Example: Sketch $y = (x + 1)^2 (x - 2)$

STEP 1. Intercepts

y int when x = 0

$y = (1)^2(-2)$
$\quad = -2$

Passes through (0, -2)

x int when y = 0

$0 = (x + 1)^2 (x - 2)$

$(x + 1)^2 = 0$ and $(x - 2) = 0$

$x + 1 = 0$ $\qquad x = -2$

$x = -1$

Passes through (-1, 0) and (-2, 0)

STEP 2. Finding turning points (T.Ps)

Let $y = u \,.\, v$

where $u = (x + 1)^2$ and $v = (x - 2)$

$\qquad u' = 2(x + 1) \qquad v' = 1$

$$\frac{dy}{dx} = u'.\, v + v'.\, u$$
$$= 2(x + 1).\,(x - 2) + 1.\,(x + 1)^2$$
$$= (x + 1)[\, 2.(x - 2) + (x + 1)]$$
$$= (x + 1)[\, 2x - 4 + x + 1]$$
$$= (x + 1)\,[3x - 3]$$

T.Ps when $\frac{dy}{dx} = \mathbf{0}$

$(x + 1)\,[3x - 3] = 0$

$(x + 1) = 0$ and $[3x - 3] = 0$

$x = -1 \qquad 3x = 3$

$\qquad\qquad x = 1$

When $x = -1$, $y = (-1+1)^2(-1+2)$
$\qquad = 0$

(-1, 0)

when $x = 1$ $y = (2)^2(-1)$
$\qquad = -4$

(1, -4)

STEP 3. Nature of the T.Ps

When $x = -2$, $\frac{dy}{dx} = +ve$

Max at (-1, 0)

When $x = 0$, $\frac{dy}{dx} = -ve$

Min at (1, -4)

When $x = 2$, $\frac{dy}{dx} = +ve$