If $z = r(\cos\theta + i\sin\theta)$ and $w = t(\cos\phi + i\sin\phi)$

$$\Rightarrow \quad zw = rt\left[\cos(\theta+\phi) + i\sin(\theta+\phi)\right]$$

For $\times$, add angles

$$\Rightarrow \quad \frac{z}{w} = \frac{r}{t}\left[\cos(\theta-\phi) + i\sin(\theta-\phi)\right]$$

For $\div$, minus angles

Also if $z = r(\cos\theta + i\sin\theta) \Rightarrow \boxed{z = re^{i\theta}}$ (exponential form)

Hyperbolics and Trig.

$$e^{i\theta} = \cos\theta + i\sin\theta$$

So :

$$\boxed{\cosh x = \frac{e^x + e^{-x}}{2}}$$

$$\boxed{\cosh(i\theta) = \frac{e^{i\theta} + e^{-i\theta}}{2}}$$

$$\boxed{\sinh x = \frac{e^x - e^{-x}}{2}}$$

So for any complex number :

$$\boxed{\cosh(z) = \cos(iz)}$$

$$\boxed{i\sinh(z) = \sin(iz)}$$

(so $\cosh 4i \Rightarrow \cos 4$ and $\sinh 4i \Rightarrow i\sin 4$)

Osborn's Rule :

$\sin\theta \to \sinh\theta$,

$\cos\theta \to \cosh\theta$ .... etc

except $\sin^2\theta \to -\sinh^2\theta$

De Moivre's Theorem :

If $z = r(\cos\theta + i\sin\theta)$, then

$z^p = \left[r(\cos\theta + i\sin\theta)\right]^p$, so

$$\boxed{z^p = r^p(\cos p\theta + i\sin p\theta)}$$

Where [illegible] $d \Rightarrow z = d^{1/n}$

where d is a complex number

[illegible] n roots of a complex number $d^{1/n}$

$$\boxed{d^{1/n} = r^{1/n}\left[\cos\frac{\theta+2\pi k}{n} + i\sin\frac{\theta+2\pi k}{n}\right]}$$

Example : solve $z^3 + 4 - 4\sqrt{3}i = 0$

$z^3 = -4 + 4\sqrt{3}i \Rightarrow z = (-4+4\sqrt{3}i)^{1/3}$

so $d = (-4+4\sqrt{3}i)$ and $n = 3$

$r = 8$ and $\theta = 2\pi/3$

Do this for each value of k

$k = 0 \Rightarrow d^{1/3} = 8^{1/3}\left[\cos\frac{2\pi/3}{3} + i\sin\frac{2\pi/3}{3}\right]$

$k = 1 \Rightarrow d^{1/3} = 8^{1/3}\left[\cos\frac{8\pi}{9} + i\sin 8\pi/9\right]$

$k = 2 \Rightarrow d^{1/3} = 8^{1/3}\left[\cos\frac{2\pi/3 + 4\pi}{3} + i\sin\frac{2\pi/3 + 4\pi}{3}\right]$

$\therefore$ roots are $2\angle 2\pi/9$, $2\angle 8\pi/9$, $2\angle 14\pi/9$

after simplification

can be written in cartesian or polar form

Errors and Percentage Change:

$$\boxed{\delta f \simeq \frac{\delta f}{\delta x}\delta x + \frac{\delta f}{\delta y}}$$

δf is called absolute Error

$\frac{\delta f}{f}$ is called ~~absolute~~ relative error

$\frac{\delta f}{f} \times 100$ is called Percentage ~~absolute~~ relative error.

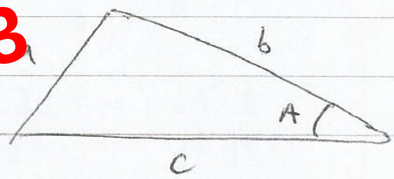
Functions of 3 or more variables:

if $f(x, y, u, v \ldots)$ and $\delta x, \delta y, \delta u, \delta v \ldots$

are small errors in $x, y, u, v \ldots$ respectively therefore:

$$\delta f \simeq \frac{\delta f}{\delta x}\delta x + \frac{\delta f}{\delta y}\delta y + \frac{\delta f}{\delta u}\delta u + \frac{\delta f}{\delta v}\delta v + \ldots$$

Example: Area $= S = \frac{1}{2} bc \sin A$



Given $b = 4m$, $c = 3m$, $A = 30°$

error in b and c is $\pm 0.005m$

error in ~~δ~~ A is $\pm 0.01°$

Calculate max. error in S?

differentiate each one with respect to each variable and sum it using formula

$$\frac{\delta s}{\delta b} = \frac{1}{2} c \sin A \qquad \frac{\delta s}{\delta c} = \frac{1}{2} b \sin A \qquad \frac{\delta s}{\delta A} = \frac{1}{2} bc \cos A.$$

$$\therefore \delta s \simeq \frac{\delta s}{\delta b}\delta b + \frac{\delta s}{\delta c}\delta c + \frac{\delta s}{\delta A}\delta A$$

$$= \frac{1}{2} c \sin A \, \delta b + \frac{1}{2} b \sin A \, \delta c + \frac{1}{2} bc \cos A \, \delta A.$$

$|\delta b|_{max} = |\delta c|_{max} = 0.005$

$|\delta A|_{max} = \pi/180 \times 0.01$

$$\therefore |\delta s|_{max} = \left(\tfrac{1}{2} \times 3 \times 1\right) \times 0.005 + \left(\tfrac{1}{2} \times 4 \times 0.5\right) \times 0.005 + \left(\tfrac{1}{2} \times 4 \times 3 \times \tfrac{\sqrt{3}}{2}\right) \times \pi/180 \times 0.01$$

$$\therefore |\delta S|_{max} = 0.0097 \ m^2$$