Department of Mathematics, UMIST

MATHEMATICAL FORMULA TABLES

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$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$

$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$

$2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$2\cos A\sin B = \sin(A+B) - \sin(A-B)$

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$-2\sin A\sin B = \cos(A+B) - \cos(A-B)$

$a\sin x + b\cos x = R\sin(x+\phi)$, where $R = \sqrt{a^2+b^2}$ and $\cos\phi = a/R$, $\sin\phi = b/R$.

If $t = \tan\frac{1}{2}x$ then $\sin x = \frac{2t}{1+t^2}$, $\cos x = \frac{1-t^2}{1+t^2}$

$\cos x = \frac{1}{2}\left(e^{ix} + e^{-ix}\right)$; $\sin x = \frac{1}{2i}\left(e^{ix} - e^{-ix}\right)$

$e^{ix} = \cos x + i\sin x$; $e^{-ix} = \cos x - i\sin x$

Z TRANSFORMS

$$Z\{f(t)\} = \tilde{f}(z) = \sum_{k=0}^{\infty} f(kT)z^{-k}$$

function	transform
$\delta_{t,nT}$	$z^{-n}(n \geq 0)$
e^{-at}	$\dfrac{z}{z - e^{-aT}}$
te^{-at}	$\dfrac{Tze^{-aT}}{(z - e^{-aT})^2}$
t^2e^{-at}	$\dfrac{T^2ze^{-aT}(z + e^{-aT})}{(z - e^{-aT})^3}$
$\sinh at$	$\dfrac{z \sinh aT}{z^2 - 2z \cosh aT + 1}$
$\cosh at$	$\dfrac{z(z - \cosh aT)}{z^2 - 2z \cosh aT + 1}$
$e^{-at} \sin \omega t$	$\dfrac{ze^{-aT} \sin \omega T}{z^2 - 2ze^{-aT} \cos \omega T + e^{-2aT}}$
$e^{-at} \cos \omega t$	$\dfrac{z(z - e^{-aT} \cos \omega T)}{z^2 - 2ze^{-aT} \cos \omega T + e^{-2aT}}$
$te^{-at} \sin \omega t$	$\dfrac{Tze^{-aT}(z^2 - e^{-2aT}) \sin \omega T}{(z^2 - 2ze^{-aT} \cos \omega T + e^{-2aT})^2}$
$te^{-at} \cos \omega t$	$\dfrac{Tze^{-aT}(z^2 \cos \omega T - 2ze^{-aT} + e^{-2aT} \cos \omega T)}{(z^2 - 2ze^{-aT} \cos \omega T + e^{-2aT})^2}$

Shift Theorem

$$Z\{f(t + nT)\} = z^n \tilde{f}(z) - \sum_{k=0}^{n-1} z^{n-k} f(kT) \quad (n > 0)$$

Initial value theorem

$$f(0) = \lim_{z \to \infty} \tilde{f}(z)$$

iii. The following moments of inertia are for uniform bodies about the axes stated:

rod, length ℓ, through mid-point, perpendicular to rod	$\frac{1}{12}m\ell^2$
hoop, radius r, through centre, perpendicular to hoop	mr^2
disc, radius r, through centre, perpendicular to disc	$\frac{1}{2}mr^2$
sphere, radius r, diameter	$\frac{2}{5}mr^2$

Work done

$$W = \int_{t_A}^{t_B} \mathbf{F}.\frac{d\mathbf{r}}{dt}dt.$$

NORMAL DISTRIBUTION

The function tabulated is the cumulative distribution function of a standard $N(0,1)$ random variable, namely

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2} dt.$$

If X is distributed $N(0,1)$ then $\Phi(x) = Pr.(X \leq x)$.

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000