5.1. Mathematical Induction

The Principle of Mathematical Induction

Given a statement $\mathcal{P}$ concerning the integer n, suppose that

1. $\mathcal{P}$ is true for some particular integer n_0;
2. If $\mathcal{P}$ is true for some particular integer $k \geq n_0$, then it is true for the next integer $k+1$.

Then $\mathcal{P}$ is true for all integers $n \geq n_0$.

Problem. Prove that $1+3+5+\ldots+(2n-1) = n^2$.

Solution. Put $n_0 = 1$. Then

$$1+3+\ldots+(2n_0-1) = 1 = (n_0)^2.$$

Now suppose that k is an integer, $k \geq 1$, and that the statement is true for $n = k$:

$$1+3+\ldots+(2k-1) = k^2.$$

We must show that

$$1+3+\ldots+(2(k+1)-1) = (k+1)^2.$$

Indeed,

$$1+3+\ldots+(2k-1)+(2(k+1)-1) =$$

$$= k^2 + (2k+1) = (k+1)^2.$$

By the Principle of Math. Induction,

$$1+3+5+\ldots+(2n-1) = n^2$$

for all integers $n \geq 1$.