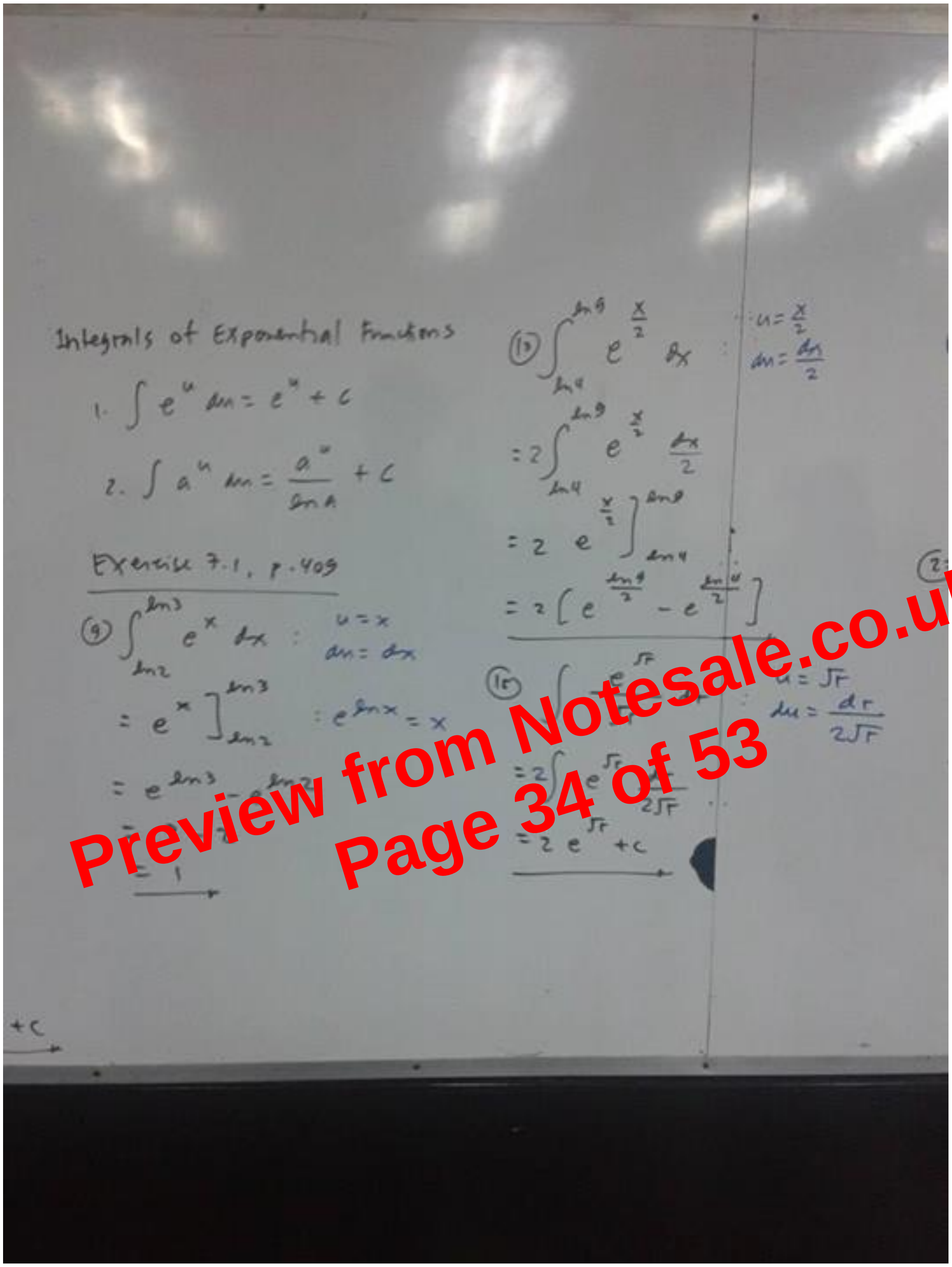
$= \int$

$= -\int$

$= -\ln$

#65 / p.334]

$= \ln$



Integrals of Exponential Functions

1. $\int e^u \, du = e^u + c$

2. $\int a^u \, du = \frac{a^u}{\ln a} + c$

Exercise 7.1, p.409

⑨ $\int_{\ln 2}^{\ln 3} e^x \, dx$: $u = x$, $du = dx$

$= e^x \Big]_{\ln 2}^{\ln 3}$: $e^{\ln x} = x$

$= e^{\ln 3} - e^{\ln 2}$

$= 3 - 2$

$= 1$

⑬ $\int_{\ln 4}^{\ln 9} e^{\frac{x}{2}} \, dx$: $u = \frac{x}{2}$, $du = \frac{dx}{2}$

$= 2\int_{\ln 4}^{\ln 9} e^{\frac{x}{2}} \frac{dx}{2}$

$= 2 \, e^{\frac{x}{2}} \Big]_{\ln 4}^{\ln 9}$

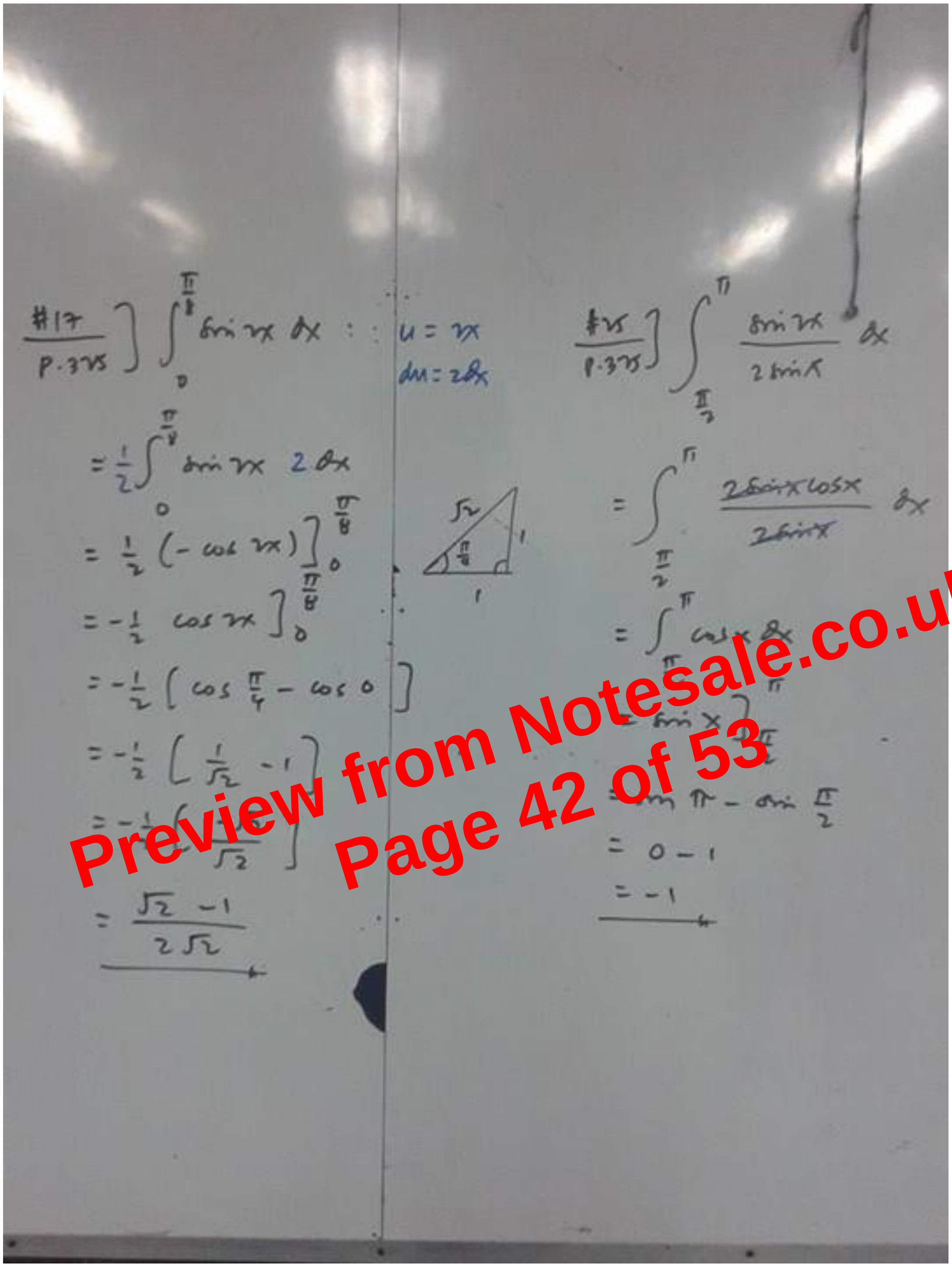
$= 2\left[e^{\frac{\ln 9}{2}} - e^{\frac{\ln 4}{2}}\right]$

⑮ $\int \frac{e^{\sqrt{r}}}{\sqrt{r}} \, dr$: $u = \sqrt{r}$, $du = \frac{dr}{2\sqrt{r}}$

$= 2\int e^{\sqrt{r}} \frac{dr}{2\sqrt{r}}$

$= 2 e^{\sqrt{r}} + c$

+c



$\frac{\#17}{p.325}\Big]$ $\int_0^{\frac{\pi}{8}} \sin 2x \, dx$: : $u = 2x$

$du = 2dx$

$= \frac{1}{2}\int_0^{\frac{\pi}{8}} \sin 2x \; 2\,dx$

$= \frac{1}{2}\left(-\cos 2x\right)\Big]_0^{\frac{\pi}{8}}$

$= -\frac{1}{2} \cos 2x \Big]_0^{\frac{\pi}{8}}$

$= -\frac{1}{2}\left[\cos \frac{\pi}{4} - \cos 0\right]$

$= -\frac{1}{2}\left[\frac{1}{\sqrt{2}} - 1\right]$

$= -\frac{1}{2}\left[\frac{[illegible]}{\sqrt{2}}\right]$

$= \frac{\sqrt{2} - 1}{2\sqrt{2}}$

$\frac{\#25}{p.325}\Big]$ $\int_{\frac{\pi}{2}}^{\pi} \frac{\sin 2x}{2\sin x}\, dx$

$= \int_{\frac{\pi}{2}}^{\pi} \frac{2\sin x \cos x}{\cancel{2\sin x}}\, dx$

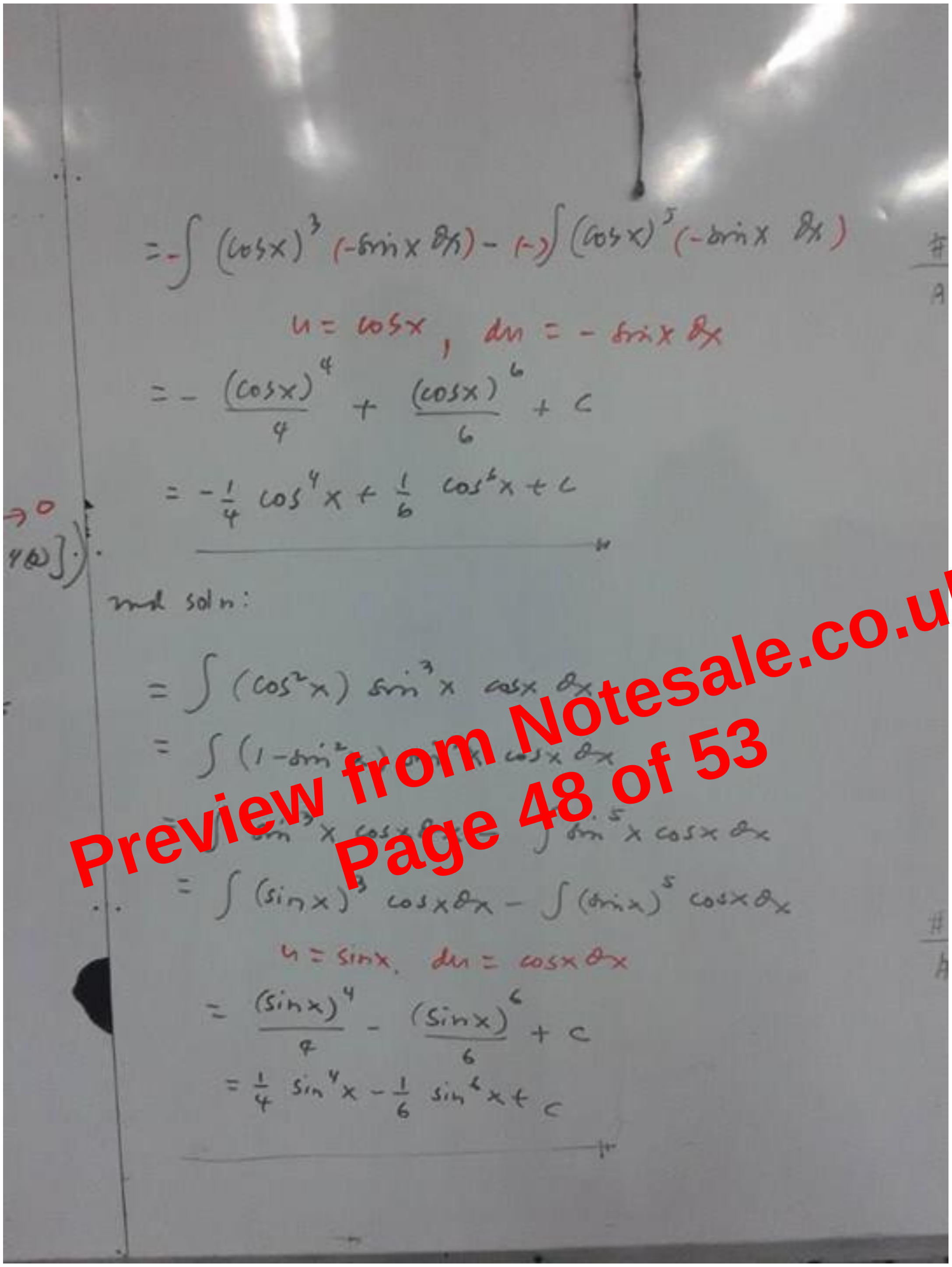
$= \int_{\frac{\pi}{2}}^{\pi} \cos x \, dx$

$= \sin x \Big]_{\frac{\pi}{2}}^{\pi}$

$= \sin \pi - \sin \frac{\pi}{2}$

$= 0 - 1$

$= -1$



$$= -\int (\cos x)^3 (-\sin x\, dx) - (-)\int (\cos x)^5 (-\sin x\, dx)$$

$$u = \cos x, \quad du = -\sin x\, dx$$

$$= -\frac{(\cos x)^4}{4} + \frac{(\cos x)^6}{6} + c$$

$$= -\frac{1}{4}\cos^4 x + \frac{1}{6}\cos^6 x + c$$

2nd soln:

$$= \int (\cos^2 x) \sin^3 x \cos x\, dx$$

$$= \int (1 - \sin^2 x) \sin^3 x \cos x\, dx$$

$$= \int \sin^3 x \cos x\, dx - \int \sin^5 x \cos x\, dx$$

$$= \int (\sin x)^3 \cos x\, dx - \int (\sin x)^5 \cos x\, dx$$

$$u = \sin x, \quad du = \cos x\, dx$$

$$= \frac{(\sin x)^4}{4} - \frac{(\sin x)^6}{6} + c$$

$$= \frac{1}{4}\sin^4 x - \frac{1}{6}\sin^6 x + c$$