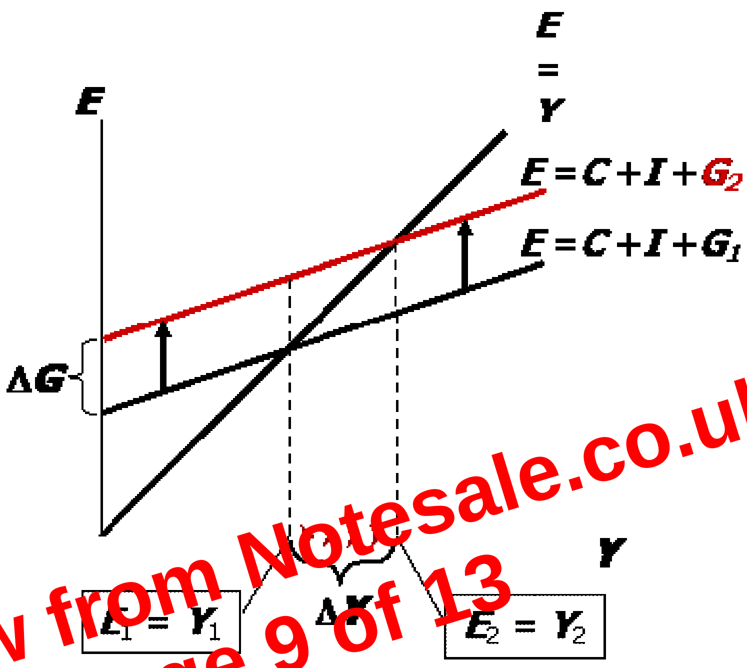
$$\Delta Y = (1 + c + c^2 + c^3 + ... + c^n + ...)\Delta G$$

The terms in the brackets form a geometric series with geometric ratio given by c and the first element given by 1. In the Appendix of Lecture Note 6 you will see that a geometric series of infinite elements with geometric ratio b and first element 1 tends to: $\frac{1}{1-c}$. The effect of an increase in government expenditure, everything else constant, can be seen graphically:



Suppose that G increases from G_1 to G_2. The initial equilibrium is Y_1. This change in G will increase the planned expenditure by the same amount, therefore, the line describing E will shift upwards by the amount ΔG. Given the multiplier effect, there will an increase in total income (larger than ΔG), and therefore, the new equilibrium will be Y_2.

b) The effect of a change in Taxes

We now ask the following question: what is the effect of the change in the taxes ΔT on total income, **everything else constant** (meaning $\Delta G = \Delta I = 0$) ?

From equation 4) we have:

$$\Delta Y = c\Delta Y - c\Delta T$$

Therefore:

Mathematical Appendix

Total Differential of a function of several variables.

Consider a function of two variables $y = f(x_1, x_2)$.

We want to see what is the change in *y* if we change at the same time x_1 and x_2 by very small amounts: dx_1 and dx_2.

The answer is given by the **Total Differential** of the function y:

$$dy = \frac{\partial f(x_1, x_2)}{\partial x1} dx_1 + \frac{\partial f(x_1, x_2)}{\partial x_2} dx_2$$

where $\frac{\partial f(x_1, x_2)}{\partial x1} dx_1$ measures the effect of changing x_1 by the amount dx_1.

For example suppose a function: $y = 3x_1 + 5x_2$

Suppose that $x_1 = 1$ and $x_2 = 2$, then $y = 13$.

Now suppose that you change the two variables by $dx_1 = 0.2$ and $dx_2 = 0.5$. The new values for the two variables are: $x_1^* = x_1 + dx_1 = 1 + 0.2 = 1.2$ and $x_2^* = x_2 + dx_2 = 2 + 0.5 = 2.5$.

The new value of y after those changes is: $y^* = 3 \times 1.2 + 5 \times 2.5 = 3.6 + 12.5 = 16.1$

Therefore the change in y induced by those changes in the x's is given by: $dy = 16.1 - 13 = 3.1$.

(Notice that we are using the symbol d in this particular example, but we should use instead the symbol Δ since we are dealing with changes that are not infinitesimal).

You can see the same effect by applying the total differential approach.

Here $\frac{\partial f(x_1, x_2)}{\partial x1} = 3$ while $\frac{\partial f(x_1, x_2)}{\partial x_2} = 5$

Therefore we should have:

$$dy = 3dx_1 + 5dx_2 = 3 \times 0.2 + 5 \times 0.5 = 0.6 + 2.5 = 3.1$$

In writing equation 4) in the lecture note we are implicitly using the idea of total differential.

We have:

$$E = C + G + I$$