2. **Three-Dimensional Geometry** **(15 Periods)**

Direction cosines and direction ratios of a line joining two points. Cartesian and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Angle between (*i*) two lines, (*ii*) two planes, (*iii*) a line and a plane. Distance of a point from a plane.

Unit V : LINEAR PROGRAMMING

(20 Periods)

1. **Linear Programming :** Introduction, related terminology such as constraints, objective function, optimization. Different types of linear programming (L.P.) problems, mathematical formulation of L.P. problems, graphical method of solution for problems in two variables, feasible and infeasible regions, feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

Unit VI : PROBABILITY

1. **Probability** **(30 Periods)**

Multiplication theorem on probability. Conditional probability, independent events, total probability, Baye's theorem, Random variable and its probability distribution, mean and variance of a random variable. Repeated independent (Bernoulli) trials and Binomial distribution.

QUESTION-WISE BREAK UP

Marks per Question	*Total Number of Questions in*	
	2013-14	*2014-15*
1	10	6
4	12	13
6	7	7
Total	**29**	**26**

CONTENTS

- Onto function (surjective) : A function $f : A \to B$ is said to be onto iff $R_f = B$ *i.e.* $\forall\ b \in B$, there exists $a \in A$ such that $f(a) = b$
- A function which is not one-one is called many-one function.
- A function which is not onto is called into function.
- **Bijective Function :** A function which is both injective and surjective is called bijective function.
- **Composition of Two Functions :** If $f : A \to B$, $g : B \to C$ are two functions, then composition of f and g denoted by gof is a function from A to C given by, $(gof)\ (x) = g\ (f\ (x))\ \ \forall\ \ x \in A$

  Clearly gof is defined if Range of $f \subset$ domain of g. Similarly fog can be defined.
- **Invertible Function :** A function $f : X \to Y$ is invertible iff it is bijective.

  If $f : X \to Y$ is bijective function, then function $g : Y \to X$ is said to be inverse of f iff $fog = I_y$ and $gof = I_x$

  when I_x, I_y are identity functions.
- g is inverse of f and is denoted by f^{-1}.
- **Binary Operation :** A binary operation '*' defined on set A is a function from $A \times A \to A$. * (a, b) is denoted by $a * b$.

- Binary operation * defined on set A is said to be commutative iff

  $$a * b = b * a \ \ \forall\ \ a, b \in A.$$
- Binary operation * defined on set A is called associative iff $a * (b * c) = (a * b) * c\ \ \forall\ \ a, b, c \in A$
- If * is Binary operation on A, then an element $e \in A$ is said to be the identity element iff $a * e = e * a\ \ \forall\ \ a \in A$
- Identity element is unique.
- If * is Binary operation on set A, then an element b is said to be inverse of $a \in A$ iff $a * b = b * a = e$
- Inverse of an element, if it exists, is unique.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Write the principal value of

(i) $\sin^{-1}\left(-\sqrt{3}/2\right)$ (ii) $\cos^{-1}\left(\sqrt{3}/2\right)$.

(iii) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ (iv) $\text{cosec}^{-1}(-2)$.

(v) $\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)$. (vi) $\sec^{-1}(-2)$.

(vii) $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{-1}{2}\right) + \tan^{-1}\left(-1/\sqrt{3}\right)$

2. What is the value of the following functions (using principal value).

(i) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) - \sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$. (ii) $\sin^{-1}\left(-\frac{1}{2}\right) - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

(iii) $\tan^{-1}(1) - \cot^{-1}(-1)$. (iv) $\text{cosec}^{-1}\left(\sqrt{2}\right) + \sec^{-1}\left(\sqrt{2}\right)$.

(v) $\tan^{-1}(1) + \cot^{-1}(1) + \sin^{-1}(1)$.

(vi) $\sin^{-1}\left(\sin\frac{4\pi}{5}\right)$. (vii) $\tan^{-1}\left(\tan\frac{5\pi}{6}\right)$.

(viii) $\text{cosec}^{-1}\left(\text{cosec}\frac{3\pi}{4}\right)$.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

3. Show that $\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right) = \frac{\pi}{4} + \frac{x}{2}$. $x \in [0, \pi]$

- **Transpose of a Matrix :** If $A = [a_{ij}]_{m \times n}$ be an $m \times n$ matrix then the matrix obtained by interchanging the rows and columns of A is called the transpose of the matrix. Transpose of A is denoted by A' or A^T.

  Properties of the transpose of a matrix.

  (i) $(A')' = A$ (ii) $(A \pm B)' = A' \pm B'$

  (iii) $(kA)' = kA'$, k is a scalar (iv) $(AB)' = B'A'$

- **Symmetric Matrix :** A square matrix $A = [a_{ij}]$ is symmetric if $a_{ij} = a_{ji}$ $\forall$ i, j. Also a square matrix A is symmetric if $A' = A$.

- **Skew Symmetric Matrix :** A square matrix $A = [a_{ij}]$ is skew-symmetric, if $a_{ij} = -a_{ji}$ $\forall$ i, j. Also a square matrix A is skew - symmetric, if $A' = -A$.

- **Determinant :** To every square matrix $A = [a_{ij}]$ of order $n \times n$, we can associate a number (real or complex) called determinant of A. It is denoted by det A or $|A|$.

  Properties

  (i) $|AB| = |A|\ |B|$

  (ii) $|kA|_{n \times n} = k^n\ |A|_{n \times n}$ where k is a scalar.

  Area of triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

  The points (x_1, y_1), (x_2, y_2), (x_3, y_3) are collinear $\Leftrightarrow \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

- **Adjoint of a Square Matrix *A*** is the transpose of the matrix whose elements have been replaced by their cofactors and is denoted as *adj A*.

  Let $A = [a_{ij}]_{n \times n}$

  $adj\ A = [A_{ji}]_{n \times n}$

11. Find the value of $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

12. If $\begin{vmatrix} 2x+5 & 3 \\ 5x+2 & 9 \end{vmatrix} = 0$, find x.

13. For what value of k, the matrix $\begin{bmatrix} k & 2 \\ 3 & 4 \end{bmatrix}$ has no inverse.

14. If $A = \begin{bmatrix} \sin 30° & \cos 30° \\ -\sin 60° & \cos 60° \end{bmatrix}$, what is $|A|$.

15. Find the cofactor of a_{12} in $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$.

16. Find the minor of a_{23} in $\begin{vmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{vmatrix}$.

17. Find the value of P, such that the matrix $\begin{bmatrix} -1 & 2 \\ 4 & P \end{bmatrix}$ is singular.

18. Find the value of x such that the points (0, 2), (1, x) and (3, 1) are collinear.

19. Area of a triangle with vertices (k, 0), (1, 1) and (0, 3) is 5 unit. Find the value (s) of k.

20. If A is a square matrix of order 3 and $|A| = -2$, find the value of $|-3A|$.

21. If $A = 2B$ where A and B are square matrices of order 3×3 and $|B| = 5$, what is $|A|$?

22. What is the number of all possible matrices of order 2×3 with each entry 0, 1 or 2.

23. Find the area of the triangle with vertices (0, 0), (6, 0) and (4, 3).

24. If $\begin{vmatrix} 2x & 4 \\ -1 & x \end{vmatrix} = \begin{vmatrix} 6 & -3 \\ 2 & 1 \end{vmatrix}$, find x.

26. If $x = ae^t (\sin t - \cos t)$

$y = ae^t (\sin t + \cos t)$ then show that $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is 1.

27. If $y = \sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right]$ then find $-\frac{dy}{dx}$.

28. If $y = x^{\log_e x} + (\log_e x)^x$ then find $\frac{dy}{dx}$.

29. Differentiate x^{x^x} w.r.t. x.

30. Find $\frac{dy}{dx}$, if $(\cos x)^y = (\cos y)^x$

31. If $y = \tan^{-1}\left(\frac{\sqrt{1+\sin x} - \sqrt{1-\sin x}}{\sqrt{1+\sin x} + \sqrt{1-\sin x}}\right)$ where $\frac{\pi}{2} < x < \pi$ find $\frac{dy}{dx}$.

Hint : $\sin\frac{x}{2} > \cos\frac{x}{2}$ *for* $x \in \left(\frac{\pi}{2}, \pi\right)$.

32. If $x = \sin\left(\frac{1}{a}\log_e y\right)$ then show that $(1 - x^2) y'' - xy' - a^2y = 0$.

33. Differentiate $(\log x)^{\log x}$, $x > 1$ w.r.t. x

34. If $\sin y = x \sin (a + y)$ then show that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$.

35. If $y = \sin^{-1}x$, find $\frac{d^2y}{dx^2}$ in terms of y.

36. If $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then show that $\frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$.

37. If $y = e^{a\cos^{-1}x}, -1 \le x \le 1$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$

38. If $y^3 = 3ax^2 - x^3$ then prove that $\frac{d^2y}{dx^2} = \frac{-2a^2x^2}{y^5}$.

52. $(-\infty, -1)$ and $(1, \infty)$.

53. $\frac{25}{3}$.

54. Increasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ Decreasing in $\left(0, \frac{\pi}{4}\right)$.

55. $a = -2$.

56. Strictly decreasing in $(1, \infty)$.

60. 0.2083

61. 2.9907

62. 0.06083

63. 0.1925

64. 5.002

65. –34.995

66. 45.46

68. 25, 10

74. Strictly increasing in $\left[0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right]$

Strictly decreasing in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$.

75. Strictly increasing in $(1, 3) \cup (3, \infty)$

Strictly decreasing in $(-\infty, -1) \cup (-1, 1)$.

76. Local maxima at $x = \frac{\pi}{6}$

Local max. value $= \frac{\sqrt{3}}{2} - \frac{\pi}{6}$

Local minima at $x = -\frac{\pi}{6}$

Local minimum value $= \frac{-\sqrt{3}}{2} + \frac{\pi}{6}$

77. Strictly increasing in $(-\infty, 2] \cup [3, \infty)$

Strictly decreasing in $(2, 3)$.

CHAPTER 7

INTEGRALS

POINTS TO REMEMBER

- Integration is the reverse process of Differentiation.

- Let $\frac{d}{dx}F(x) = f(x)$ then we write $\int f(x)dx = F(x) + c$.

- These integrals are called indefinite integrals and c is called constant of integration.

- From geometrical point of view an indefinite integral is collection of family of curves each of which is obtained by translating one of the curves parallel to itself upwards or downwards along y-axis.

STANDARD FORMULAE

1. $$\int x^n dx = \begin{cases} \dfrac{x^{n+1}}{n+1} + c & n \neq -1 \\ \log|x| + c & n = -1 \end{cases}$$

2. $$\int (ax+b)^n \, dx = \begin{cases} \dfrac{(ax+b)^{n+1}}{(n+1)a} + c & n \neq -1 \\ \dfrac{1}{a}\log|ax+b| + c & n = -1 \end{cases}$$

3. $\int \sin x \; dx = -\cos x + c.$ 4. $\int \cos x \; dx = \sin x + c.$

5. $\int \tan x \, . dx = -\log|\cos x| + c = \log|\sec x| + c.$

(v) $2\left[\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}\right] + c$

(vi) $\left(\dfrac{x^4 - 1}{4}\right)\tan^{-1}x - \dfrac{x^3}{12} + \dfrac{x}{4} + c.$

(vii) $\dfrac{1}{2}e^{2x}\tan x + c.$

(viii) $\dfrac{e^x}{2x} + c.$

(ix) $\dfrac{x - a}{2}\sqrt{2ax - x^2} + \dfrac{a^2}{2}\sin^{-1}\left(\dfrac{x - a}{a}\right) + c$

(x) $e^x\left(\dfrac{x - 1}{x + 1}\right) + c.$

(xi) $e^x \tan x + c.$

(xii) $x\log\left|\log x\right| - \dfrac{x}{\log x} + c.$

(xiii) $-2\left(6 + x - x^2\right)^{3/2}$
$+8\left[\dfrac{2x - 1}{4}\sqrt{6 + x - x^2} + \dfrac{25}{8}\sin^{-1}\left(\dfrac{2x - 1}{5}\right)\right] + c$

(xiv) $\dfrac{1}{3}\log\left|x + 1\right| - \dfrac{1}{6}\log\left|x^2 - x + 1\right| + \dfrac{1}{\sqrt{3}}\tan^{-1}\left(\dfrac{2x - 1}{\sqrt{3}}\right) + c$

(xv) $\dfrac{2}{3}\left(x^2 - 4x + 3\right)^{3/2} - \left(\dfrac{x - 2}{2}\right)\sqrt{x^2 - 4x + 3}$
$+\dfrac{1}{2}\log\left|x - 2 + \sqrt{x^2 - 4x + 3}\right| + c$

(xvi) $\left(\dfrac{x - 2}{2}\right)\sqrt{x^2 - 4x + 8} + 2\log\left|(x - 2) + \sqrt{x^2 - 4x + 8}\right| + c$

59. (i) $\frac{2(2x-1)}{\pi}\sin^{-1}\sqrt{x}+\frac{2\sqrt{x-x^2}}{\pi}-x+c$

(ii) $-2\sqrt{1-x}+\cos^{-1}\sqrt{x}+\sqrt{x-x^2}+c$

(iii) $-\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2}\left[\log\left(1+\frac{1}{x^2}\right)-\frac{2}{3}\right]+c$

(iv) $\frac{\sin x-x\cos x}{x\sin x+\cos x}+c$

(v) $(x+a)\tan^{-1}\sqrt{\frac{x}{a}}-\sqrt{ax}+c$

(vi) $2\sin^{-1}\frac{\sqrt{3}-1}{2}$

(vii) 0

(viii) $\frac{3}{\pi}+\frac{1}{\pi^2}.$

60. (i) $x-4\log|x|-\frac{3}{4}\log|x-1|+\frac{3}{4}\log|x+1|$
$+\log|x^2+1|-\frac{1}{2}\tan^{-1}x+c.$

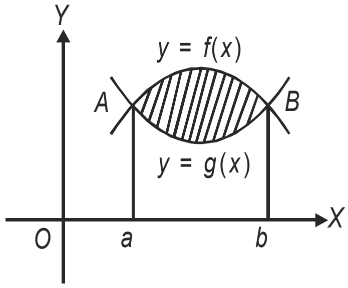
(ii) $\frac{1}{5}\log|x-1|-\frac{1}{10}\log\left|x^2+4\right|-\frac{1}{10}\tan^{-1}\left(\frac{x}{2}\right)+c.$

(iii) $2x-\frac{1}{8}\log|x+1|+\frac{81}{8}\log|x-3|-\frac{27}{2(x-3)}+c.$

(iv) $x+\frac{1}{2}\log\left|\frac{x-2}{x+2}\right|-\tan^{-1}\left(\frac{x}{2}\right)+c.$

(v) $\pi/\sqrt{2}.$

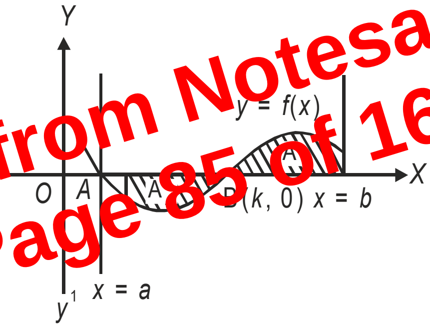
- Area bounded by two curves $y = f(x)$ and $y = g(x)$ such that $0 \le g(x) \le f(x)$ for all $x \in [a, b]$ and between the ordinate at $x = a$ and $x = b$ is given by



$$\text{Area} = \int_a^b [f(x) - g(x)]\,dx$$

- Required Area

$$= \left|\int_a^k f(x)dx\right| + \int_k^b f(x)dx.$$



LONG ANSWER TYPE QUESTIONS (6 MARKS)

1. Find the area enclosed by circle $x^2 + y^2 = a^2$.

2. Find the area of region bounded by $\left\{(x, y) : |x - 1| \le y \le \sqrt{25 - x^2}\right\}$.

3. Find the area enclosed by the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

(vii) $e^{\tan^{-1} x}$

4.(i) 2 (ii) 1

(iii) 2 (iv) 1

(v) 1 (vi) 2

(vii) 2

5.(vi) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$ (vii) $x\left(\frac{dy}{dx}\right)^2 + xy\frac{d^2y}{dx^2} = y\frac{dy}{dx}$

(viii) $2\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$

6.(i) $y \sin x = \frac{2\sin^3 x}{3} + c$ (ii) $y = \frac{x^2(4\log_e x - 1)}{16} + \frac{c}{x^2}$

(iii) $y = \sin x + \frac{c}{x},\ x > 0$ (iv) $y = \tan x - 1 + ce^{-\tan x}$

(v) $xy = \frac{y^4}{4} + c$ (vi) $x = -y^2e^{-y} + cy^2$

7.(i) $cy = (x + 2)(1 - 2y)$ (ii) $(e^x + 2)\sec y = c$

(iii) $\sqrt{1 - x^2} + \sqrt{1 - y^2} = c$

(iv) $\frac{1}{2}\log\left|\frac{\sqrt{1 - y^2} - 1}{\sqrt{1 - y^2} + 1}\right| = \sqrt{1 - x^2} - \sqrt{1 - y^2} + c$

(v) $(x^2 + 1)(y^2 + 1) = 2$

(vi) $\log y = -\frac{1}{4}\cos^4 x + \frac{1}{6}\cos^6 x + xe^x - e^x + c$

$$= \frac{1}{16}\left[\frac{\cos^3 2x}{3} - \cos 2x\right] + (x-1)e^x + c$$

(vii) $\log|\tan y| - \frac{\cos 2x}{4} = c$

8.(i) $\frac{-x^3}{3y^3} + \log|y| = c$ (ii) $\tan^{-1}\left(\frac{y}{x}\right) = \log|x| + c$

(iii) $x^2 + y^2 = 2x$

(iv) $y = ce^{\cos(x/y)}$

(v) $\sin\left(\frac{y}{x}\right) = cx$ (vi) $c\left(x^2 - y^2\right) = y$

(vii) $-e^{-y} = e^x + \frac{x^3}{3} + c$ (viii) $\sin^{-1} y = \sin^{-1} x + c$

(ix) $\left|y^2 + 2xy\right| = \frac{c}{x^2}$

9.(i) $x^2 - y^2 + 2xy\frac{dy}{dx} = 0$ (ii) $2y = x\frac{dy}{dx}$, $y = 2x\frac{dy}{dx}$

(iii) $x^2 - y^2 + 2xy\frac{dy}{dx} = 0$

(iv) $(x-y)^2(1+y')^2 = (x+yy')^2$

10. $\log\left|x^2 + xy + y^2\right| = 2\sqrt{3}\tan^{-1}\left(\frac{x+2y}{\sqrt{3}x}\right) + c$

11. $\frac{x^3}{x^2+y^2} = \frac{c}{x}(x+y)$

$\vec{a}$ and $\vec{b}$ $(0 \le \theta \le \pi)$ and $\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ such that $\overrightarrow{a}, \overrightarrow{b}$ *and* $\hat{n}$ form a right handed system.

- Cross product of two vectors is not commutative i.e., $\overrightarrow{a} \times \overrightarrow{b} \neq \overrightarrow{b} \times \overrightarrow{a}$, but $\overrightarrow{a} \times \overrightarrow{b} = -\left(\overrightarrow{b} \times \overrightarrow{a}\right)$.

- $\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{o} \Leftrightarrow \overrightarrow{a} = \overrightarrow{o},\ \overrightarrow{b} = \overrightarrow{o}$ *or* $\overrightarrow{a} \parallel \overrightarrow{b}$.

- $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \overrightarrow{o}$.

- $\hat{i} \times \hat{j} = \hat{k},\ \hat{j} \times \hat{k} = \hat{i},\ \hat{k} \times \hat{i} = \hat{j}$ and $\hat{j} \times \hat{i} = -\hat{k},\ \hat{k} \times \hat{j} = -\hat{i},\ \hat{i} \times \hat{k} = -\hat{j}$

- If $\overrightarrow{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\overrightarrow{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then

$$\overrightarrow{a} \times \overrightarrow{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

- Unit vector perpendicular to both $\overrightarrow{a}$ and $\overrightarrow{b} = \pm\left(\dfrac{\left(\overrightarrow{a} \times \overrightarrow{b}\right)}{\left|\overrightarrow{a} \times \overrightarrow{b}\right|}\right)$.

- $\left|\overrightarrow{a} \times \overrightarrow{b}\right|$ is the area of *parallelogram* whose adjacent sides are $\overrightarrow{a}$ and $\overrightarrow{b}$.

- $\dfrac{1}{2}\left|\overrightarrow{a} \times \overrightarrow{b}\right|$ is the area of parallelogram where diagonals are $\overrightarrow{a}$ and $\overrightarrow{b}$.

- If $\overrightarrow{a}, \overrightarrow{b}$ and $\overrightarrow{c}$ forms a triangle, then *area of the triangle.*

$$= \frac{1}{2}\left|\overrightarrow{a} \times \overrightarrow{b}\right| = \frac{1}{2}\left|\overrightarrow{b} \times \overrightarrow{c}\right| = \frac{1}{2}\left|\overrightarrow{c} \times \overrightarrow{a}\right|.$$

- Scalar triple product of three vectors $\overrightarrow{a}, \overrightarrow{b}$ and $\overrightarrow{c}$ is defined as $\overrightarrow{a}.\left(\overrightarrow{b} \times \overrightarrow{c}\right)$ and is denoted as $\left[\overrightarrow{a}\ \overrightarrow{b}\ \overrightarrow{c}\right]$

57. For any three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$, prove that $\vec{a} - \vec{b}$, $\vec{b} - \vec{c}$ and $\vec{c} - \vec{a}$ are coplanar.

ANSWERS

1. $-\frac{5\sqrt{3}}{2}, \frac{5}{2}$.

2. $a = \pm\frac{1}{3}$

3. $\vec{x}$ and $\vec{y}$ are like parallel vectors.

4. $\sqrt{126}$ sq units.

5. $\frac{\pi}{3}$

6. $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k}$

7. (6, 11)

8. $\left(5, \frac{14}{3}, -6\right)$

9. $4\hat{i} + 6\hat{j} + 4\sqrt{3}\,\hat{k}$.

10. $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$.

11. $\cos^{-1}\left(\frac{3}{7}\right)$.

12. $\frac{3\hat{i} + 4\hat{j} - \hat{k}}{\sqrt{26}}$.

13. 0

14. 4

15. –9

16. 2

17. $\frac{\pi}{2}$.

18. $\sqrt{5}$

19. $\frac{3}{2}$ sq. units.

20. $\sqrt{13}$

21. $\frac{2\pi}{3}$

17. Write the vector equation of the plane which is at a distance of 8 units from the origin and is normal to the vector $\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$.

18. What is equation of the plane if the foot of perpendicular from origin to this plane is (2, 3, 4)?

19. Find the angles between the planes $\vec{r} \cdot \left(\hat{i} - 2\hat{j} - 2\hat{k}\right) = 1$ and $\vec{r} \cdot \left(3\hat{i} - 6\hat{j} + 2\hat{k}\right) = 0$.

20. What is the angle between the line $\dfrac{x+1}{3} = \dfrac{2y-1}{4} = \dfrac{2-z}{-4}$ and the plane $2x + y - 2z + 4 = 0$?

21. If O is origin $OP = 3$ with direction ratios proportional to $-1, 2, -2$ then what are the coordinates of P?

22. What is the distance between the line $\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda\left(\hat{i} + \hat{j} + 4\hat{k}\right)$ from the plane $\vec{r} \cdot \left(-\hat{i} + 5\hat{j} - \hat{k}\right) + 5 = 0$.

23. Write the line $2x = 3y = 4z$ in vector form.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

24. The line $\dfrac{x-4}{1} = \dfrac{2y-4}{2} = \dfrac{k-z}{-2}$ lies exactly in the plane $2x - 4y + z = 7$. Find the value of k.

25. Find the equation of a plane containing the points (0, –1, –1), (–4, 4, 4) and (4, 5, 1). Also show that (3, 9, 4) lies on that plane.

26. Find the equation of the plane which is perpendicular to the plane $\vec{r} \cdot \left(5\hat{i} + 3\hat{j} + 6\hat{k}\right) + 8 = 0$ & which is containing the line of intersection of the planes $\vec{r} \cdot \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = 4$ and $\vec{r} \cdot \left(2\hat{i} + \hat{j} - \hat{k}\right) + 5 = 0$.

27. If l_1, m_1, n_1, and l_2, m_2, n_2 are direction cosines of two mutually perpendicular lines, show that the direction cosines of line perpendicular to both of them are

$$m_1n_2 - n_1m_2,\ n_1l_2 - l_1n_2,\ l_1m_2 - m_1l_2.$$

49. Find shortest distance between the lines :

$$\vec{r} = (1-\lambda)\hat{i} + (\lambda - 2)\hat{j} + (3 - 2\lambda)\hat{k}$$

$$\vec{r} = (\mu + 1)\hat{i} + (2\mu - 1)\hat{j} - (2\mu + 1)\hat{k}.$$

50. A variable plane is at a constant distance $3p$ from the origin and meets the coordinate axes in A, B and C. If the centroid of ΔABC is (α, β, γ), then show that $\alpha^{-2} + \beta^{-2} + \gamma^{-2} = p^{-2}$.

51. A vector $\vec{n}$ of magnitude 8 units is inclined to x–axis at 45°, y axis at 60° and an acute angle with z-axis. If a plane passes through a point $\left(\sqrt{2}, -1, 1\right)$ and is normal to $\vec{n}$, find its equation in vector form.

52. Find the foot of perpendicular from the point $2\hat{i} - \hat{j} + 5\hat{k}$ on the line $\vec{r} = \left(11\hat{i} - 2\hat{j} - 8\hat{k}\right) + \lambda\left(10\hat{i} - 4\hat{j} - 11\hat{k}\right)$. Also find the length of the perpendicular.

53. A line makes angles α, β, λ, δ with the four diagonals of a cube. Prove that

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}.$$

54. Find the equation of the plane passing through the intersection of planes $2x + 3y - z = -1$ and $x + y - 2z + 3 = 0$ and perpendicular to the plane $3x - y - 2z = 4$. Also find the inclination of this plane with xy-plane.

ANSWERS

1. $\sqrt{b^2 + c^2}$

2. 90°

3. $\dfrac{x-2}{3} = \dfrac{y+3}{4} = \dfrac{z-5}{-1}$.

4. $\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + 3\hat{k}\right)$

5. $\lambda = 2$

6. 2

7. $\dfrac{x-1}{0} = \dfrac{y+1}{2} = \dfrac{z}{-1}$.

8. $\pm\dfrac{1}{3}, \mp\dfrac{2}{3}, \pm\dfrac{2}{3}$

10. A random variable X, taking values 0, 1, 2 has the following probability distribution for some number k.

$$P(X) = \begin{cases} k & \text{if } X = 0 \\ 2k & \text{if } X = 1 \\ 3k & \text{if } X = 2 \end{cases}, \text{ find } k.$$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

11. A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. What is the probability that the problem is solved.

12. A die is rolled. If the outcome is an even number, what is the probability that it is a prime?

13. If A and B are two events such that
$P(A) = \frac{1}{4}, P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$. Find P (not A and not B).

14. In a class of 25 students with roll numbers 1 to 25, a student is picked up at random to answer a question. Find the probability that the roll number of the selected student is either a multiple of 5 or of 7.

15. A can hit a target 4 times in 5 shots B three times in 4 shots and C twice in 3 shots. They fire a volley. What is the probability that atleast two shots hit.

16. Two dice are thrown once. Find the probability of getting an even number on the first die or a total of 8.

17. A and B throw a die alternatively till one of them throws a '6' and wins the game. Find their respective probabilities of winning, if A starts the game.

18. If A and B are events such that $P(A) = \frac{1}{2}, P(A \cup B) = \frac{3}{5}$ and $P(B) = p$ find p if events

    (i) are mutually exclusive,

    (ii) are independent.

transport are respectively $\frac{3}{10}$, $\frac{1}{5}$, $\frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{12}$ if he comes by train, bus and scooter respectively but if comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that he comes by train?

30. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is six. Find the probability that it is actually a six. What is the importance of "Always Speak the Truth"?

31. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident is 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver? Which mode of transport would you suggest to a student and why?

32. Two cards from a pack of 52 cards are lost. One card is drawn from the remaining cards. If drawn card is heart, find the probability that the lost cards were both hearts.

33. A box *X* contains 2 white and 3 red balls and a bag *Y* contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag *Y*.

34. In answering a question on a multiple choice, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be incorrect with probability $\frac{1}{4}$. What is the probability that the student knows the answer, given that he answered correctly?

35. Suppose a girl throws a die. If she gets 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4 she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head. What is the probability that she throws 1, 2, 3 or 4 with the die?

36. In a bolt factory machines *A, B* and *C* manufacture 60%, 30% and 10% of the total bolts respectively, 2%, 5% and 10% of the bolts produced by

13. Using properties of determinants, prove that

$$\begin{vmatrix} 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix} = (x+y+z)^3$$

14. Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ with respect to

$\cos^{-1}\left(2x\sqrt{1-x^2}\right)$, when $x \neq 0$

15. If $y = x^x$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$

16. Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

is (a) Strictly Increasing

(b) Strictly Decreasing

OR

Find the equations of the tangent and normal to the curve

$$x = a\sin^3\theta \text{ and } y = a\cos^3\theta \text{ at } \theta = \frac{\pi}{4}$$

17. Evaluate : $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x}\, dx$

OR

Evaluate : $\int (x-3)\sqrt{x^2+3x-18}\, dx$

18. Find the particular solution of the differential equation

$e^x\sqrt{1-y^2}\, dx + \frac{y}{x}\, dy = 0$, given that $y = 1$ when $x = 0$

$$\text{or } = \frac{1}{3}\left(x^2 + 3x - 18\right)^{3/2}$$

$$-\frac{9}{8}\left\{(2x+3)\sqrt{x^2+3x-18} - \frac{81}{2}\log\left|x + \frac{3}{2} + \sqrt{x^2+3x-18}\right|\right\} + c$$

18. $e^x\sqrt{1-y^2}\,dx = \frac{-y}{x}\,dy \Rightarrow xe^x\,dx = \frac{-y}{\sqrt{1-y^2}}\,dy$ 1m

Integrating both sides

$$\int xe^x\,dx = \frac{1}{2}\int \frac{-2y}{\sqrt{1-y^2}}\,dy$$

$\Rightarrow xe^x - e^x = \sqrt{1-y^2} + c$ 1+1m

For $x = 0$ $y = 1$, $c = -1$ solution is : $e^x(x-1) = \sqrt{1-y^2} - 1$ ½+½m

19. Given differential equation can be written as

$\frac{dy}{dx} + \frac{2x}{x^2-1}y = \frac{2}{(x^2-1)^2}$ 1m

Integrating factor $= e^{\int \frac{2x}{x^2-1}dx} = e^{\log(x^2-1)} = x^2 - 1$ 1m

$\therefore$ Solution is $y.\left(x^2-1\right) = \int \frac{2}{\left(x^2-1\right)^2}.\left(x^2-1\right)dx + c$ 1m

$$\Rightarrow y\left(x^2-1\right) = 2\int \frac{1}{x^2-1}\,dx + c$$

$\Rightarrow y\left(x^2-1\right) = \log\left|\frac{x-1}{x+1}\right| + c$ 1m

20. $\left[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}\right] = \left(\vec{a}+\vec{b}\right).\left\{\left(\vec{b}+\vec{c}\right)\times\left(\vec{c}+\vec{a}\right)\right\}$ ½m

$= \left(\vec{a}+\vec{b}\right)\left\{\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{c}+\vec{c}\times\vec{a}\right\}$ 1m

$= \vec{a}.\left(\vec{b}\times\vec{c}\right)+\vec{a}.\left(\vec{b}\times\vec{a}\right)+\vec{a}.\left(\vec{c}\times\vec{a}\right)+\vec{b}.\left(\vec{b}\times\vec{c}\right)$
$+\vec{b}.\left(\vec{b}\times\vec{a}\right)+\vec{b}.\left(\vec{c}\times\vec{a}\right)$ 1½m

$\left\{\vec{a}.\left(\vec{b}\times\vec{a}\right) = \vec{a}.\left(\vec{c}\times\vec{a}\right) = \vec{b}.\left(\vec{b}\times\vec{c}\right) = \vec{b}.\left(\vec{b}\times\vec{a}\right) = 0\right\}$

$= 2\left\{\vec{a}.\left(\vec{b}\times\vec{c}\right)\right\} = 2\left[\vec{a},\vec{b},\vec{c}\right]$ 1m

OR

$\vec{a}+\vec{b}+\vec{c} = \vec{0} \quad \therefore\ \vec{a}+\vec{b} = -\vec{c}$ ½m

$\Rightarrow \left(\vec{a}+\vec{b}\right)^2 = \left(-\vec{c}\right)^2 = \left(\vec{c}\right)^2$ ½m

$\Rightarrow \left|\vec{a}\right|^2+\left|\vec{b}\right|^2+2\,\vec{a}.\vec{b}+\left|\vec{c}\right|^2$ 1m

$\Rightarrow .9+25+2\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta = 49$, θ being angle between $\vec{a}$ & $\vec{b}$ 1m

$\therefore\ \cos\theta = \dfrac{15}{2.3.5} = \dfrac{1}{2} \Rightarrow \theta = \dfrac{\pi}{3}$ 1m

21. Let $\dfrac{x+1}{3} = \dfrac{y+3}{5} = \dfrac{z+5}{7} = u;\ \dfrac{x-2}{1} = \dfrac{y-4}{3} = \dfrac{z-6}{5} = v$

General points on the lines are

$(3u-1,\ 5u-3,\ 7u-5)$ & $(v+2,\ 3v+4.\ 5v+6)$ 1m

lines intersect if

$3u-1 = v+2,\ 5u-3 = 3v+4,\ 7u-5 = 5v+6$ for some u & v 1m

or $3u-v = 3$.........(1), $5u-3v = 7$............(2), $7u-5v = 11$.............(3)

Solving equations (1) and (2), we get $u = \dfrac{1}{2}, v = -\dfrac{3}{2}$ ½m

Putting u & v in equation (3), $7.\dfrac{1}{2}-5\left(-\dfrac{3}{2}\right) = 11$ $\therefore$ lines intersect ½m

Probability distribution is :

X :	0	1	2	3	4
$P(X)$:	$\frac{16}{81}$	$\frac{32}{81}$	$\frac{24}{81}$	$\frac{8}{81}$	$\frac{1}{81}$
$X\,P(X)$:	0	$\frac{32}{81}$	$\frac{48}{81}$	$\frac{24}{81}$	$\frac{4}{81}$

2½+½m

Mean $= \sum X\,P(X) = \frac{108}{81}$ or $\frac{4}{3}$ 1m

4. If $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $\vec{a}\,.\,\vec{b} = \dfrac{-15}{2}$ then

write the angle between $\vec{a}$ and $\vec{b}$.

5. What is the area of a parallelogram whose adjacent

sides are given by vectors $\hat{c} + \hat{j}$ and $2\hat{i} - 3\hat{k}$?

6. If the lines $\dfrac{x-1}{4} = \dfrac{y+1}{2} = \dfrac{1-z}{-2}$ and $\vec{r} = \hat{i} + \lambda\left(2\hat{i} - \hat{j} + 3p\hat{k}\right)$ are perpendicular, then write the value of 'p'.

SECTION B

Question number 7 to 19 carry 4 marks each.

7. Using elementary transformations find the inverse of the matrix

$$A = \begin{pmatrix} 8 & 3 \\ 5 & 2 \end{pmatrix}$$

8. Two schools A and B decided to award prizes to their students for two values honesty and punctuality. School A decided to award a total of Rs. 18000 for two values to 2 and 3 students respectively while School B decided to award Rs. 22000 for two values to 6 and 1 students respectively. What is the amount given for honesty and for punctuality. Solve using matrix method.

Which value you prefer to be rewarded most and why?

9. If $\begin{vmatrix} x+7 & 18 & y+a \\ x+8 & 25 & y+b \\ x+9 & 32 & y+c \end{vmatrix} = 0$ and $a + c \neq 2b$

then find the value of 'x'.

10. Prove that

$$\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{4}{5}\right) = \tan^{-1}\left(\frac{63}{16}\right)$$

15. $\frac{x^3}{3}\tan^{-1}x = \frac{1}{3}\left(\frac{x^2}{2} - \frac{1}{2}\log\left|x^2+1\right|\right) + c$

16. $2y = \frac{d^2y}{dx^2} - \frac{dy}{dx}$ 17. $\left|\vec{a}\times\vec{b}\right| = 84$

18. $\sqrt{24}$ units

19.

X	0	1	2
P(X)	1/16	3/8	9/16

Section - C

20. $f^{-1}(x) = \frac{3x+3}{x-2}$ 21. Each side is $r\sqrt{3}$ cm.

22. $\left(\frac{32}{3\pi} - \frac{4\sqrt{3}}{3}\right)$ sq. units. ***OR*** $\frac{8}{[illegible]}$ sq. units.

23. $x + ye^{xy} = 2$

24. Equation of plane is $8x - 6y + z = 23$

OR

(1, 6, 0), $2\sqrt{6}$ unit & image is (–3, 8, –2)

25. 4/9, If a person speaks truth then integrity and character develops & he/she rises in life.

26. 30 km in 1 hr. ($\frac{50}{3}$ km at 25 km/hr and $\frac{40}{3}$ km at 40 km/hr) The values promoted are the safety of life and saving petrol (energy).

17. $-e^{-y} = e^{x} + \frac{x^{3}}{3} + c$

18. $\lambda = 1$

19. $x = 1, 2, -15$

Section - C

20. $\frac{9}{13}$

21. ' * ' is commutative and associative. Identity element is 4, Inverse of 8 is 2.

22. Units of type A = 3, Units of type B = 8

Maximum Revenue = Rs. 1900

Yes. We all should give equal rights to men and women.

23. length $= \left(\frac{20}{4 + \pi}\right) m$, breadth $= \left(\frac{10}{4 + \pi}\right) m$

24. $\left(\frac{8\pi}{3} - 2\sqrt{3}\right)$ sq. units.

OR

$\frac{1}{3}$ sq. units.

25. (1, 0, 7)

26. ***OR*** $\frac{27}{2}$