indirect taxes (such as value added taxes). In general indirect taxes are paid with some temporal lag respect to the time of the taxable event occurred. If inflation is high, there will be a deterioration of the real value of taxes collected by the government.

For example, if the taxable event occurred in period t but the government collect the taxes about that event only in period t+1 (e.g. for bureaucratic reasons), the real value of the tax will depend on the price level in t+1. If price level is particularly high at t+1 compared to t, the government is loosing some real income from those taxes.

d) *General inconvenience*

Inflation makes it harder to compare nominal values from different time periods.

This complicates long-range financial planning.

Examples:

- Parents trying to decide how much to save for the future college expenses of their (now) young child.
- Workers trying to decide how much to save for retirement.
- The CEO of a big corporation trying to decide whether to build a new factory, which will yield a revenue stream for 20 years or more.

2) <u>The costs of unexpected inflation</u>

a) *Arbitrary redistribution of purchasing power*

Many long-term contracts not indexed, but based on π^e.

If π turns out different from π^e, then some gain at others' expense.

Example: borrowers and lenders.

Suppose that at time t you want to borrow some money and you need to pay back at time t+1. You need to repay the amount borrowed plus a nominal interest rate *i*.

The nominal interest you pay at t+1 is decided at time t when you borrow the money.

The nominal interest rate depends on the inflation expected at time t+1.

Now you can have the following situation at t+1:

a) If $\pi > \pi^e$, then purchasing power is transferred from lenders to borrowers. The borrower repays the loan with less valuable dollars for example.

b) If $\pi < \pi^e$, then purchasing power is transferred from borrowers to lenders.

For example, at time t you borrow \$1000 and you agree to pay back \$1000 plus a nominal interest rate of 10% at time t+1. Suppose the real interest rate is constant in both periods and known with certainty by the borrower and the lender and it is equal

Mathematical Appendix

1) **<u>The Chain Rule of basic calculus:</u>**

Consider a function $f(x)$ where $x = g(z)$.

Suppose we want to find $\frac{df}{dz}$, that is the derivative of function *f* with respect to *z*. That derivative is given by:

$$\frac{df}{dz} = \frac{df}{dx}\frac{dx}{dz}$$

For example: $f(x) = x^2$ and $x = z - 1$. Here we have:

$\frac{df}{dx} = 2x$ and $\frac{dx}{dz} = 1$. Therefore:

$$\frac{df}{dz} = 2x$$

Using the definition of x into the above equation we obtain:

$$\frac{df}{dz} = 2(z-1)$$

In this simple case we could have substituted the definition of x directly into *f(x)*. In this case we will obtain $f(z) = (z-1)^2$. Calculating directly the derivative $\frac{df}{dz}$ gives you exactly the same result as before.

In many cases, simple substitution is not possible, because for example we have functions in general form.

Consider for example a general function of two variables: $f(x, y)$

Assume that $y = 3z + 2w$

Suppose you want to calculate $\frac{\partial f}{\partial z}$ (here we use the notation of partial differentiation ∂ since we deal with a function with more than one variable)

According to the chain rule, and knowing that $\frac{\partial y}{\partial z} = 3$ we have:

$$\frac{\partial f}{\partial z} = \frac{\partial f}{\partial y}\frac{\partial y}{\partial z} = 3\frac{\partial f}{\partial y}$$

2) **<u>Properties of the logs</u>**

The log with base b of a number y is defined as

Take logs on both sides:

$$\ln(M) - \ln(P) = \alpha \ln(Y)$$

Or written differently:

$$\ln(M) = \ln(P) + \alpha \ln(Y)$$

Now the parameter α is the elasticity of money with respect to income:

$$\frac{\partial \ln(M)}{\partial \ln(Y)} = \frac{Y}{M}\frac{\partial M}{\partial Y} = \alpha$$

where we use the concept of partial derivative since we have a function of two variables (M is a function of P and Y).

Logarithms and the growth rate of a variable:

From property f.1) we have: $\frac{d\ln(x)}{dx} = \frac{1}{x}$

We can rewrite that expression as:

$$d\ln(x) = \frac{dx}{x}$$

Now, we have defined $\frac{dx}{x}$ as the instantaneous growth rate of the variable x. This means that instantaneous growth rate of x is equal to the absolute change in the natural log of that variable $d\ln(x)$.

Consider now the discrete rate of growth of the variable x (when the change in the variable dx is not infinitesimally small): $\frac{\Delta x}{x} = \frac{x_t - x_{t-1}}{x_{t-1}}$

This expression is approximately equal to $\Delta \ln(x)$ the absolute discrete change in the logarithm of x. That is:

$$\Delta \ln(x) \cong \frac{\Delta x}{x} \qquad \text{A1)}$$

where the symbol $\cong$ means approximately equal.

Expression A1) implies that:

$$\frac{\Delta x}{x} = \frac{x_t - x_{t-1}}{x_{t-1}} \cong \ln(x_t) - \ln(x_{t-1}) = \Delta \ln(x)$$

Example: consider the following data for the variable x:

Year	x	ln(x)	Δx/x	$\ln(x_t)-\ln(x_{t-1})$
2000	10	2,302585		
2001	12	2,484907	0,2	0,182322
2002	14	2,639057	0,166667	0,154151