Where a is the first element of an infinte geometric series.

In our case, if all the payments or receipts are the same, we have:

$$PV = \frac{y}{(1+r)} + \frac{y}{(1+r)^2} + \ldots + \frac{y}{(1+r)^n} + \ldots$$

that is a geometric series with first element $\frac{y}{(1+r)}$ and geometric ratio $R = \frac{1}{(1+r)}$

Using those facts into result B):

$$\sum_{\infty} = \frac{\frac{y}{(1+r)}}{1 - \frac{1}{(1+r)}} = \frac{\frac{y}{(1+r)}}{\frac{1+r-1}{(1+r)}} = \frac{y}{(1+r)} \frac{(1+r)}{r} = \frac{y}{r}$$

b) Implicit Differentiation

Implicit Differentiation is an important result that is given by the **implicit function theorem** (we are not going to state or prove this theorem).

An Implicit Function is a function like:

$$F(x, y) = 0$$

meaning that we do not know how to express EXPLICITLY (or simply we do not want to) one variable (for example y) as a function of the other (for example x).

An example of an EXPLICIT function is: $y = a + bx^2$.

An example of an IMPLICIT function is: $(x^2 + y^2)^2 - (x^2 + y^2) = 0$

In the second case to express directly y as a function of x can be particularly difficult (in this particular case it is impossible).

However, from an implicit function we can find some properties of the relationship between y and x even if we do not know this relationship!

In particular we can calculate the derivative $\frac{dy}{dx}$ even if we do not know the exact function that relates the two variables.

Implicit Differentiation Rule: given an implicit function of n variables $F(x_1, x_2, \ldots, x_n)$, we can calculate the derivative:

$$\frac{\partial x_i}{\partial x_j} = -\frac{F_j}{F_i}$$