$$\sim \begin{bmatrix} 1 & 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & 0 & -1 & 2 & -2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & 2 & -2 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -2 & 2 & -1 \end{bmatrix}$$

Thus

$$A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ 0 & -1 & 1 \\ -2 & 2 & -1 \end{bmatrix}.$$

4. Compute the following determinant by row operations (20pts)

$$\begin{vmatrix} 4 & -2 & 1 & -3 \\ 8 & 2 & 1 & -3 \\ 4 & 1 & 1 & -2 \\ 8 & -1 & 1 & -3 \end{vmatrix}$$

Solution. By elementary row operations, we have

$$\begin{vmatrix} 4 & -2 & 1 & -3 \\ 8 & 2 & 1 & -3 \\ 4 & 1 & 1 & -2 \\ 8 & -1 & 1 & -3 \end{vmatrix} = \begin{vmatrix} 4 & -2 & 1 & -3 \\ 0 & 6 & -1 & 3 \\ 0 & 3 & 0 & 1 \\ 0 & 3 & -1 & 3 \end{vmatrix} = 4 \begin{vmatrix} 6 & -1 & 3 \\ 3 & 0 & 1 \\ 3 & -1 & 3 \end{vmatrix}$$

$$= 4 \begin{vmatrix} 6 & -1 & 3 \\ 3 & 0 & 1 \\ -3 & 0 & 0 \end{vmatrix} = 4 \begin{vmatrix} 3 & 1 \\ -3 & 0 \end{vmatrix} = 12.$$