5. solve the differential equation for y:

$y' - 3y = e^{2x}$, where $y = f(x)$, for $y(2) = -1$

thus, for $y' + a(x) \cdot y = b(x)$, $a(x) = -3$ & $b(x) = e^{2x}$.

given the formula $$y = \frac{\int e^{\int a(x)\,dx} \cdot b(x)\,dx + C}{e^{\int a(x)\,dx}},$$

we may write that $$y = \frac{\int e^{\int -3\,dx} \cdot e^{2x}\,dx + C}{e^{\int -3\,dx}} =$$

$$\frac{\int e^{-x}\,dx + C}{e^{-3x}} = \frac{-e^{-x} + C}{e^{-3x}} = e^{3x}\left(-e^{-x} + C\right) = -e^{2x} + C \cdot e^{3x}$$

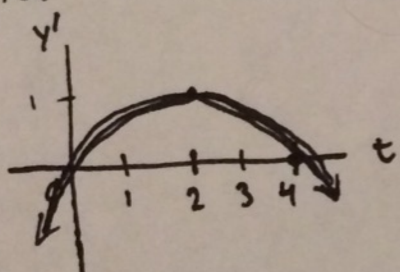
when $y(2) = -1$, $y = -1$ and $x = 2$.

thus, since $y = -e^{2x} + C \cdot e^{3x}$, we know $-1 = -e^4 + C \cdot e^6 \Rightarrow$

$$e^4 - 1 = C \cdot e^6 \Rightarrow \frac{e^4 - 1}{e^6} = C \Rightarrow e^{-2} - e^{-6} = C$$

thus, $y = -e^{2x} + e^{3x}\left(e^{-2} - e^{-6}\right)$

6. graph the direction field of $y' = y - \frac{1}{4}y^2$, for $y = f(t)$, for the curves starting at $y = 1$ & $y = -1$:



first we graph y' & y on one plane, so we can see the value of y' for any y. it looks like $y' > \emptyset$ on $\emptyset < y < 4$.

~~[illegible]~~ from there, we graph the direction field, keeping in mind that we're looking for slopes not values: