Basic Concepts Behind the Binary System

To understand binary numbers, begin by recalling elementary school math. When we first learned about numbers, we were taught that, in the decimal system, things are organized into columns:

```
H | T | O
1 | 9 | 3
```

such that "H" is the hundreds column, "T" is the tens column, and "O" is the ones column. So the number "193" is 1-hundreds plus 9-tens plus 3-ones.

Years later, we learned that the ones column meant 10^0, the tens column meant 10^1, the hundreds column 10^2 and so on, such that

```
10^2|10^1|10^0
  1 |  9 |  3
```

the number 193 is really {(1*10^2)+(9*10^1)+(3*10^0)}.

As you know, the decimal system uses the digits 0-9 to represent numbers. If we wanted to put a larger number in column 10^n (e.g., 10), we would have to multiply 10*10^n, which would give 10^(n+1), and be carried a column to the left. For example, putting ten in the 10^0 column is impossible, so we put a 1 in the 10^1 column, and a 0 in the 10^0 column, thus using two columns. Twelve would be 12*10^0, or 10^0(10+2), or 10^1+2*10^0, which also uses an additional column to the left (12).

The binary system works under the exact same principles as the decimal system, only it operates in base 2 rather than base 10. In other words, instead of columns being

```
10^2|10^1|10^0
```

they are

```
2^2|2^1|2^0
```

Instead of using the digits 0-9, we only use 0-1 (again, if we used anything larger it would be like multiplying 2*2^n and getting 2^n+1, which would not fit in the 2^n column. Therefore, it would shift you one column to the left. For example, "3" in binary cannot be put into one column. The first column we fill is the right-most column, which is 2^0, or 1. Since 3>1, we need to use an extra column to the left, and indicate it as "11" in binary (1*2^1) + (1*2^0).

Examples: What would the binary number 1011 be in decimal notation?

Try a few examples of binary addition:

```
                    1                 1
      111             111             111
     +110                  +110            +110
    ______              ______          _____
           1                    01            1101

      1                        11             1
 101          101     101
    +111                  +111             +111
    _____               ____             _____
          0                    00             1100

          1                    1                      1
   111     111        111
      +111                 +111                   +111
      _____               _____                  _____
     0                    10                   1110
```

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Using the regular algorithm for binary adition, add (5+12), (-5+12), (-12+-5), and (12+-12) in each system. Then convert back to decimal numbers.

```
Signed Magnitude:

    5+12          -5+12          12+-5           12+-12

 00000101       10000101       10001100        00001100
 00001100       00001100       10000101        10001100
__________     ________       ________        _________
 00010001       10010001       00010000        10011000

    17            -17             16              -24

One' Complement:

 00000101       11111010       11110011        00001100
 00001100       00001100       11111010        11110011
_________      ________       ________        ________
 00010001       00000110       11101101        11111111

    17              6            -18               0

Two's Complement:

 00000101       11111011       11110100        00001100
 00001100       00001100       11111011        11110100
 ________      ________       ________        ________
 00010001       00000111       11101111        00000000

     17             7            -17                0

Signed Magnitude:
```