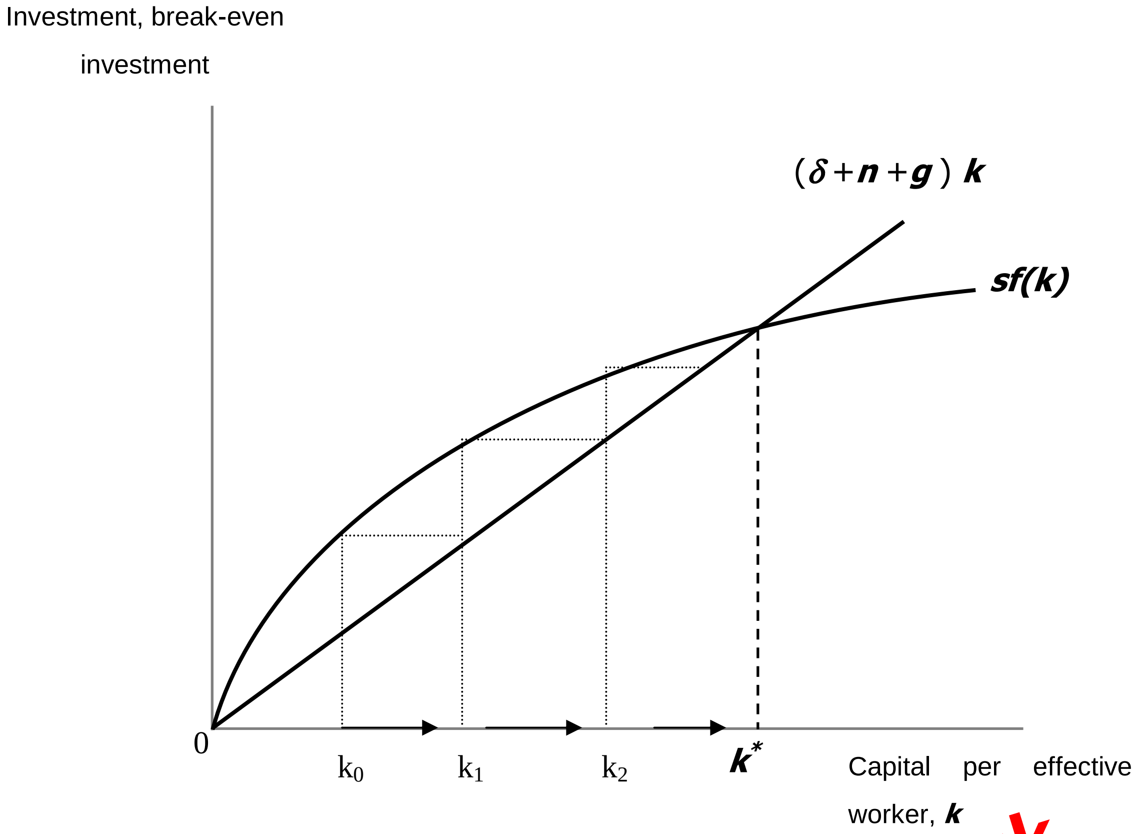


We can show that a same convergence will happen if we start with a level of capital $k_0 > k^*$. In that case the initial capital is too high and it will decrease towards k^*.

This means that the economy, independently of the starting capital level will always converge to the steady state.

The Balanced Growth Path in the Solow Model

We want to know how the main variables of the model behave when $k = k^*$ in terms of their growth rates at the steady state.

a) ***Capital per effective labour***: $\frac{\Delta k^*}{k^*} = 0$. In equilibrium capital per effective worker is constant at k^* and therefore its steady state growth rate is zero.

b) ***Capital stock*** K: $\frac{\Delta K}{K} = n + g$. At the steady state this is given by ALk^*. Effective labour grows at rate $n + g$, k^* is constant and therefore the capital stock grows at rate $n + g$.

c) ***Output per effective worker***: $\frac{\Delta y}{y} = 0$. Output per effective worker must be constant at the steady state since we have $y = f(k^*)$ and k^* is constant.

d) ***Output per worker***: $\frac{\Delta(Y/L)}{Y/L} = g$. To see this remember that $y = \frac{Y}{AL}$ and so $\frac{Y}{L} = Ay$. We know that the growth rate of y is zero while A grows at rate g, therefore Ay grows at rate $g + 0 = g$.

b) it reduces $(1-s)$ and so it should reduce consumption;

Can we find a saving rate and so a capital level that maximizes the steady state level of consumption per effective worker?

In steady state consumption is given by: $c^* = y^* - i^*$, where the asterisk says that the variables are at the steady state level (where $k = k^*$).

In steady state: $y^* = f(k^*)$ and $i_t^* = (n+g+\delta)k^*$ (investment must be equal to break-even investment).

Therefore consumption per effective worker at the steady state is given by:

$$c^* = f(k^*) - (n+g+\delta)k^* \qquad 10)$$

What is the steady state capital level k^* that maximises consumption c^*?

We need to find $\frac{dc^*}{dk^*}$ (the derivative of consumption with respect to capital) from 10) and set it equal to zero.

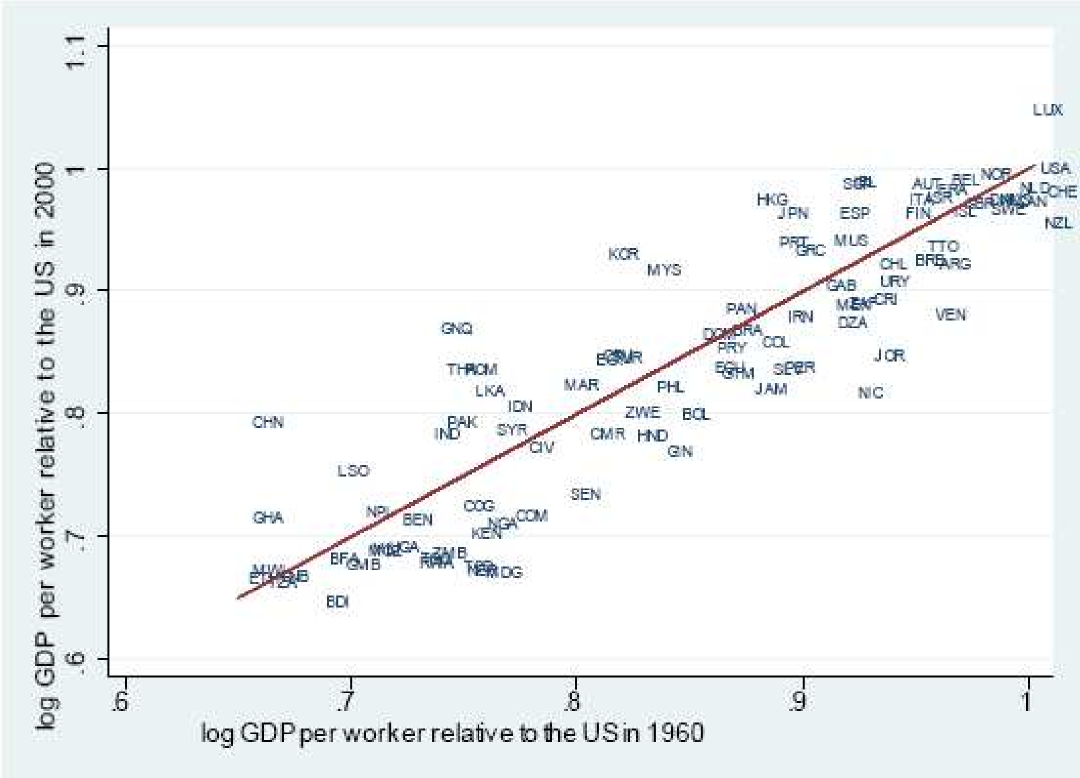
$$\frac{dc^*}{dk^*} = f'(k^*) - (n+g+\delta) = 0 \Rightarrow f'(k^*) = n+g+\delta \qquad 11)$$

The condition 11) says that consumption per effective worker is maximised when the steady state level of capital is such that the *Marginal Productivity of Capital* per effective worker ($f'(k^*) = \frac{df(k^*)}{dk^*}$) is equal to $n+g+\delta$.

The condition $f'(k^*) = n+g+\delta$ is called the ***Golden Rule***. The capital level k^*_{gold} that satisfies that condition is called the ***Golden Rule Level of Capital***. The saving rate associated with k^*_{gold} is the ***Golden Rule Saving Rate*** called s_{gold}.

Graphically: $f'(k^*) = \frac{df(k^*)}{dk^*}$ represents the slope of the curve $f(k)$ at k^*. The term $n+g+\delta$ is the slope of the line $(n+g+\delta)k$. If $f'(k^*) = n+g+\delta$ it means that the curve $f(k)$ and the line $(n+g+\delta)k$ must be parallel at the steady state where consumption is maximised.

This is shown graphically below. From the graph you should notice that there is nothing that assures that steady state level of capital that the economy converges to is equal to the golden rule level. In the graph below for example the current steady state level of capital is larger than the golden state level. This means that there is too much capital in the economy. By decreasing the level of capital in the economy, consumption per capita will increase. Once we know the golden rule capital level we know the golden rule saving rate associated with. A government can use policy to affect the saving rate in order for the economy to be closer to the golden rule capital level. In the graph the saving rate is currently higher than the golden rule level. Any decrease in the saving rate towards s_{gold} will improve consumption per capita for all generations. It will be a Pareto-improvement.



Countries that were poorer relative to the US in 1960 (horizontal axis) are still poorer relative to the US 40 years later (vertical axis).
Does this mean the Solow model fails? No, because "other things" may not be equal. Countries can differ in saving rates and population growth. In samples of countries with similar savings and population growth rates, income gaps shrink about 2% per year. What the Solow model really predicts is ***conditional convergence***: different countries do not need to converge to the same steady state but they may converge to different steady states. Conditional converge implies that countries that start further below their balanced growth path (countries that are poor relative to their balanced growth path) should grow faster than rich countries (relative to their balanced growth path). Data for full sample of countries lend support to the conditional convergence hypothesis.