General term

In the expansion of $(x+y)^n$ $(r+1)^{th}$ term is called the general term which can be represented by T_{r+1}

$T_{r+1} = {}^nC_r\, x^{n-r}\, a^r = {}^nC_r$ (first term)$^{n-r}$ (second term)^r.

Independent term or Constant term

Independent term or constant term of a binomial expansion is the term in which exponent of the variable is zero.

Condition : $(n-r)$ [Power of x] + r [Power of y] = 0, in the expansion of $[x+y]^n$.

Number of terms in the expansion of $(a+b+c)^n$ and $(a+b+c+d)^n$

$(a+b+c)^n$ can be expanded as : $(a+b+c)^n = \{(a+b)+c\}^n$

$$= (a+b)^n + {}^nC_1(a+b)^{n-1}(c)^1 + {}^nC_2(a+b)^{n-2}(c)^2 + \ldots + {}^nC_n c^n$$

$$= (n+1)\text{term} + n\,\text{term} + (n-1)\text{term} + \ldots + 1\,\text{term}$$

$\therefore$ Total number of terms

$$= (n+1)+(n)+(n-1)+\ldots\ldots+1 = \frac{(n+1)(n+2)}{2}.$$

Similarly, number of terms in the expansion of

$$(a+b+c+d)^n = \frac{(n+1)(n+2)(n+3)}{6}.$$

Middle term

The middle term depends upon the value of n.

(1) **When n is even**, then total number of terms in the expansion of $(x+y)^n$ is $n+1$ (odd). So there is only one middle term i.e., $\left(\frac{n}{2}+1\right)^{th}$ term is the middle term. $T_{\left[\frac{n}{2}+1\right]} = {}^nC_{n/2}\, x^{n/2} y^{n/2}$

(2) **When n is odd**, then total number of terms in the expansion of $(x+y)^n$ is $n+1$ (even). So, there are two middle terms i.e., $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+3}{2}\right)^{th}$ are two middle terms.

$$T_{\left(\frac{n+1}{2}\right)} = {}^nC_{\frac{n-1}{2}}\, x^{\frac{n+1}{2}}\, y^{\frac{n-1}{2}} \text{ and } T_{\left(\frac{n+3}{2}\right)} = {}^nC_{\frac{n+1}{2}}\, x^{\frac{n-1}{2}}\, y^{\frac{n+1}{2}}$$

- When there are two middle terms in the expansion then their binomial coefficients are equal.
- Binomial coefficient of middle term is the greatest binomial coefficient.

To determine a particular term in the expansion

In the expansion of $\left(x^{\alpha} \pm \frac{1}{x^{\beta}}\right)^n$, if x^m occurs in T_{r+1}, then r is given by $n\alpha - r(\alpha+\beta) = m \Rightarrow r = \frac{n\alpha - m}{\alpha+\beta}$

Thus in above expansion if constant term which is independent of x, occurs in T_{r+1} then r is determined by

$$n\alpha - r(\alpha+\beta) = 0 \Rightarrow r = \frac{n\alpha}{\alpha+\beta}$$

Greatest term and Greatest coefficient

(1) **Greatest term :** If T_r and T_{r+1} be the r^{th} and $(r+1)^{th}$ terms in the expansion of $(1+x)^n$, then

$$\frac{T_{r+1}}{T_r} = \frac{{}^nC_r x^r}{{}^nC_{r-1} x^{r-1}} = \frac{n-r+1}{r} x$$

Let numerically, T_{r+1} be the greatest term in the above expansion. Then $T_{r+1} \geq T_r$ or $\frac{T_{r+1}}{T_r} \geq 1$.

$$\therefore \frac{n-r+1}{r}|x| \geq 1 \text{ or } r \leq \frac{(n+1)}{(1+|x|)}|x| \qquad \ldots\text{(i)}$$

Now substituting values of n and x in (i), we get $r \leq m+f$ or $r \leq m$, where m is a positive integer and f is a fraction such that $0 < f < 1$.

When n is even T_{m+1} is the greatest term, when n is odd T_m and T_{m+1} are the greatest terms and both are equal.

Short cut method : To find the greatest term (numerically) in the expansion of $(1+x)^n$.

(i) Calculate $m = \left|\frac{x(n+1)}{x+1}\right|$

(ii) If m is integer, then T_m and T_{m+1} are equal and both are greatest term.

(iii) If m is not integer, then $T_{[m]+1}$ is the greatest term, where [.] denotes the greatest integral part.

(2) **Greatest coefficient**

(i) If n is even, then greatest coefficient is ${}^nC_{n/2}$

(ii) If n is odd, then greatest coefficient are ${}^nC_{\frac{n+1}{2}}$ and ${}^nC_{\frac{n+3}{2}}$.

To find a term from the end in the expansion of $(x+a)^n$

It can be easily seen that in the expansion of $(x+a)^n$

$(r+1)^{th}$ term from end $= (n-r+1)^{th}$ term from beginning

i.e. $T_{r+1}(E) = T_{n-r+1}(B)$

$\therefore T_r(E) = T_{n-r+2}(B)$.

Properties of binomial coefficients

In the binomial expansion of $(1+x)^n$,

$$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots\ldots + {}^nC_r x^r + \ldots + {}^nC_n x^n$$

where ${}^nC_0, {}^nC_1, {}^nC_2, \ldots\ldots, {}^nC_n$ are the coefficients of various powers of x and called binomial coefficients, and they are written as $C_0, C_1, C_2, \ldots C_n$.

Hence, $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots\ldots + C_r x^r + \ldots\ldots + C_n x^n \ldots\ldots$(i)

(1) The sum of binomial coefficients in the expansion of $(1+x)^n$ is 2^n.

Putting $x=1$ in (i), we get $2^n = C_0 + C_1 + C_2 + \ldots\ldots + C_n \ldots$(ii)

(2) Sum of binomial coefficients with alternate signs : Putting $x=-1$ in (i)

Preview from Notesale.co.uk
Page 2 of 5

Scanned by CamScanner