of the general power system load flow problem and describe its solution using four techniques: Newton-Raphson, Gauss-Seidel, Genetic Algorithm and Simulated Annealing. Also included are illustrative numerical results relating to the particular power system configuration analysed in the Workbook.

## 2. Formulation of the Load Flow Problem

Load flow studies are based on a nodal voltage analysis of a power system. As an example, consider the very simple system represented by the single-line diagram in Fig. 1. Here two generators (1 and 2) are interconnected by one transmission line and are separately connected to a load (3) by two other lines. If the phasor currents injected into the system are $\boldsymbol{I_1}$, $\boldsymbol{I_2}$, and $\boldsymbol{I_3}$, and the lines are modelled by simple series admittances, then it is possible to draw the equivalent circuit for one representative phase of the balanced three-phase system, as shown in Fig. 2.

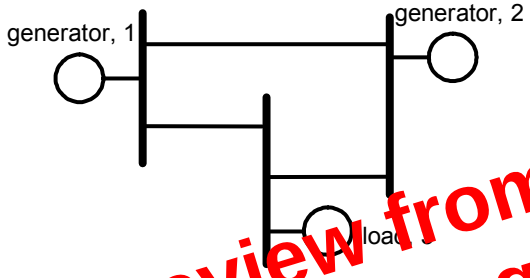


Fig. 1 Single-line diagram of a simple example power system

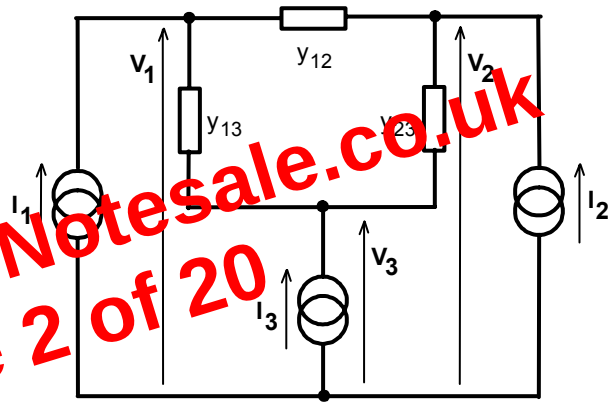


Fig. 2 Equivalent circuit for one phase of the system shown in Fig.1



For the circuit in Fig. 2 the nodal voltage equations can be written directly. For example, at node 1:

$$\boldsymbol{I_1} = (y_{12} + y_{13})\boldsymbol{V_1} - y_{12}\boldsymbol{V_2} - y_{13}\boldsymbol{V_3} \qquad (1)$$

In general, for a system with r nodes, then at node n:

$$\boldsymbol{I_n} = Y_{n1}\boldsymbol{V_1} + Y_{n2}\boldsymbol{V_2} + ..... + Y_{nn}\boldsymbol{V_n} + ..... + Y_{nr}\boldsymbol{V_r} = \sum_{k=1}^{r} Y_{nk}\boldsymbol{V_k} \qquad (2)$$

where: $Y_{nn}$ = sum of all admittances connected to node n

$Y_{nk}$ = - (sum of all admittances connected between nodes n and k) = $Y_{kn}$

$\boldsymbol{I_n}$ = current injected at node n

For the complete system of r nodes:

where the superscript $^*$ denotes the complex conjugate. Substituting from (2) with all complex variables written in polar form:

$$\boldsymbol{S}_n = \boldsymbol{V}_n \sum_{k=1}^{r} Y_{nk}^* \boldsymbol{V}_k^* = \sum_{k=1}^{r} |V_n||V_k||Y_{nk}| \angle\{\delta_n - \delta_k - \theta_{nk}\} \tag{6}$$

The power and reactive power inputs at node n are derived by taking the real and imaginary parts of the complex power:

$$P_n = \Re\{\boldsymbol{S}_n\} = \sum_{k=1}^{r} |V_n||V_k||Y_{nk}| \cos\{\delta_n - \delta_k - \theta_{nk}\} \tag{7}$$

$$Q_n = \Im\{\boldsymbol{S}_n\} = \sum_{k=1}^{r} |V_n||V_k||Y_{nk}| \sin\{\delta_n - \delta_k - \theta_{nk}\} \tag{8}$$

The load flow problem is to find values of voltage magnitude and phase angle, which, when substituted into (7) and (8), produce values of power and reactive power equal to the specified set values at that node, $P_{ns}$ and $Q_{ns}$.

The first step in the solution is to make initial estimates of all the variables: $|V_n^0|, \delta_n^0$ where the superscript $^0$ indicates the number of iterative cycles completed. Using these estimates, the power and reactive power input at each node can be calculated from (7) and (8). These values are compared with the specified values to give a power and reactive power error. For node n:



$$\Delta P_n^0 = P_{ns} - \sum_{k=1}^{r} |V_n^0||V_k^0||Y_{nk}| \cos\{\delta_n^0 - \delta_k^0 - \theta_{nk}\} \tag{9}$$

$$\Delta Q_n^0 = Q_{ns} - \sum_{k=1}^{r} |V_n^0||V_k^0||Y_{nk}| \sin\{\delta_n^0 - \delta_k^0 - \theta_{nk}\} \tag{10}$$

The power and reactive power errors at each node are related to the errors in the voltage magnitudes and phase angles, e.g. $\Delta|V_n|^0, \Delta\delta_n^0$, by the first order approximations:

$$\begin{bmatrix} \vdots \\ \Delta P_n^0 \\ \vdots \\ \hline \vdots \\ \Delta Q_n^0 \\ \vdots \end{bmatrix} = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots \\ \vdots & \frac{\partial P_n}{\partial \delta_{n-1}} & \frac{\partial P_n}{\partial \delta_n} & \frac{\partial P_n}{\partial \delta_{n+1}} & \cdots & \frac{\partial P_n}{\partial |V_{n-1}|} & \cdots & \frac{\partial P_n}{\partial |V_n|} & \frac{\partial P_n}{\partial |V_{n+1}|} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots & \cdots \\ \hline \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots & \cdots \\ \vdots & \frac{\partial Q_n}{\partial \delta_{n-1}} & \frac{\partial Q_n}{\partial \delta_n} & \frac{\partial Q_n}{\partial \delta_{n+1}} & \cdots & \frac{\partial Q_n}{\partial |V_{n-1}|} & \cdots & \frac{\partial Q_n}{\partial |V_n|} & \frac{\partial Q_n}{\partial |V_{n+1}|} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots & \cdots \end{bmatrix} \begin{bmatrix} \Delta\delta_{n-1}^0 \\ \Delta\delta_n^0 \\ \Delta\delta_{n+1}^0 \\ \hline \Delta|V_{n-1}|^0 \\ \Delta|V_n|^0 \\ \Delta|V_{n+1}|^0 \end{bmatrix} \tag{11}$$

The cycle of chromosome evaluation and breeding continues through many generations. Evaluation of the error function for each chromosome requires relatively simple calculations, so the genetic algorithm can continue through many generations until the error has fallen to acceptable levels. A typical variation of error with generation number is shown in Fig. 3. The genetic algorithm quickly reduces the error, but a very large number of generations is needed to bring the error close to zero. In the Excel Workbook, accelerated convergence is obtained after 100 generations, by redefining the search space so that it is centred on the best available solution at that stage and is reduced in size. The effect of this redefinition is apparent in Fig. 3, where the error reduction receives fresh impetus after the 100$^{th}$ generation.

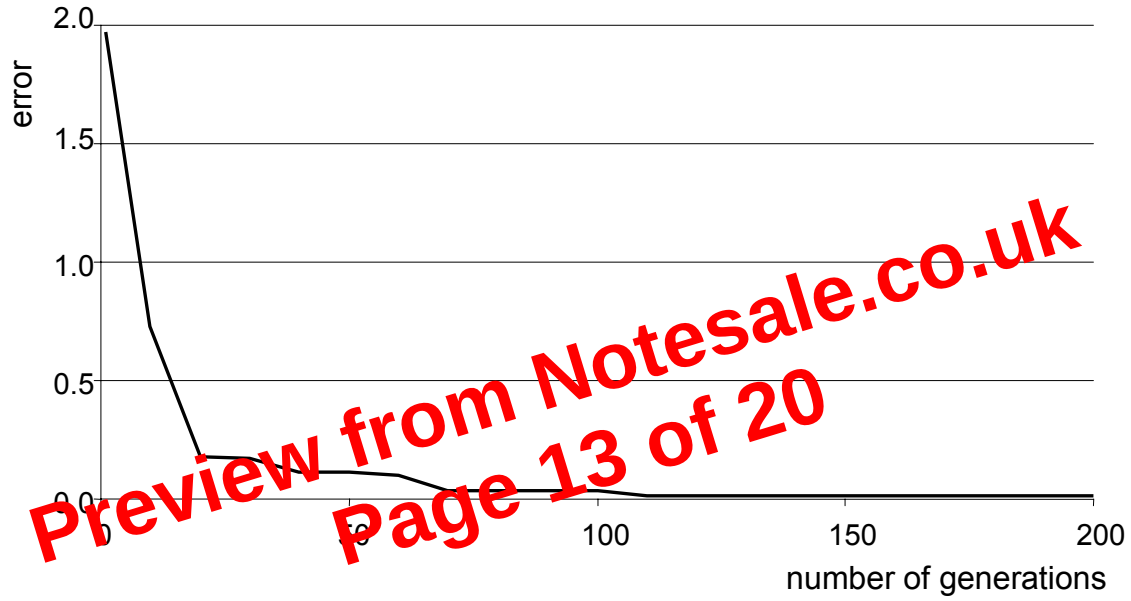




Fig. 3 Typical error associated with the best chromosome as a function of the number of generations

### 5.3 Simulated Annealing

Simulated annealing is a global search technique in which a randomly-generated potential solution, Y, to a problem is compared to an existing solution, X. The probability of Y being accepted for investigation depends on the proximity of Y to X and the extent to which the solution has been developed, as represented by a 'temperature' parameter, T, which reduces throughout the annealing process. Both potential solutions are investigated and Y is chosen to replace X as the existing