Unit 1 - 1 The Straight Line

Recognising equations of straight lines

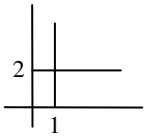
Straight lines only have **x** and **y** appearing with no higher power than 1.

If there are any terms such as x^2, x^3, x^4 or y^2, y^3, y^4 terms etc – it is **NOT** a straight line.

e.g. $y = x^2 + 2$ is not a straight line

$x - y = 3$ is a straight line

Equations of lines parallel to OX and OY



$x = 1$ is a line parallel to the y-axis passing through x = 1

$y = 2$ is a line parallel to the x-axis passing through y = 2

The equation of a line of gradient m through a point (a, b)

Use $m = \dfrac{y-b}{x-a}$

or $y - b = m(x - a)$

where m is the gradient and (a, b) the point

When you do not have the y-intercept, but only the gradient and a point, you should proceed as follows. Recall the gradient formula, you need two points on the line to obtain the gradient.

Since you have the gradient and a point, then to obtain the equation, choose another general point (x, y). Use the gradient formula again in this form.

Example: Find the equation of the line with gradient 3, passing through P(1, 2)

$3 = \dfrac{y-2}{x-1}$ so $3(x - 1) = (y - 2)$ $3x - 3 = y - 2$

simplify $\Rightarrow$ $y = 3x - 1$ or $y - 3x + 1 = 0$ or $y - 3x = -1$

Intersecting Lines:

Treat the equations of the lines as simultaneous equations.

To find the co-ordinates of the point where two lines intersect, treat the equations of the lines as simultaneous equations and solve them.

The solution (x, y) is the point of intersection

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Some useful bits and pieces:

Median of a triangle - the line drawn from a vertex to the **middle** of the opposite side.

Altitude of a triangle – the line drawn from a vertex perpendicular to the opposite side. (Measures the **height**)

Perpendicular bisector of a line is a perpendicular line passing through the mid-point.

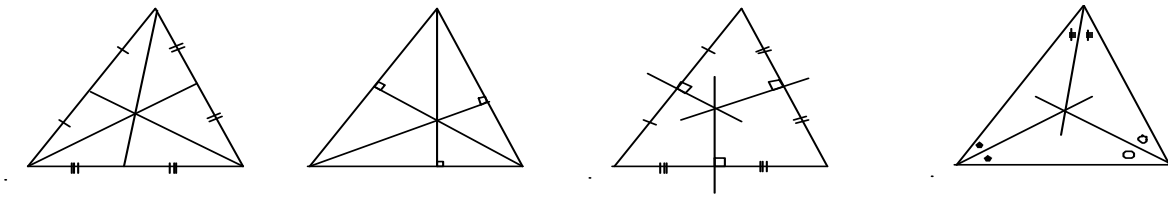
Perpendicular at right angles to

Intersect where lines cut each other

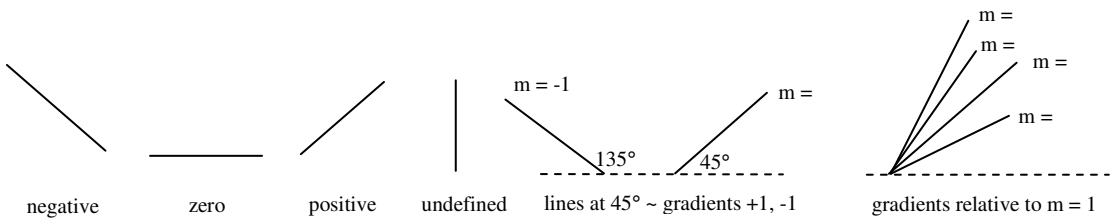
Bisect lines cut each other exactly in **half**.

Locus the path traced out according to some specified condition.
By using this condition, you can find the equation of this path which is called the locus.

Image of a line under reflection, translation or rotation is the position where line moves to under the transformation.



Gradients



Unit 1 - 2.1 Composite and Inverse Functions

Definition of a function

A function is defined from a set A to a set B as a rule which links each member of A to exactly one member of B.

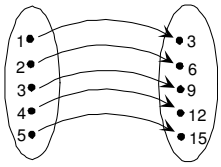
Notation:

y = f(x) or f : x → y (f maps x to y)

Domain and Range

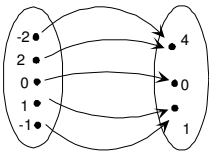
The **domain** of a function is the **input** – the variable the function operates upon.

The **range** of a function is the **output** – the value of the function.



Domain Range

f(x) = 3x or f : x → 3x



Domain Range

g(x) = x^2 or g : x → x^2

Domain of h(x)=√x and k(x) = $\frac{1}{x-1}$

We write this as:

domain of **h(x) is {x : x∈ℜ : x ≥ o }**

and:

domain of **k(x) is: {x : x∈ℜ : x ≠ 1 }**

You should always be aware of the danger of dividing by zero.

The largest domain of h(x) = √x is the set of real numbers greater than or equal to zero, since you cannot take the square root of a negative number.

The largest domain of $k(x) = \frac{1}{x-1}$ is the set of real numbers except x = 1

(since this would make the denominator zero ~ you cannot divide by zero).

Undefined functions

Functions may be undefined for particular values of x ~ in particular:

- where you would need to take the square root of a negative number
- where you would need to divide by 0

$f(x) = \sqrt{x-1}$ is undefined when x < 1
(requires square root of negative number)

$h(x) = \frac{x}{x-3}$ is undefined when x = 3
(results in division by zero).

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Related Functions

Given f(x), what is f(x+1) or f(x^2) or f(2x) etc.

To find f(x+1),

simply replace the 'x' in f(x) with 'x+1' etc.

and simplify.

Example: f(x) = 3x + 1 what is f(x+1)
Solution: f(x+1) = 3(x+1) + 1 ⇒ **3x + 4**

Example: h(x) = x^2 – 3x what is h(2x)
Solution: h(2x) = $(2x)^2$ – 3(2x) ⇒ **4 x^2 – 6x**

Example: f(x) = $2x^2$ + 3x what is f(x+1)
Solution: f(x+1) = $2(x+1)^2$ + 3(x+1) ⇒ **$2x^2$ + 7x + 5**

Evaluating functions:

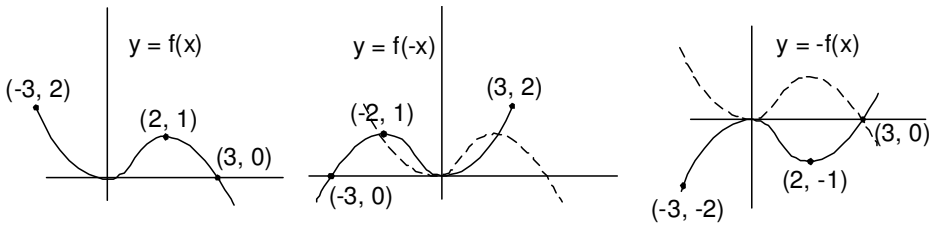
Given f(x), what is f(1) or f(0) or f(-2) etc.

To evaluate a function f(x) at x = 2 (say), calculate what the value of the function is when you replace x by 2

Example: If f(x) = x^2 + 3x – 1 Evaluate f(-1)
Solution: f(-1) = $(-1)^2$ + 3(-1) – 1 ⇒ **-3**

Example: If f(x) = $3x^3$ – 5x + 2 What is f(a)
Solution: f(a) = $3a^3$ – 5a + 2

Examples of graphs of related functions



The exponential function and its graph

Any function of the form

$f(x) = a^x$ where $a > 0$ and $a \neq 1$

is called an *exponential function* with *base a.*

The graph of the function has equation $y = a^x$

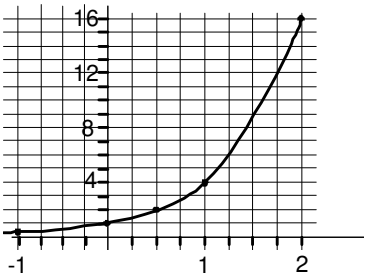
Note: In **all cases** the graph passes through (0, 1) since $a^0 = 1$ for all values of a and the line y = 0 is an **asymptote** to the graph $y = a^x$

The graph of the function $f(x) = 4^x$ is shown here

$f(2) = 4^2 = 16$ $\quad f(1) = 4^1 = 4$

$f(0) = 4^0 = 1$ $\quad f\left(\frac{1}{2}\right) = 4^{\frac{1}{2}} = 2$

$f(-1) = 4^{-1} = 0.25$

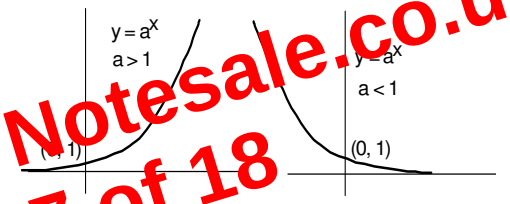


Decreasing and increasing exponential functions

For the function $f(x) = a^x$

if ***a* > 1**
then the function is an **increasing** function.

if ***a* < 1**
then the function is a **decreasing** function.



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Sketching graphs of exponential functions

Method:

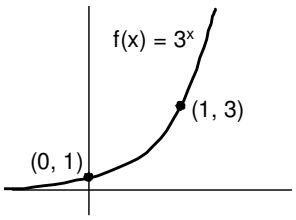
The graph must pass through (0, 1)

Is it decreasing or increasing (is $a > 1$ or $a < 1$)

Pick a suitable point (e.g. x = 1 or x = 2) to get an idea of the steepness

Example: Sketch the graph of $f(x) = 3^x$

The graph passes through (0, 1)
It is increasing, since $a = 3$
Choose x = 1 so f(x) = 3
You have 2 points ~ (0, 1) and (1, 3)
You know it is increasing



The Logarithmic Function and its Graph

$f(x) = a^x$ has an inverse function $f(x) = \log_a x$

These are related as follows:

$$y = a^x \quad \Leftrightarrow \quad x = \log_a y$$

The graph of $y = \log_a x$ is the mirror image of the graph $y = a^x$ in the line y = x.

Note that the line **x = 0** is an **asymptote** to the graph $y = \log_a x$