Scratch Paper

You will receive a supply of scratch paper before you begin the test. You can replenish your supply of scratch paper as necessary throughout the test by asking the test administrator.

Breaks

There is a 10-minute break following the third section and a one-minute break between the other test sections. You might want to replenish your supply of scratch paper during a scheduled break. Section timing will not stop if you take an unscheduled break, so you should proceed with your test without interruption once it begins.

Test-taking Strategies for the Paper-based Test

Analytical Writing Sections

In the paper-based revised General Test, the topics in the Analytical Writing measure will be presented in the test book, and you will handwrite your essay responses in the test book in the space provided.

It is important to budget your time. Within the 30-minute time limit for each section, you'll need to allow sufficient time to think about the topic, plan a response, and compose your essay. Although GRE readers understand the time constraints under which you write and will consider your response a first draft, you will still want to produce the best possible example of your writing.

Save a few minutes at the end of each timed section to check for obvious errors. Although an occasional spelling or grammatical error will not affect your score, severe and persistent errors will detract from the overall effectiveness of your writing and lower your score.

During the actual administration of the revised General Test, you may work only on the particular writing section the test center supervisor designates and only for the time allowed. You may *not* go back to an earlier section of the test after the supervisor announces, "Please stop work" for that section. The supervisor is authorized to dismiss you from the center for doing so.

Verbal Reasoning and Quantitative Reasoning Sections

The questions in the Verbal Reasoning and Quantitative Reasoning measures have a variety of formats. Some require you to select a single answer choice; others require you to select one or more answer choices, and yet others require you to enter a numeric answer. Make sure when answering a question that you understand what response is required. A calculator will be provided at the test center for use during the Quantitative Reasoning sections.

When taking a Verbal Reasoning or Quantitative Reasoning section, you are free, within that section, to skip questions that you might have difficulty answering and come back to them later during the time provided to work on that section. Also during that time you may change the answer to any question in that section by erasing it completely and filling in an alternative answer. Be careful not to leave any stray marks in

minor variations in difficulty among the different test editions. Thus, a given scaled score for a particular measure reflects the same level of performance regardless of which edition of the test was taken.

Score Reporting

Three scores will be reported on the revised General Test:

- a Verbal Reasoning score reported on a 130–170 score scale, in 1-point increments
- a Quantitative Reasoning score reported on a 130–170 score scale, in 1-point increments
- an Analytical Writing score reported on a 0–6 score scale, in half-point increments

If you do not answer any questions at all for a measure (Verbal Reasoning, Quantitative Reasoning, or Analytical Writing), you receive a No Score (NS) for that measure.

Descriptions of the analytical writing abilities characteristic of particular score levels are available in this publication on page 41, and on the GRE website at **www.ets.org/gre/awscoredescriptions**.

The *ScoreSelect*℠ Option

Introduced in July 2012, the new *ScoreSelect*℠ option is available for both the GRE revised General Test and GRE Subject Tests and can be used by anyone with reportable scores from the last five years. This option lets you decide which test scores to send to the institutions you designate. You can send scores from your most recent test administration or scores from all of the times you've taken a GRE test as part of your four free score reports. After test day, you can send scores from your *Most Recent*, *All*, or *Any* specific test administration(s) for a fee when ordering Additional Score Reports. Just remember, scores for a test administration must be reported in their entirety. For more information, visit **www.ets.org/gre/scoreselect**.

Score Reporting Time Frames

Scores from computer-based GRE revised General Test administrations are reported approximately 10 to 15 days after the test date. Scores from paper-based administrations are reported within six weeks after the test date. If you are applying to a graduate or business school program, be sure to review the appropriate admissions deadlines and plan to take the test in time for your scores to reach the institution.

For more information on score reporting, visit the GRE website at **www.ets.org/gre/scores/get**.

2 GRE® Analytical Writing

Your goals for this chapter

- Recognize the two types of Analytical Writing tasks
- Study examples of each type of writing task
- Learn strategies for responding to the writing tasks
- Review actual student responses and ETS reader commentary

Overview of the Analytical Writing Measure

The Analytical Writing measure assesses your critical thinking and analytical writing skills. It assesses your ability to articulate and support complex ideas, construct and evaluate arguments, and sustain a focused and coherent discussion. It does not assess specific content knowledge.

The Analytical Writing measure consists of two separately timed analytical writing tasks:

- a 30-minute "Analyze an Issue" task
- a 30-minute "Analyze an Argument" task

The Issue task presents an opinion on an issue of broad interest followed by specific instructions on how to respond to that issue. You are required to evaluate the issue, considering its complexities, and develop an argument with reasons and examples to support your views.

The Argument task presents a different challenge from that of the Issue task: it requires you to evaluate a given argument according to specific instructions. You will need to consider the logical soundness of the argument rather than to agree or disagree with the position it presents.

The two tasks are complementary in that one requires you to construct your own argument by taking a position and providing evidence supporting your views on the issue, whereas the other requires you to evaluate someone else's argument by assessing its claims and evaluating the evidence it provides.

- think about the claim and instructions in relation to your own ideas and experiences, to events you have read about or observed, and to people you have known; this is the knowledge base from which you will develop compelling reasons and examples in your argument that reinforce, negate, or qualify the claim in some way
- decide what position on the issue you want to take and defend
- decide what compelling evidence (reasons and examples) you can use to support your position

Remember that this is a task in critical thinking and persuasive writing. The most successful responses will explore the complexity of the claim and instructions. As you prepare for the Issue task, you might find it helpful to ask yourself the following questions:

- What precisely is the central issue?
- What precisely are the instructions asking me to do?
- Do I agree with all or with any part of the claim? Why or why not?
- Does the claim make certain assumptions? If so, are they reasonable?
- Is the claim valid only under certain conditions? If so, what are they?
- Do I need to explain how I interpret certain terms or concepts used in the claim?
- If I take a certain position on the issue, what reasons support my position?
- What examples—either real or hypothetical—could I use to illustrate those reasons and advance my point of view? Which examples are most compelling?

Once you have decided on a position to defend, consider the perspective of others who might not agree with your position. Ask yourself:

- What reasons might someone use to refute or undermine my position?
- How should I acknowledge or defend against those views in my essay?

To plan your response, you might want to summarize your position and make brief notes about how you will support the position you're going to take. When you've done this, look over your notes and decide how you will organize your response. Then write a response developing your position on the issue. Even if you don't write a full response, you should find it helpful to practice with a few of the Issue topics and to sketch out your possible responses. After you have practiced with some of the topics, try writing responses to some of the topics within the 30-minute time limit so that you have a good idea of how to use your time in the actual test.

It would probably be helpful to get some feedback on your response from an instructor who teaches critical thinking or writing or to trade papers on the same topic with other students and discuss one another's responses in relation to the scoring guide. Try to determine how each paper meets or misses the criteria for each score point in the guide. Comparing your own response to the scoring guide will help you see how and where you might need to improve.

The Form of Your Response

You are free to organize and develop your response in any way that you think will effectively communicate your ideas about the issue and the instructions. Your response may, but need not, incorporate particular writing strategies learned in English composition or writing-intensive college courses. GRE readers will not be looking for a particular developmental strategy or mode of writing; in fact, when GRE readers are trained, they review hundreds of Issue responses that, although highly diverse in content and form, display similar levels of critical thinking and persuasive writing. Readers will see, for example, some Issue responses at the 6 score level that begin by briefly summarizing the writer's position on the issue and then explicitly announcing the main points to be argued. They will see others that lead into the writer's position by making a prediction, asking a series of questions, describing a scenario, or defining critical terms in the quotation. The readers know that a writer can earn a high score by giving multiple examples or by presenting a single, extended example. Look at the sample Issue responses, particularly at the 5 and 6 score levels, to see how other writers have successfully developed and organized their arguments.

You should use as many or as few paragraphs as you consider appropriate for your argument—for example, you will probably need to create a new paragraph whenever your discussion shifts to a new cluster of ideas. What matters is not the number of examples, the number of paragraphs, or the form your argument takes but, rather, the cogency of your ideas about the issue and the clarity and skill with which you communicate those ideas to academic readers.

Sample Issue Task

> As people rely more and more on technology to solve problems, the ability of humans to think for themselves will surely deteriorate.
>
> Discuss the extent to which you agree or disagree with the statement and explain your reasoning for the position you take. In developing and supporting your position, you should consider ways in which the statement might or might not hold true and explain how these considerations shape your position.

Strategies for This Topic

In this task, you are asked to discuss the extent to which you agree or disagree with the statement. Thus, responses may range from strong agreement or strong disagreement, to qualified agreement or qualified disagreement. You are also instructed to explain your reasoning and consider ways in which the statement might or might not hold true. A successful response need not comment on all or any one of the points listed below and may well discuss other reasons or examples not mentioned here in support of its position.

Although this topic is accessible to respondents of all levels of ability, for any response to receive a top score, it is particularly important that you remain focused on the task and provide clearly relevant examples and/or reasons to support the point of view you are expressing. Lower level responses may be long and full of examples of modern technology, but those examples may not be clearly related to a particular position. For example, a respondent who strongly disagrees with the statement may choose

Essay Responses and Reader Commentary

Score 6 Response *

The statement linking technology negatively with free thinking plays on recent human experience over the past century. Surely there has been no time in history where the lived lives of people have changed more dramatically. A quick reflection on a typical day reveals how technology has revolutionized the world. Most people commute to work in an automobile that runs on an internal combustion engine. During the workday, chances are high that the employee will interact with a computer that processes information on silicon bridges that are .09 microns wide. Upon leaving home, family members will be reached through wireless networks that utilize satellites orbiting the earth. Each of these common occurences would have been inconceivable at the turn of the 19th century.

The statement attempts to bridge these dramatic changes to a reduction in the ability for humans to think for themselves. The assumption is that an increased reliance on technology negates the need for people to think creatively to solve previous quandaries. Looking back at the introduction, one could argue that without a car, computer, or mobile phone, the hypothetical worker would need to find alternate methods of transport, information processing, and communication. Technology short circuits this thinking by making the problems obsolete.

However, this reliance on technology does not necessarily preclude the creativity that marks the human species. The prior examples reveal that technology allows for convenience. The car, computer, and phone all release additional time for people to live more efficiently. This efficiency does not preclude the need for humans to think for themselves. In fact, technology frees humanity to not only tackle new problems, but may itself create new issues that did not exist without technology. For example, the proliferation of automobiles has introduced a need for fuel conservation on a global scale. With increasing energy demands from emerging markets, global warming becomes a concern inconceivable to the horse-and-buggy generation. Likewise dependence on oil has created nation-states that are not dependent on taxation, allowing ruling parties to oppress minority groups such as women. Solutions to these complex problems require the unfettered imaginations of maverick scientists and politicians.

In contrast to the statement, we can even see how technology frees the human imagination. Consider how the digital revolution and the advent of the internet has allowed for an unprecedented exchange of ideas. WebMD, a popular internet portal for medical information, permits patients to self research symptoms for a more informed doctor visit. This exercise opens pathways of thinking that were previously closed off to the medical layman. With increased interdisciplinary interactions, inspiration can arrive from the most surprising corners. Jeffrey Sachs, one of the architects of the UN Millenium Development Goals, based his ideas on emergency care triage techniques. The unlikely marriage of economics and medicine has healed tense, hyperinflation environments from South America to Eastern Europe.

This last example provides the most hope in how technology actually provides hope to the future of humanity. By increasing our reliance on technology, impossible goals can now be achieved. Consider how the late 20th century witnessed the complete elimination of smallpox. This disease had ravaged the human race since prehistorical days, and yet with the technology of vaccines, free thinking humans dared to imagine a

*All responses in this publication are reproduced exactly as written, including errors, misspellings, etc., if any.

Write down each of these thoughts as a brief note. When you've gone as far as you can with your evaluation, look over the notes and put them in a good order for discussion (perhaps by numbering them). Then write an evaluation according to the specific instructions by fully developing each point that is relevant to those instructions. Even if you choose not to write a full essay response, you should find it very helpful to practice evaluating a few of the arguments and sketching out your responses. When you become quicker and more confident, you should practice writing some Argument responses within the 30-minute time limit so that you will have a good sense of how to pace yourself in the actual test. For example, you will not want to discuss one point so exhaustively or to provide so many equivalent examples that you run out of time to make your other main points.

You might want to get feedback on your response(s) from a writing instructor, a philosophy teacher, or someone who emphasizes critical thinking in his or her course. It can also be very informative to trade papers on the same topic with fellow students and discuss one another's responses in terms of the scoring guide. Focus not so much on giving the "right scores" as on seeing how the papers meet or miss the performance standards for each score point and what you therefore need to do in order to improve.

How to Interpret Numbers, Percentages, and Statistics in Argument Topics

Some arguments contain numbers, percentages, or statistics that are offered as evidence in support of the argument's conclusion. For example, an argument might claim that a certain community event is less popular this year than it was last year because only 100 people attended this year in comparison with 150 last year, a 33 percent decline in attendance. It is important to remember that you are not being asked to do a mathematical task with the numbers, percentages, or statistics. Instead you should evaluate these as evidence intended to support the conclusion. In the example above, the conclusion is that a community event has become less popular. You should ask yourself: does the difference between 100 people and 150 people support that conclusion? Note that, in this case, there are other possible explanations; for example, the weather might have been much worse this year, this year's event might have been held at an inconvenient time, the cost of the event might have gone up this year, or there might have been another popular event this year at the same time. Each of these could explain the difference in attendance, and thus would weaken the conclusion that the event was "less popular." Similarly, percentages might support or weaken a conclusion depending on what actual numbers the percentages represent. Consider the claim that the drama club at a school deserves more funding because its membership has increased by 100 percent. This 100 percent increase could be significant if there had been 100 members and now there are 200 members, whereas the increase would be much less significant if there had been 5 members and now there are 10. Remember that any numbers, percentages, or statistics in Argument tasks are used only as evidence in support of a conclusion, and you should always consider whether they actually support the conclusion.

The Form of Your Response

You are free to organize and develop your response in any way that you think will effectively communicate your evaluation of the argument. Your response may, but need not, incorporate particular writing strategies learned in English composition or writing-intensive college courses. GRE readers will not be looking for a particular develop-

Paragraph 1 offers reasons why the first assumption is questionable (e.g., residents may not have the necessary time or money for water sports). Similarly, paragraphs 2 and 3 explain that riverside recreational facilities may already be adequate and may, in fact, produce additional income if usage increases. Thus, the response is adequately developed and satisfactorily organized to show how the argument depends on questionable assumptions.

This essay does not, however, rise to a score of 5 because it fails to consider several other unstated assumptions (e.g., that the survey is reliable or that the efforts to clean the river will be successful). Furthermore, the final paragraph makes some extraneous, unsupported assertions of its own. Mason City may actually have a budget surplus so that cuts to other projects will not be necessary, and cleaning the river may provide other real benefits even if it is not used more for water sports.

This response is generally free of errors in grammar and usage and displays sufficient control of language to support a score of 4.

Score 3 Response

> Surveys are created to speak for the people; however, surveys do not always speak for the whole community. A survey completed by Mason City residents concluded that the residents enjoy water sports as a form of recreation. If that is so evident, why has the river not been used? The blame can not be soley be placed on the city park department. The city park department can only do as much as they observe. The real issue is not the residents use of the river, but their desire for a more pleasant smell and a more pleasant sight. If the city government cleans the river, it might take years for the smell to go away. If the budget is changed to accomodate the clean up of the Mason River, other problems will arise. The residents will then begin to complain about other issues in their city that will be ignored because of the great emphasis being placed on Mason River. If more money is taken out of the budget to clean the river an assumption can be made. This assumption is that the budget for another part of city maintenance or building will be tapped into to. In addition, to the budget being used to clean up Mason River, it will also be allocated in increasing riverside recreational facilites. The government is trying to appease its residents, and one can warrant that the role of the government is to please the people. There are many assumptions being made; however, the government can not make the assumption that people want the river to be cleaned so that they can use it for recreational water activities. The government has to realize the long term effects that their decision will have on the monetary value of their budget.

Reader Commentary

Even though much of this essay is tangential, it offers some relevant examination of the argument's assumptions. The early sentences mention a questionable assumption (that the survey results are reliable) but do not explain how the survey might have been flawed. Then the response drifts to irrelevant matters—a defense of the city park department, a prediction of budget problems, and the problem of pleasing city residents. Some statements even introduce unwarranted assumptions that are not part of the original argument (e.g., "The residents will then begin to complain about other issues," and "This assumption is that the budget for another part of city maintenance or building will be tapped into."). Near the end, the response does correctly note that city government

GRE Scoring Guide: Analyze an Issue

Score 6

In addressing the specific task directions, a 6 response presents a cogent, well-articulated analysis of the issue and conveys meaning skillfully.

A typical response in this category

- articulates a clear and insightful position on the issue in accordance with the assigned task
- develops the position fully with compelling reasons and/or persuasive examples
- sustains a well-focused, well-organized analysis, connecting ideas logically
- conveys ideas fluently and precisely, using effective vocabulary and sentence variety
- demonstrates superior facility with the conventions of standard written English (i.e., grammar, usage, and mechanics) but may have minor errors

Score 5

In addressing the specific task directions, a 5 response presents a generally thoughtful, well-developed analysis of the issue and conveys meaning clearly.

A typical response in this category

- presents a clear and well-considered position on the issue in accordance with the assigned task
- develops the position with logically sound reasons and/or well-chosen examples
- is focused and generally well organized, connecting ideas appropriately
- conveys ideas clearly and well, using appropriate vocabulary and sentence variety
- demonstrates facility with the conventions of standard written English but may have minor errors

Score 4

In addressing the specific task directions, a 4 response presents a competent analysis of the issue and conveys meaning with acceptable clarity.

A typical response in this category

- presents a clear position on the issue in accordance with the assigned task
- develops the position with relevant reasons and/or examples
- is adequately focused and organized
- demonstrates sufficient control of language to express ideas with acceptable clarity
- generally demonstrates control of the conventions of standard written English but may have some errors

Reading Comprehension Questions

Reading Comprehension questions are designed to test a wide range of abilities required to read and understand the kinds of prose commonly encountered in graduate school. Those abilities include

- understanding the meaning of individual words
- understanding the meaning of individual sentences
- understanding the meaning of paragraphs and larger bodies of text
- distinguishing between minor and major points
- summarizing a passage
- drawing conclusions from the information provided
- reasoning from incomplete data, inferring missing information
- understanding the structure of a text, how the parts relate to one another
- identifying the author's perspective
- identifying the author's assumptions
- analyzing a text and reaching conclusions about it
- identifying strengths and weaknesses
- developing and considering alternative explanations

As this list implies, reading and understanding a piece of text requires far more than a passive understanding of the words and sentences it contains—it requires active engagement with the text, asking questions, formulating and evaluating hypotheses, and reflecting on the relationship of the particular text to other texts and information.

Each Reading Comprehension question is based on a passage, which may range in length from one paragraph to several paragraphs. The test contains approximately ten passages; the majority of the passages in the test are one paragraph in length, and only one or two are several paragraphs long. Passages are drawn from the physical sciences, the biological sciences, the social sciences, the arts and humanities, and everyday topics, and are based on material found in books and periodicals, both academic and nonacademic.

Typically, about half of the questions on the test will be based on passages, and the number of questions based on a given passage can range from one to six. Questions can cover any of the topics listed above, from the meaning of a particular word to assessing evidence that might support or weaken points made in the passage. Many, but not all, of the questions are standard multiple-choice questions, in which you are required to select a single correct answer; others ask you to select multiple correct answers, and still others ask you to select a sentence from the passage. These question types are presented in more detail below, and you should make sure that you are familiar with the differences among them.

General Advice

Reading passages are drawn from many different disciplines and sources, so you may encounter material with which you are not familiar. Do not be discouraged when this happens; all the questions can be answered on the basis of the information provided in the passage, and you are not expected to rely on any outside knowledge. If, however, you encounter a passage that seems particularly hard or unfamiliar, you may want to save it for last.

Here, we must determine what word would indicate something that the author is praised for not permitting. The only answer choice that fits the case is "obscure," since enhancing and underscoring are generally good things to do, not things one should refrain from doing. Choosing "obscure" clarifies the choice for the first blank; the only choice that fits well with "obscure" is "overshadowed." Notice that trying to fill blank (i) without filling blank (ii) first is very hard—each choice has at least some initial plausibility. Since the third blank requires a phrase that matches "enormous gaps" and "sparseness of our observations," the best choice is "superficiality of our theories."

Thus the correct answer is **overshadowed** (Choice A), **obscure** (Choice E), and **superficiality of our theories** (Choice I).

2. Vain and prone to violence, Caravaggio could not handle success: the more his (i)_____ as an artist increased, the more (ii)_____ his life became.

Blank (i)	Blank (ii)
(A) temperance	(D) tumultuous
(B) notoriety	(E) providential
(C) eminence	(F) dispassionate

Explanation

In this sentence, what follows the colon must explain or spell out what precedes it. So roughly what the second part must say is that as Caravaggio became more successful, his life got more out of control. When one looks at words to fill the blanks, it becomes clear that "tumultuous" is the best fit for blank (ii), since neither of the other choices suggests being out of control. And for blank (i), the best choice is "eminence," since to increase in eminence is a consequence of becoming more successful. It is true that Caravaggio might also increase in notoriety, but an increase in notoriety as an artist is not as clear a sign of success as an increase in eminence.

Thus the correct answer is **eminence** (Choice C) and **tumultuous** (Choice D).

3. In parts of the Arctic, the land grades into the landfast ice so _____ that you can walk off the coast and not know you are over the hidden sea.

(A) permanently
(B) imperceptibly
(C) irregularly
(D) precariously
(E) relentlessly

Explanation

The word that fills the blank has to characterize how the land grades into the ice in a way that explains how you can walk off the coast and over the sea without knowing it. The word that does that is "imperceptibly"; if the land grades imperceptibly into the ice, you might well not know that you had left the land. Describing the shift from land to ice as permanent, irregular, precarious, or relentless would not help to explain how you would fail to know.

Thus the correct answer is **imperceptibly** (Choice B).

4. No other contemporary poet's work has such a well-earned reputation for (i)_____, and there are few whose moral vision is so imperiously unsparing. Of late, however, the almost belligerent demands of his severe and densely forbidding poetry have taken an improbable turn. This new collection is the poet's fourth book in six years—an ample output even for poets of sunny disposition, let alone for one of such (ii)_____ over the previous 50 years. Yet for all his newfound (iii)_____, his poetry is as thorny as ever.

Blank (i)	Blank (ii)	Blank (iii)
(A) patent accessibility	(D) penitential austerity	(G) taciturnity
(B) intrinsic frivolity	(E) intractable prolixity	(H) volubility
(C) near impenetrability	(F) impetuous prodigality	(I) pellucidity

5. Managers who think that strong environmental performance will (i)_____ their company's financial performance often (ii)_____ claims that systems designed to help them manage environmental concerns are valuable tools. By contrast, managers who perceive environmental performance to be (iii)_____ to financial success may view an environmental management system as extraneous. In either situation, and whatever their perceptions, it is a manager's commitment to achieving environmental improvement rather than the mere presence of a system that determines environmental performance.

Blank (i)	Blank (ii)	Blank (iii)
(A) eclipse	(D) uncritically accept	(G) complementary
(B) bolster	(E) appropriately acknowledge	(H) intrinsic
(C) degrade	(F) hotly dispute	(I) peripheral

6. Philosophy, unlike most other subjects, does not try to extend our knowledge by discovering new information about the world. Instead it tries to deepen our understanding through (i)_____ what is already closest to us—the experiences, thoughts, concepts, and activities that make up our lives but that ordinarily escape our notice precisely because they are so familiar. Philosophy begins by finding (ii)_____ the things that are (iii) _____.

Blank (i)	Blank (ii)	Blank (iii)
(A) attainment of	(D) essentially irrelevant	(G) most prosaic
(B) rumination on	(E) utterly mysterious	(H) somewhat hackneyed
(C) detachment from	(F) thoroughly commonplace	(I) refreshingly novel

3. Political advertising may well be the most (i)_____ kind of advertising: political candidates are usually quite (ii)_____, yet their campaign advertisements often hide important differences behind smoke screens of smiles and empty slogans.

Blank (i)	Blank (ii)
(A) polemical	(D) interchangeable
(B) effective	(E) dissimilar
(C) deceptive	(F) vocal

Explanation

Looking at Blank (i), it is hard to select a correct answer, since all three answer choices fit the immediate context well. Looking to the second part of the sentence, however, we can see such expressions as "hide" and "smoke screens," both of which suggest that the correct answer for Blank (i) is "deceptive." Making that assumption, we can go on to see that the answer for Blank (ii) is "dissimilar," since what is deceptive about political advertisements is that they hide important differences. Reading the sentence again with "deceptive" and "dissimilar" in place confirms those choices.

Thus the correct answer is **deceptive** (Choice C) and **dissimilar** (Choice E).

4. Richard M. Russell said 52 percent of the nation's growth since the Second World War had (i)_____ invention. He said (ii)_____ research, the government's greatest role in assuring continuing innovation is promoting a strong, modern patent office. "Unless we can (iii)_____ original ideas, we will not have invention," Mr. Russell said. Speculating on the state of innovation over the next century, several inventors agreed that the future lay in giving children the tools to think creatively and the motivation to invent.

Blank (i)	Blank (ii)	Blank (iii)
(A) been at the expense of	(D) in addition to restricting	(G) evaluate
(B) no bearing on	(E) aside from supporting	(H) protect
(C) come through	(F) far from exaggerating	(I) disseminate

Explanation

A quick overview of the paragraph shows that its topic is the encouragement of invention and innovation. This implies that Blank (i) should be filled with "come through," which emphasizes the importance of invention; the other choices suggest that invention is irrelevant or somehow harmed by growth. Again, the only one of the choices for Blank (ii) that continues the theme of encouraging invention is "aside from supporting." Finally, the second sentence emphasizes the importance for innovation of a strong patent office, and this thought is reaffirmed in the following quotation from Mr. Russell, which requires "protect" in Blank (iii).

Thus the correct answer is **come through** (Choice C), **aside from supporting** (Choice E), and **protect** (Choice H).

"demanding." The words that meet this requirement are "exacting" (Choice A) and "meticulous" (Choice F), and they produce sentences that are alike in meaning. Although "acerbic analyses" means close to the same thing as "scathing analyses," both "acerbic" and "scathing" have meanings that are quite different from "demanding," so neither fits well in the blank.

Thus the correct answer is **exacting** (Choice A) and **meticulous** (Choice F).

7. Her _____ should not be confused with miserliness; as long as I have known her, she has always been willing to assist those who are in need.

   A stinginess
   B diffidence
   C frugality
   D illiberality
   E intolerance
   F thrift

Explanation

The sentence explains that the person spoken of is not miserly, since she is quite prepared to be generous. So for the sentence to make sense, the word filling the blank has to be something that is consistent with generosity and yet might, by those without a full understanding of her behavior, be mistaken for miserliness. The words "frugality" and "thrift" fulfill this requirement and yield two sentences that are alike in meaning, so that pair forms the correct answer. Neither "stinginess" nor "illiberality" makes sense in the sentence, since they are synonymous with "miserliness" and inconsistent with generosity. Other choices, such as "diffidence," might perhaps make a sensible sentence if placed in the blank, but do not form part of the correct answer since they have no companion word that would make a sentence of similar meaning.

Thus the correct answer is **frugality** (Choice C) and **thrift** (Choice F).

8. A misconception frequently held by novice writers is that sentence structure mirrors thought: the more convoluted the structure, the more _____ the ideas.

   A complicated
   B engaged
   C essential
   D fanciful
   E inconsequential
   F involved

Explanation

Because the second half of the sentence illustrates the idea that "structure mirrors thought," any word that fills the blank must be similar in meaning to "convoluted." The two words that are similar to "convoluted" are "complicated" and "involved" (Choices A and F), which produce sentences alike in meaning. "Fanciful," while somewhat similar in meaning to "convoluted," is not as similar to either "complicated" or "involved" as those words are to each other. The other answer choices are not similar in meaning to "convoluted," and thus do not produce coherent sentences.

Thus the correct answer is **complicated** (Choice A) and **involved** (Choice F).

Explanation

The presence of predators of zooplankton feeding near the surface during the day would suggest that *D. galeata* is consumed at a higher rate than *D. hyalina*: this would explain why *D. hyalina* is often more numerous, so **Choice B** is correct. Choices C and E are incorrect because although they help to explain why the two zooplankton reproduce at different rates, they do not help to resolve the apparent paradox. Choices A and D are incorrect because nothing is said in the paragraph to show the relevance of the presence of other species of zooplankton, nor of the habit of clustering under vegetation, to the relative population size of the two species.

Questions 4 and 5 are based on the following reading passage.

Tocqueville, apparently, was wrong. Jacksonian America was not a fluid, egalitarian society where individual wealth and poverty were ephemeral conditions. At least so argues E. Pessen in his iconoclastic study of the very rich in the United States between 1825 and 1850.

Pessen does present a quantity of examples, together with some refreshingly intelligible statistics, to establish the existence of an inordinately wealthy class. Though active in commerce or the professions, most of the wealthy were not self-made but had inherited family fortunes. In no sense mercurial, these great fortunes survived the financial panics that destroyed lesser ones. Indeed, in several cities the wealthiest one percent constantly increased its share until by 1850 it owned half of the community's wealth. Although these observations are true, Pessen overestimates their importance by concluding from them that the undoubted progress toward inequality in the late eighteenth century continued in the Jacksonian period and that the United States was a class-ridden, plutocratic society even before industrialization.

Description

The passage describes Pessen's argument that Jacksonian America was not fluid and egalitarian but class ridden and plutocratic, and criticizes it for leaping to an unjustified conclusion.

4. According to the passage, Pessen indicates that all of the following were true of the very wealthy in the United States between 1825 and 1850 EXCEPT:

   (A) They formed a distinct upper class.
   (B) Many of them were able to increase their holdings.
   (C) Some of them worked as professionals or in business.
   (D) Most of them accumulated their own fortunes.
   (E) Many of them retained their wealth in spite of financial upheavals.

Explanation

For this question, you are to identify the one statement that CANNOT be correctly attributed to Pessen. Therefore, you must first determine which of the statements given can be attributed to Pessen. According to the passage, Pessen maintains all of the following: there was a class of "inordinately wealthy" Americans (Choice A); in some places that class "constantly increased its share" (Choice B); its members were "active in commerce or the professions" (Choice C); and "these great fortunes survived the financial panics that destroyed lesser ones" (Choice E). However, Pessen also maintains, in contradiction to Choice D, that "most of the wealthy were not self-made but had inherited family fortunes." Therefore, **Choice D** is correct.

For the following question, consider each of the choices separately and select all that apply.

3. It can be inferred that the process described in the passage makes use of which of the following?

   A The tendency of hot air to rise
   B The directional movement of wind
   C The temperature differential between the sea and the desert

Explanation

Choices B and C are correct. This question asks the reader which of the three phenomena listed in the answer choices is used in the process described in the passage.

Choice A is incorrect: the passage does not indicate that the tendency of hot air to rise is used in the process, and in fact says that heated air is drawn down, not up, as part of the greenhouse design.

Choice B is correct: the second sentence describes the orientation of a perforated cardboard wall toward the prevailing wind so that hot air blows in and is moistened.

Choice C is correct: the passage describes the use of seawater to cool hot desert air and to provide moisture that is absorbed by heated air and then condensed on a seawater-cooled surface for the purpose of irrigating the plants.

For the following question, consider each of the choices separately and select all that apply.

4. It can be inferred that the greenhouse roof is designed to allow for which of the following?

   A The avoidance of intense solar heat inside the greenhouse
   B The entry of sunlight into the greenhouse to make the plants grow
   C The mixture of heated air with greenhouse air to enhance the collection of moisture

Explanation

All three choices are correct. This question asks the reader which of the three effects listed in the answer choices are intended as part of the design of the greenhouse roof.

Choice A is correct: the purpose of the double-layered roof is to trap solar heat before it gets inside the greenhouse proper.

Choice B is correct: the coating on the inner layer of the roof allows visible sunlight into the greenhouse.

Choice C is correct: the last two sentences of the passage describe how heated air from the roof is drawn down to mix with greenhouse air, resulting in the collection of distilled water for irrigation purposes.

For the following question, consider each of the choices separately and select all that apply.

8. The author of the passage suggests which of the following about *Hamlet*?

   A *Hamlet* has usually attracted critical interpretations that tend to stiffen into theses.

   B *Hamlet* has elements that are not amenable to an all-encompassing critical interpretation.

   C *Hamlet* is less open to an all-encompassing critical interpretation than is *Wuthering Heights*.

Explanation

Choice B is correct. This question asks the reader which of the three statements about *Hamlet* listed in the answer choices are suggested by the author of the passage.

Choice A is incorrect: the passage does not provide information about the characteristics of the usual critical interpretations of *Hamlet*.

Choice B is correct: *Hamlet* is mentioned only in the final sentence of the passage, which refers to "this respect" in which *Hamlet* and *Wuthering Heights* are similar. The previous sentence reveals the point of similarity referred to: *Wuthering Heights* has elements that resist inclusion in an all-encompassing interpretive framework.

Choice C is incorrect: the passage mentions only a feature shared between *Hamlet* and *Wuthering Heights*. It does not suggest anything about a difference in their openness to a particular critical interpretation.

The following sample questions focus on simplifying the comparison.

$$y > 4$$

	Quantity A	Quantity B
6.	$\frac{3y + 2}{5}$	y

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ The two quantities are equal.
Ⓓ The relationship cannot be determined from the information given.

Explanation

Set up the initial comparison:

$$\frac{3y + 2}{5} \boxed{?} y$$

Then simplify:

Step 1: Multiply both sides by 5 to get

$$3y + 2 \boxed{?} 5y$$

Step 2: Subtract $3y$ from both sides to get

$$2 \boxed{?} 2y$$

Step 3: Divide both sides by 2 to get

$$1 \boxed{?} y$$

The comparison is now simplified as much as possible. In order to compare 1 and y, note that you are given the information $y > 4$ (above Quantities A and B). It follows from $y > 4$ that $y > 1$, or $1 < y$, so that in the comparison $1 \boxed{?} y$, the placeholder $\boxed{?}$ represents *less than* ($<$): $1 < y$.

However, the problem asks for a comparison between Quantity A and Quantity B, not a comparison between 1 and y. To go from the comparison between 1 and y to a comparison between Quantities A and B, start with the last comparison, $1 < y$, and carefully consider each simplification step in reverse order to determine what each comparison implies about the preceding comparison, all the way back to the comparison between Quantities A and B if possible. Since step 3 was "*divide* both sides by 2," *multiplying* both sides of the comparison $1 < y$ by 2 implies the preceding comparison $2 < 2y$, thus reversing step 3. Each simplification step can be reversed as follows:

- Reverse step 3: *multiply* both sides by 2.
- Reverse step 2: *add* $3y$ to both sides.
- Reverse step 1: *divide* both sides by 5.

When each step is reversed, the relationship remains *less than* ($<$), so Quantity A is less than Quantity B.

Thus, the correct answer is Choice B, Quantity B is greater.

Explanation

Set up the initial comparison:

$$7w - 4 \boxed{?} 2w + 5$$

Then simplify:

Step 1: Subtract $2w$ from both sides and add 4 to both sides to get

$$5w \boxed{?} 9$$

Step 2: Divide both sides by 5 to get

$$w \boxed{?} \frac{9}{5}$$

The comparison cannot be simplified any further. Although you are given that $w > 1$, you still don't know how w compares to $\frac{9}{5}$, or 1.8. For example, if $w = 1.5$, then $w < 1.8$, but if $w = 2$, then $w > 1.8$. In other words, the relationship between w and $\frac{9}{5}$ cannot be determined. Note that each of these simplification steps is reversible, so in reverse order, each simplification step implies that the *relationship cannot be determined* in the preceding comparison. Thus, the relationship between Quantities A and B cannot be determined.

The correct answer is Choice D, the relationship cannot be determined from the information given.

The strategy of simplifying the comparison works most efficiently when you note that a simplification step is reversible while actually taking the step. Here are some common steps that are always reversible:

- Adding any number or expression to both sides of a comparison
- Subtracting any number or expression from both sides
- Multiplying both sides by any nonzero number or expression
- Dividing both sides by any nonzero number or expression

Remember that if the relationship is an inequality, multiplying or dividing both sides by any *negative* number or expression will yield the opposite inequality. Be aware that some common operations like squaring both sides are generally not reversible and may require further analysis using other information given in the question in order to justify reversing such steps.

Multiple-choice Questions—Select One Answer Choice

Description

These questions are multiple-choice questions that ask you to select only one answer choice from a list of five choices.

Tips for Answering

- **Use the fact that the answer is there.** If your answer is not one of the five answer choices given, you should assume that your answer is incorrect and do the following:
  - Reread the question carefully—you may have missed an important detail or misinterpreted some information.
  - Check your computations—you may have made a mistake, such as mis-keying a number on the calculator.
  - Reevaluate your solution method—you may have a flaw in your reasoning.
- **Examine the answer choices.** In some questions you are asked explicitly which of the choices has a certain property. You may have to consider each choice separately, or you may be able to see a relationship between the choices that will help you find the answer more quickly. In other questions, it may be helpful to work backward from the choices, say, by substituting the choices in an equation or inequality to see which one works. However, be careful, as that method may take more time than using reasoning.
- **For questions that require approximations, scan the answer choices to see how close an approximation is needed.** In other questions, too, it may be helpful to scan the choices briefly before solving the problem to get a better sense of what the question is asking. If computations are involved in the solution, it may be necessary to carry out all computations exactly and round only your final answer in order to get the required degree of accuracy. In other questions, you may find that estimation is sufficient and will help you avoid spending time on long computations.

Sample Questions

Select a single answer choice.

1. If $5x + 32 = 4 - 2x$, what is the value of x ?

   (A) -4
   (B) -3
   (C) 4
   (D) 7
   (E) 12

Explanation

Solving the equation for x, you get $7x = -28$, and so $x = -4$. **The correct answer is Choice A, −4.**

Sample Questions

Select one or more answer choices according to the specific question directions.

If the question does not specify how many answer choices to select, select all that apply.

- **The correct answer may be just one of the choices or as many as all of the choices, depending on the question.**
- **No credit is given unless you select all of the correct choices and no others.**

If the question specifies how many answer choices to select, select exactly that number of choices.

1. Which two of the following numbers have a product that is between -1 and 0 ?

   Indicate both of the numbers.

   A -20
   B -10
   C 2^{-4}
   D 3^{-2}

Explanation

For this question, you must select a pair of answer choices. The product of the pair must be negative, so the possible products are $(-20)(2^{-4})$, $(-20)(3^{-2})$, $(-10)(2^{-4})$, and $(-10)(3^{-2})$. The product must also be greater than -1. The first product is $\frac{-20}{2^4} = -\frac{20}{16} < -1$, the second product is $\frac{-20}{3^2} = -\frac{20}{9} < -1$, and third product is $\frac{-10}{2^4} = -\frac{10}{16} > -1$, so you can stop there.

The correct answer consists of Choices B (-10) and C (2^{-4}).

---

2. Which of the following integers are multiples of both 2 and 3 ?

   Indicate all such integers.

   A 8
   B 9
   C 12
   D 18
   E 21
   F 36

Explanation

You can first identify the multiples of 2, which are 8, 12, 18, and 36, and then among the multiples of 2 identify the multiples of 3, which are 12, 18, and 36. Alternatively, if you realize that every number that is a multiple of 2 and 3 is also a multiple of 6, you can check which choices are multiples of 6.

The correct answer consists of Choices C (12), D (18), and F (36).

- **Examine your answer to see if it is reasonable with respect to the information given.** You may want to use estimation or another solution path to double-check your answer.

Sample Questions

Enter your answer as an integer or a decimal if there is a single answer box OR as a fraction if there are two separate boxes—one for the numerator and one for the denominator.

To enter an integer or a decimal, either type the number in the answer box using the keyboard or use the Transfer Display button on the calculator.

- **First, click on the answer box—a cursor will appear in the box—and then type the number.**
- **To erase a number, use the Backspace key.**
- **For a negative sign, type a hyphen. For a decimal point, type a period.**
- **To remove a negative sign, type the hyphen again and it will disappear; the number will remain.**
- **The Transfer Display button on the calculator will transfer the calculator display to the answer box.**
- **Equivalent forms of the correct answer, such as 2.5 and 2.50, are all correct.**
- **Enter the exact answer unless the question asks you to round your answer.**

To enter a fraction, type the numerator and the denominator in the respective boxes using the keyboard.

- **For a negative sign, type a hyphen; to remove it, type the hyphen again. A decimal point cannot be used in a fraction.**
- **The Transfer Display button on the calculator cannot be used for a fraction.**
- **Fractions do not need to be reduced to lowest terms, though you may need to reduce your fraction to fit in the boxes.**

1. One pen costs \$0.25 and one marker costs \$0.35. At those prices, what is the total cost of 18 pens and 100 markers?

\$ [ ]

Explanation

Multiplying \$0.25 by 18 yields \$4.50, which is the cost of the 18 pens; and multiplying \$0.35 by 100 yields \$35.00, which is the cost of the 100 markers. The total cost is therefore \$4.50 + \$35.00 = \$39.50. Equivalent decimals, such as \$39.5 or \$39.500, are considered correct.

Thus, the correct answer is \$39.50 (or equivalent).

Note that the dollar symbol is in front of the answer box, so the symbol \$ does not need to be entered in the box. In fact, only numbers, a decimal point, and a negative sign can be entered in the answer box.

5. Convert 6 miles per hour to feet per second.

Explanation

The solution to this problem uses the conversion factors 1 mile = 5,280 feet and 1 hour = 3,600 seconds as follows:

$$\left(\frac{6 \text{ miles}}{1 \text{ hour}}\right)\left(\frac{5{,}280 \text{ feet}}{1 \text{ mile}}\right)\left(\frac{1 \text{ hour}}{3{,}600 \text{ seconds}}\right) = ? \frac{\text{feet}}{\text{second}}$$

Enter 6 [×] 5280 [÷] 3600 [=] to get 8.8. Alternatively, enter 6 [×] 5280 [=] to get the result 31,680, and then enter [÷] 3600 [=] to get **8.8 feet per second.**

---

6. At a fund-raising event, 43 participants donated $60 each, 21 participants donated $80 each, and 16 participants donated $100 each. What was the average (arithmetic mean) donation per participant, in dollars?

Explanation

The solution to this problem is to compute the weighted mean $\frac{(43)(60) + (21)(80) + (16)(100)}{43 + 21 + 16}$. You can use the memory buttons and parentheses for this computation as follows:

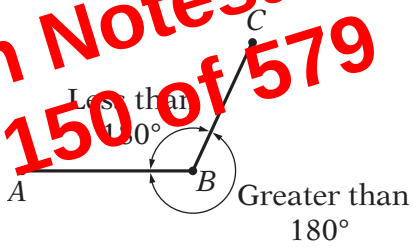
Enter 43 [×] 60 [=] [M+] 21 [×] 80 [=] [M+] 16 [×] 100 [=] [M+] [MR] [÷] [(] 43 [+] 21 [+] 16 [)] [=] to get 73.25 or **$73.25 per participant**.

When the [M+] button is first used, the number in the calculator display is stored in memory and an **M** appears to the left of the display to show that the memory function is in use. Each subsequent use of the [M+] button adds the number in the current display to the number stored in memory and replaces the number stored in memory by the sum. When the [MR] button is pressed in the computation above, the current value in memory, 5,860, is displayed. To clear the memory, use the [MC] button, and the **M** next to the display disappears.

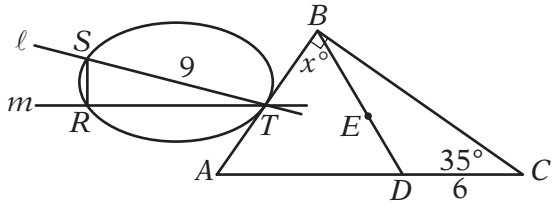
- The phrase *area of a rectangle* means the area of the region enclosed by the rectangle. The same terminology applies to circles, triangles, and other closed figures.
- The *distance between a point and a line* is the length of the perpendicular line segment from the point to the line, which is the shortest distance between the point and the line. Similarly, the *distance between two parallel lines* is the distance between a point on one line and the other line.
- In a geometric context, the phrase *similar triangles* (or other figures) means that the figures have the same shape. See the Geometry section of the *Math Review* for further explanation of the terms *similar* and *congruent*.

Geometric Figures

- Geometric figures consist of points, lines, line segments, curves (such as circles), angles, and regions; also included are labels, and markings or shadings that identify these objects or their sizes. A point is indicated by a dot, a label, or the intersection of two or more lines or curves. Points, lines, angles, etc., that are shown as distinct are indeed distinct. All figures are assumed to lie in a plane unless otherwise indicated.
- If points A, B, and C do not lie on the same line, then line segments AB and BC form two angles with vertex B—one angle with measure less than 180° and the other with measure greater than 180°, as shown below. Unless otherwise indicated, angle ABC, also denoted by $\angle ABC$ or $\angle B$, refers to the *smaller* of the two angles.



- The notation AB may mean the line segment with endpoints A and B, or it may mean the length of the line segment. The meaning can be determined from the context.
- Geometric figures **are not necessarily** drawn to scale. That is, you should **not** assume that quantities such as lengths and angle measures are as they appear in a figure. However, you should assume that lines shown as straight are actually straight, and when curves are shown, you should assume they are not straight. Also, assume that points on a line or a curve are in the order shown, points shown to be on opposite sides of a line or curve are so oriented, and more generally, assume all geometric objects are in the relative positions shown. For questions with geometric figures, you should base your answers on geometric reasoning, not on estimating or comparing quantities by sight or by measurement.
- To illustrate some of these conventions, consider the geometric figure below.



6 GRE® Quantitative Reasoning Practice Questions

Your goals for this chapter

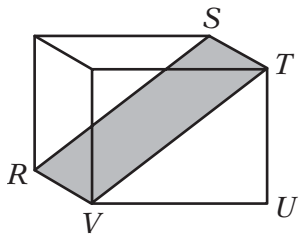
- Practice answering *GRE*® Quantitative Reasoning questions on your own
- Study answers and explanations, particularly for questions you answered incorrectly

This chapter contains four sets of GRE Quantitative Reasoning practice questions. Each of the first three practice sets consists of Quantitative Comparison questions, both types of Multiple-choice questions, and Numeric Entry questions. These three sets are arranged in order of increasing difficulty. The first is easy, the second is medium, and the third is hard. The fourth practice set consists of Data Interpretation questions of varying levels of difficulty.

Following the last set is an answer key for quick reference. Then, at the end of the chapter, you will find complete explanations for every question. Each explanation is presented with the corresponding question, so that you can easily see what was asked and what the various answer choices or Numeric Entry answer boxes were.

Sharpen your GRE Quantitative Reasoning skills by working your way through these question sets. For the Discrete question sets, begin with the easy sets and then move on to the medium and hard sets. Review the answers and explanations carefully, paying particular attention to explanations for questions that you answered incorrectly.

For the practice questions in this chapter, use the directions that begin on the following page.



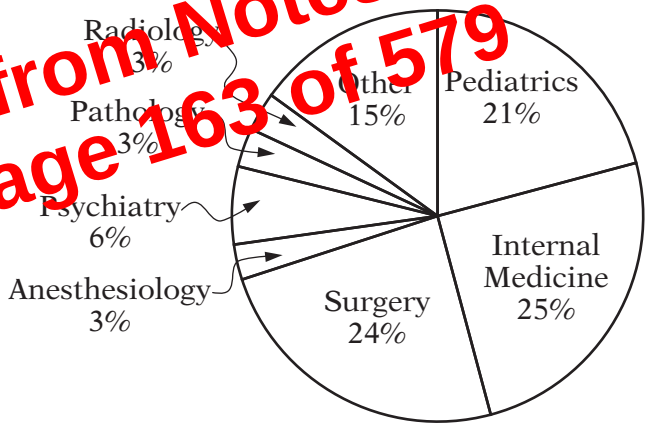
12. In the rectangular solid above, $TU = 3$, $UV = 4$, and $VR = 2$. What is the area of the shaded rectangular region?

13. A list of numbers has a mean of 8 and a standard deviation of 2.5. If x is a number in the list that is 2 standard deviations above the mean, what is the value of x ?

$x =$

Multiple-choice Questions—Select One or More Answer Choices

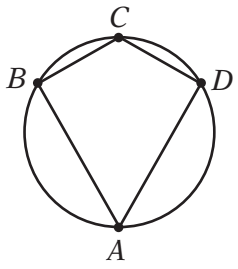
For Question 14, select all the answer choices that apply.



14. The circle graph above shows the distribution of 200,000 physicians by specialty. Which of the following sectors of the circle graph represent more than 40,000 physicians?

Indicate all such sectors.

- A Pediatrics
- B Internal Medicine
- C Surgery
- D Anesthesiology
- E Psychiatry



In the figure above, the diameter of the circle is 10.

	Quantity A	Quantity B	
3.	The area of quadrilateral $ABCD$	40	(A) (B) (C) (D)

$$x^2y > 0$$
$$xy^2 < 0$$

	Quantity A	Quantity B	
4.	x	y	(A) (B) (C) (D)

Among the 9,000 people attending a football game at College T, there were x students from College C and y students who were not from College C.

	Quantity A	Quantity B	
5.	The number of people attending the game who were not students	$9{,}000 - x - y$	(A) (B) (C) (D)

Multiple-choice Questions—Select One Answer Choice

For Questions 6 to 10, select a single answer choice.

6. If $x \neq 0$, which of the following is equivalent to $\frac{x(x^2)^3}{x^2}$?

(A) x^2
(B) x^3
(C) x^4
(D) x^5
(E) x^6

Numeric Entry

For Question 13, enter your answer in the answer box(es) below the question.

- **Your answer may be an integer, a decimal, or a fraction, and it may be negative.**
- **If a question asks for a fraction, there will be two boxes—one for the numerator and one for the denominator.**
- **Equivalent forms of the correct answer, such as 2.5 and 2.50, are all correct. Fractions do not need to be reduced to lowest terms.**
- **Enter the exact answer unless the question asks you to round your answer.**

Y	Frequency
$\frac{1}{2}$	2
$\frac{3}{4}$	7
$\frac{5}{4}$	8
$\frac{3}{2}$	8
$\frac{7}{4}$	9

13. The table above shows the frequency distribution of the values of a variable Y. What is the mean of the distribution?

Give your answer to the nearest 0.01.

Multiple-choice Questions—Select One or More Answer Choices

For Questions 14 and 15, select all the answer choices that apply.

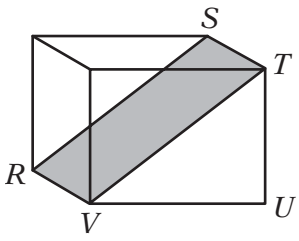
14. Let S be the set of all positive integers n such that n^2 is a multiple of both 24 and 108. Which of the following integers are divisors of every integer n in S ?

Indicate all such integers.

- [A] 12
- [B] 24
- [C] 36
- [D] 72

Numeric Entry

For Questions 12 and 13, use the directions for Numeric Entry questions.



12. In the rectangular solid above, $TU = 3$, $UV = 4$, and $VR = 2$. What is the area of the shaded rectangular region?

Explanation

To find the area of the shaded rectangular region, you need to multiply the length of the rectangular region by its width. In this question you are given the lengths of three edges: $TU = 3$, $UV = 4$, and $VR = 2$. Note that VT is the length of the shaded rectangle. To find the width of the shaded rectangle, you need to find either RS or VT. Note that VT lies on the front face of the rectangular solid. It is the hypotenuse of right triangle VUT. You know that $UV = 4$ and $TU = 3$, so by the Pythagorean theorem you can conclude that $VT = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$. Therefore the area of the shaded rectangular region is $(5)(2) = 10$. The correct answer is **10**.

13. A list of numbers has a mean of 8 and a standard deviation of 2.5. If x is a number in the list that is 2 standard deviations above the mean, what is the value of x ?

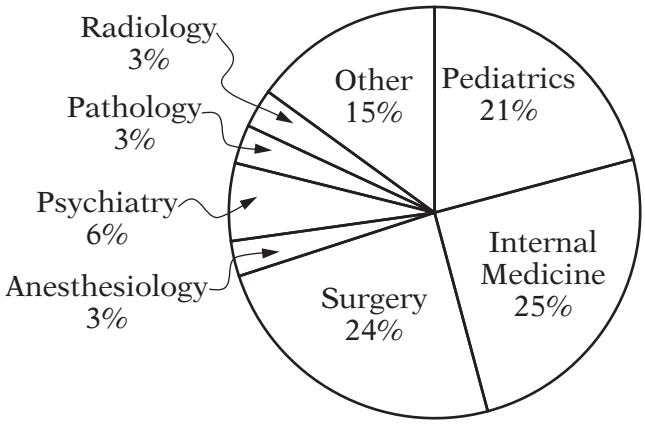
$x =$

Explanation

You are given that x is 2 standard deviations above the mean, 8. Because the standard deviation of the numbers in the list is 2.5, it follows that x is (2)(2.5), or 5 units above the mean 8. Therefore, $x = 8 + 5 = 13$, and the correct answer is **13**.

Multiple-choice Questions—Select One or More Answer Choices

For Question 14, select all the answer choices that apply.



14. The circle graph above shows the distribution of 200,000 physicians by specialty. Which of the following sectors of the circle graph represent more than 40,000 physicians?

Indicate all such sectors.

- A Pediatrics
- B Internal Medicine
- C Surgery
- D Anesthesiology
- E Psychiatry

Explanation

One approach to solve this problem is to find out what percent of 200,000 is 40,000 and then compare this percent with the percents given in the circle graph. Because $\frac{40{,}000}{200{,}000} = 0.2$, it follows that 40,000 is 20% of 200,000, and any specialty that has more than 20% of the distribution has more than 40,000 physicians. This is true for the specialties of pediatrics, internal medicine, and surgery. The correct answer consists of **Choices A, B, and C**.

Frequency Distribution for List X

Number	1	2	3	5
Frequency	10	20	18	12

Frequency Distribution for List Y

Number	6	7	8	9
Frequency	24	17	10	9

List X and list Y each contain 60 numbers. Frequency distributions for each list are given above. The average (arithmetic mean) of the numbers in list X is 2.7, and the average of the numbers in list Y is 7.1. List Z contains 120 numbers: the 60 numbers in list X and the 60 numbers in list Y.

	Quantity A	Quantity B	
2.	The average of the 120 numbers in list Z	The median of the 120 numbers in list Z	(A) (B) (C) (D)

Explanation

In this problem you are asked to compare the average with the median of the 120 numbers in list Z. Since list Z consists of the numbers in lists X and Y combined, it is reasonable to try to use the information about lists X and Y to calculate the average and the median of the numbers in list Z.

To determine the average of the 120 numbers in list Z, you can use the information given about the individual averages of the numbers in lists X and Y. Because lists X and Y each contain 60 numbers, the average of the numbers in list Z is the average of the individual averages of the numbers in lists X and Y. Thus, the average of the numbers in list Z is $\frac{2.7 + 7.1}{2}$, or 4.9.

To determine the median of the 120 numbers in list Z, first note that list Z contains an even number of numbers, so the median of the numbers in list Z is the average of the middle two numbers when the numbers are listed in increasing order. If you look at the numbers in the two lists, you will see that the 60 numbers in list X are all less than or equal to 5, and the 60 numbers in list Y are all greater than or equal to 6. Thus, the two middle numbers in list Z are 5 and 6, and the average of these numbers is $\frac{5 + 6}{2}$, or 5.5. Therefore, the median of the numbers in list Z is 5.5, and this is greater than the average of 4.9. The correct answer is **Choice B**.

Similarly, the number of ways 2 members can be selected from among the 700 members is (700)(699) divided by 2. Thus, the desired probability is

$$\frac{\frac{(580)(579)}{2}}{\frac{(700)(699)}{2}} = \frac{(580)(579)}{(700)(699)}$$

Since the answer choices are all tenths, you need to approximate the value of this fraction to the nearest tenth. There are several ways to do this approximation. One way is to use your calculator to convert the fraction to a decimal and round the decimal to the nearest tenth.

Another way is to approximate the value of the fraction as follows.

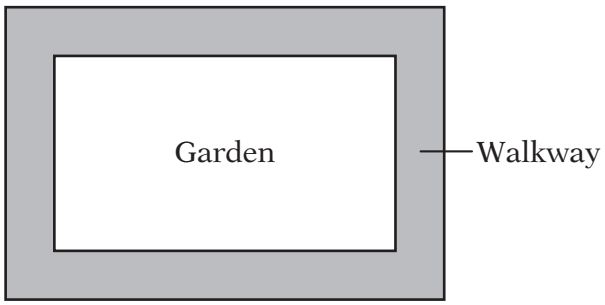
$$\frac{(580)(579)}{(700)(699)} \approx \frac{(600)(600)}{(700)(700)} = \left(\frac{6}{7}\right)^2 = \frac{36}{49} \approx \frac{36}{50} = 0.72$$

Either way, the answer choice that is closest to the value of the fraction is 0.7. The correct answer is **Choice C**.

Another approach to this problem is to consider the random selections as two separate but successive events. The probability of selecting a first member who is not a lawyer is $\frac{580}{700}$, because there are 580 members out of the 700 members who are not lawyers. For the second selection, there are only 699 members left to select from, because one member has already been selected. If the first member selected is not a lawyer, then there are only 579 members left who are not lawyers. So the probability of selecting a second member who is not a lawyer, given the condition that the first member selected was not a lawyer, is $\frac{579}{699}$. The probability that both members selected will not be lawyers is the product of the two probabilities, or $\left(\frac{580}{700}\right)\left(\frac{579}{699}\right)$, which is approximated above as 0.72. The correct answer is **Choice C.**

Numeric Entry

For Questions 11 and 12, use the directions for Numeric Entry questions.



11. The figure above represents a rectangular garden with a walkway around it. The garden is 18 feet long and 12 feet wide. The walkway is uniformly 3 feet wide, and its edges meet at right angles. What is the area of the walkway?

square feet

The coordinates of the first point are $(-4, 0)$, since the x-coordinate is -4 and the y-coordinate of every point on the x-axis is 0. For the second point, the midpoint of the line segment is halfway between the endpoints $(2, 9)$ and $(2, 0)$. Thus, the midpoint has x-coordinate 2 and y-coordinate $\frac{9}{2}$, the number halfway between 9 and 0. Based on the coordinates $(-4, 0)$ and $\left(2, \frac{9}{2}\right)$, the slope of line k is

$$\frac{\frac{9}{2} - 0}{2 - (-4)} = \frac{\frac{9}{2}}{6} = \frac{3}{4}$$

The correct answer is $\frac{3}{4}$ (or any equivalent fraction).

Multiple-choice Questions—Select One or More Answer Choices

For Questions 13 and 14, select all the answer choices that apply.

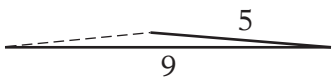
13. If the lengths of two sides of a triangle are 5 and 9, respectively, which of the following could be the length of the third side of the triangle?

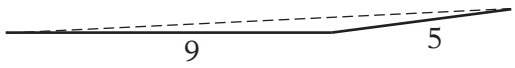
    Indicate all such lengths.

    A 3
    B 5
    C 8
    D 15

Explanation

A good way to approach this problem is to think about how much the length of the third side of a triangle with two fixed side lengths can vary. If you think about it a bit, you will see that the smaller the interior angle between the two sides of the triangle is, the smaller the length of the third side is; and the larger the interior angle between the two sides of the triangle is, the larger the length of the third side is. This suggests drawing two triangles, one in which the angle between the two sides is close to 0 degrees and one in which the angle between the two sides is close to 180 degrees, like the triangles below.

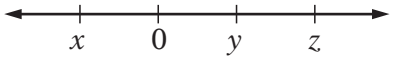




In the triangle in which the angle between the sides of length 5 and 9 is small, you can see that the length of the third side is a bit greater than $9 - 5$, or 4. If it were equal to 4, the triangle would degenerate into a line segment.

In the triangle in which the angle between the sides of length 5 and 9 is large, you can see that the length of the third side is a bit less than $9 + 5$, or 14. If it were equal to 14, the triangle would degenerate into a line segment.

Therefore, the length of the third side of the triangle must be greater than 4 and less than 14. Furthermore, it is intuitive that any length between these two numbers can be achieved by some triangle. The correct answer consists of **Choices B and C**.



14. On the number line shown above, the tick marks are equally spaced. Which of the following statements about the numbers x, y, and z must be true?

Indicate all such statements.

A $xyz < 0$
B $x + z = y$
C $z(y - x) > 0$

Explanation

You can see from their positions on the number line that x is less than 0 and both y and z are greater than 0. Because the tick marks are equally spaced, you can also see that $x = -y$ and $z = 2y$. You need to evaluate each answer choice separately to determine whether it must be true.

Choice A says that the product of the three numbers x, y, and z is less than 0. Recall that the product of three numbers is negative under either of the following two conditions.

- All three numbers are negative.
- One of the numbers is negative and the other two numbers are positive.

Choice A must be true, since x is negative and y and z are positive.

Choice B is the equation $x + z = y$. To see whether the equation must be true, it is a good idea to express two of the variables in terms of the third (that is, to "get rid of" two of the variables). The equations $x = -y$ and $z = 2y$ give x and z in terms of y, so the equation $x + z = y$ can be rewritten, substituting $-y$ for x and $2y$ for z, as $-y + 2y = y$. In this form you can quickly conclude that the equation must be true.

Choice C says that the product of the two numbers z and $y - x$ is greater than 0. Recall that the product of two numbers is positive under either of the following two conditions.

- Both numbers are positive.
- Both numbers are negative.

Since you already know that z is positive, you can conclude that the product $z(y - x)$ will be positive if $y - x$ is positive. By adding x to both sides of the inequality $y - x > 0$, you can see that it is equivalent to the inequality $y > x$, which is clearly true from the number line. Since $y - x$ is positive, the product $z(y - x)$ must be positive.

Therefore, the correct answer consists of **Choices A, B, and C**.

Explanation

To determine which of the graphs is the correct answer, you first need to determine all values of x that satisfy the inequality. To do that you need to simplify the inequality until you isolate x.

You can begin by multiplying both sides of the inequality by 3 to obtain $(3)(2 - 5x) \leq -(6x - 5)$. Note that when you multiply by 3, the right-hand side of the inequality becomes $-(6x - 5)$, not $-6x - 5$.

The rest of the simplification is as follows.

$$\begin{aligned}(3)(2 - 5x) &\leq -6x + 5\\ 6 - 15x &\leq -6x + 5\\ -15x &\leq -6x - 1\\ -9x &\leq -1\\ x &\geq \frac{1}{9}\end{aligned}$$

Note that when an inequality is multiplied (or divided) by a negative number, the direction of the inequality reverses.

The graphs in the answer choices are number lines on which only the number 0 is indicated. Therefore, you do not need to locate $\frac{1}{9}$ on the number line; it is enough to know that $\frac{1}{9}$ is a positive number. Choice C is the only choice in which the shaded part of the line is equal to or greater than a positive number. Therefore, the correct answer is **Choice C**.

9. If $1 + x + x^2 + x^3 = 60$, then the average (arithmetic mean) of x, x^2, x^3, and x^4 is equal to which of the following?

Ⓐ $12x$
Ⓑ $15x$
Ⓒ $20x$
Ⓓ $30x$
Ⓔ $60x$

Explanation

A quick inspection of the answer choices shows that it is not necessary to solve the equation $1 + x + x^2 + x^3 = 60$ for x to answer this question. You are being asked to express the average of the four quantities x, x^2, x^3, and x^4 in terms of x. To express this average in terms of x, you need to add the 4 quantities and divide the result by 4; that is, $\frac{x + x^2 + x^3 + x^4}{4}$.

The only information given in the question is that the sum of the 4 quantities, $1 + x + x^2 + x^3$, is 60, so you need to think of a way to use this information to simplify the expression $\frac{x + x^2 + x^3 + x^4}{4}$.

Note that the numerator of the fraction is a sum of 4 quantities, each of which has an x term raised to a power. Thus, the expression in the numerator can be

can be viewed in three different ways. The result can be viewed as a fraction or as a decimal, both of which are discussed later, or the result can be viewed as a **quotient** with a **remainder**, where both are integers. Each view is useful, depending on the context. Fractions and decimals are useful when the result must be viewed as a single number, while **quotients** with **remainders** are useful for describing the result in terms of integers only.

Regarding quotients with remainders, consider two positive integers a and b for which b is *not* a divisor of a; for example, the integers 19 and 7. When 19 is divided by 7, the result is greater than 2, since $(2)(7) < 19$, but less than 3, since $19 < (3)(7)$. Because 19 is 5 more than (2)(7), we say that the result of 19 divided by 7 is the quotient 2 with remainder 5, or simply "2 remainder 5." In general, when a positive integer a is divided by a positive integer b, you first find the greatest multiple of b that is less than or equal to a. That multiple of b can be expressed as the product qb, where q is the quotient. Then the remainder is equal to a minus that multiple of b, or $r = a - qb$, where r is the remainder. The remainder is always greater than or equal to 0 and less than b.

Here are examples that illustrate a few different cases of division resulting in a quotient and remainder.

- 100 divided by 45 is 2 remainder 10, since the greatest multiple of 45 that's less than or equal to 100 is (2)(45), or 90, which is 10 less than 100.
- 24 divided by 4 is 6 remainder 0, since the greatest multiple of 4 that's less than or equal to 24 is 24 itself, which is 0 less than 24. In general, the remainder is 0 if and only if a is divisible by b.
- 6 divided by 24 is 0 remainder 6, since the greatest multiple of 24 that's less than or equal to 6 is (0)(24), or 0, which is 6 less than 6.

Here are some other examples:

- 100 divided by 3 is 33 remainder 1, since 100 = (33)(3) + 1.
- 100 divided by 25 is 4 remainder 0, since 100 = (4)(25) + 0.
- 80 divided by 100 is 0 remainder 80, since 80 = (0)(100) + 80.
- When you divide 100 by 2, the remainder is 0.
- When you divide 99 by 2, the remainder is 1.

If an integer is divisible by 2, it is called an **even integer**; otherwise it is an **odd integer**. Note that when a positive odd integer is divided by 2, the remainder is always 1. The set of even integers is $\{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}$, and the set of odd integers is $\{\ldots, -5, -3, -1, 1, 3, 5, \ldots\}$. There are several useful facts regarding the sum and product of even and odd integers.

- The sum of two even integers is an even integer.
- The sum of two odd integers is an even integer.
- The sum of an even integer and an odd integer is an odd integer.
- The product of two even integers is an even integer.
- The product of two odd integers is an odd integer.
- The product of an even integer and an odd integer is an even integer.

A **prime number** is an integer greater than 1 that has only two positive divisors: 1 and itself. The first ten prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29. The integer 14 is not a prime number, since it has four positive divisors: 1, 2, 7, and 14. The integer 1 is not a prime number, and the integer 2 is the only prime number that is even.

Although the discussion about percent so far assumes a context of a *part* and a *whole*, it is not necessary that the part be less than the whole. In general, the whole is called the **base** of the percent. When the numerator of a percent is greater than the base, the percent is greater than 100%. For example, 15 is 300% of 5, since

$$\frac{15}{5} = \frac{300}{100}$$

and 250% of 16 is $\left(\frac{250}{100}\right)(16) = (2.5)(16) = 40$. Note that the decimal equivalent of 250% is 2.5.

It is also not necessary for the part to be related to the whole at all, as in the question, "a teacher's salary is what percent of a banker's salary?"

When a quantity changes from an initial positive amount to another positive amount, for example, an employee's salary that is raised, you can compute the amount of change as a percent of the initial amount. This is called **percent change**. If a quantity increases from 600 to 750, then the **percent increase** is found by dividing the amount of increase, 150, by the base, 600, which is the initial number given:

$$\frac{\textit{amount of increase}}{\textit{base}} = \frac{750 - 600}{600} = \frac{150}{600} = \frac{25}{100} = 0.25 = 25\%$$

We say the percent increase is 25%. Sometimes this computation is written as

$$\left(\frac{750 - 600}{600}\right)(100\%) = \left(\frac{150}{600}\right)(100\%) = 25\%$$

If a quantity doubles in size, then the percent increase is 100%. For example, if a quantity changes from 150 to 300, then the percent increase is

$$\frac{\textit{change}}{\textit{base}} = \frac{300 - 150}{150} = \frac{150}{150} = 100\%$$

If a quantity decreases from 500 to 400, calculate the **percent decrease** as follows.

$$\frac{\textit{change}}{\textit{base}} = \frac{500 - 400}{500} = \frac{100}{500} = \frac{20}{100} = 0.20 = 20\%$$

The quantity decreased by 20%.

When computing a percent *increase*, the base is the *smaller* number. When computing a percent *decrease*, the base is the *larger* number. In either case, the base is the initial number, before the change.

Example 1.7.5: An investment in a mutual fund increased by 12% in a single day. If the value of the investment before the increase was $1,300, what was the value after the increase?

Solution: The percent increase is 12%. Therefore, the value of the increase is 12% of $1,300, or, using the decimal equivalent, the increase is (0.12)($1,300) = $156. Thus, the value of the investment after the change is

$$\$1{,}300 + \$156 = \$1{,}456$$

Because the final result is the sum of the initial investment—100% of $1,300—and the increase—12% of $1,300—the final result is 100% + 12% = 112% of $1,300.

2. ALGEBRA

Basic algebra can be viewed as an extension of arithmetic. The main concept that distinguishes algebra from arithmetic is that of a **variable**, which is a letter that represents a quantity whose value is unknown. The letters x and y are often used as variables, although any letter can be used. Variables enable you to present a word problem in terms of unknown quantities by using algebraic expressions, equations, inequalities, and functions. This section reviews these algebraic tools and then progresses to several examples of applying them to solve real-life word problems. The section ends with coordinate geometry and graphs of functions as other important algebraic tools for solving problems.

2.1 Operations with Algebraic Expressions

An **algebraic expression** has one or more variables and can be written as a single **term** or as a sum of terms. Here are some examples of algebraic expressions.

$$2x \qquad y - \frac{1}{4} \qquad w^3z + 5z^2 - z^2 + 6 \qquad \frac{8}{n+p}$$

In the examples above, $2x$ is a single term, $y - \frac{1}{4}$ has two terms, $w^3z + 5z^2 - z^2 + 6$ has four terms, and $\frac{8}{n+p}$ has one term. In the expression $w^3z + 5z^2 - z^2 + 6$, the terms $5z^2$ and $-z^2$ are called **like terms** because they have the same variables, and the corresponding variables have the same exponents. A term that has no variable is called a **constant** term. A number that is multiplied by variables is called the **coefficient** of a term. For example, in the expression $2x^2 + 7x - 5$, 2 is the coefficient of the term $2x^2$, 7 is the coefficient of the term $7x$, and -5 is a constant term.

The same rules that govern operations with numbers apply to operations with algebraic expressions. One additional rule, which helps in simplifying algebraic expressions, is that like terms can be combined by simply adding their coefficients, as the following examples show.

$$2x + 5x = 7x$$

$$w^3z + 5z^2 - z^2 + 6 = w^3z + 4z^2 + 6$$

$$3xy + 2x - xy - 3x = 2xy - x$$

A number or variable that is a factor of each term in an algebraic expression can be factored out, as the following examples show.

$$4x + 12 = 4(x + 3)$$

$$15y^2 - 9y = 3y(5y - 3)$$

$$\frac{7x^2 + 14x}{2x + 4} = \frac{7x(x + 2)}{2(x + 2)} = \frac{7x}{2} \quad \text{(where } x \neq -2\text{, since division by 0 is not defined)}$$

To multiply two algebraic expressions, each term of the first expression is multiplied by each term of the second expression, and the results are added, as the following examples show.

Linear Equations in One Variable

To solve a linear equation in one variable, simplify each side of the equation by combining like terms. Then use the rules for producing simpler equivalent equations.

Example 2.3.1:

$$\begin{aligned} 11x - 4 - 8x &= 2(x + 4) - 2x \\ 3x - 4 &= 2x + 8 - 2x && \text{(like terms combined)} \\ 3x - 4 &= 8 && \text{(simplified)} \\ 3x - 4 + 4 &= 8 + 4 && \text{(4 added to both sides)} \\ 3x &= 12 \\ \frac{3x}{3} &= \frac{12}{3} && \text{(both sides divided by 3)} \\ x &= 4 \end{aligned}$$

You can always check your solution by substituting it into the original equation.

Note that it is possible for a linear equation to have no solutions. For example, the equation $2x + 3 = 2(7 + x)$ has no solution, since it is equivalent to the equation $3 = 14$, which is false. Also, it is possible that what looks to be a linear equation turns out to be an identity when you try to solve it. For example, $3x - 6 = -3(2 - x)$ is true for all values of x, so it is an identity.

Linear Equations in Two Variables

A linear equation in two variables, x and y, can be written in the form

$$ax + by = c$$

where a, b, and c are real numbers and a and b are not both zero. For example, $3x + 2y = 8$ is a linear equation in two variables.

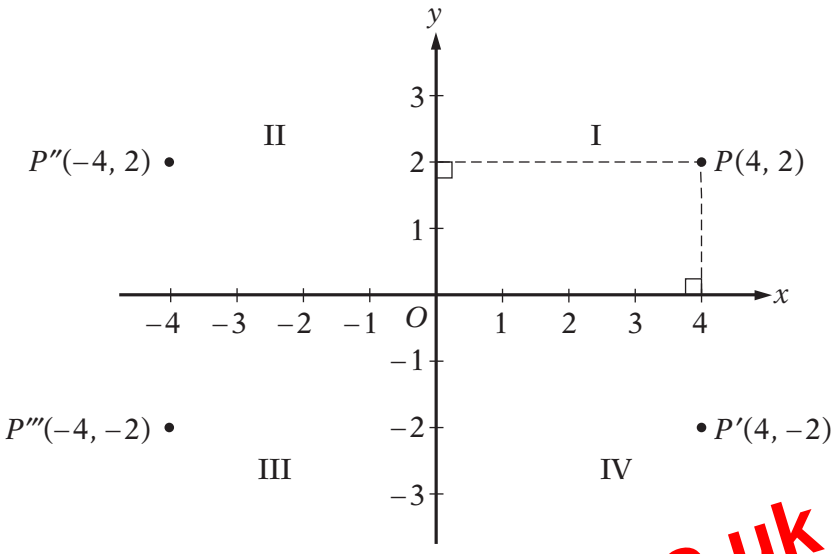
A solution of such an equation is an **ordered pair** of numbers (x, y) that makes the equation true when the values of x and y are substituted into the equation. For example, both $(2, 1)$ and $(-\frac{2}{3}, 5)$ are solutions of the equation $3x + 2y = 8$, but $(1, 2)$ is not a solution. A linear equation in two variables has infinitely many solutions. If another linear equation in the same variables is given, it is usually possible to find a unique solution of both equations. Two equations with the same variables are called a **system of equations**, and the equations in the system are called **simultaneous equations**. To solve a system of two equations means to find an ordered pair of numbers that satisfies *both* equations in the system.

There are two basic methods for solving systems of linear equations, by **substitution** or by **elimination**. In the substitution method, one equation is manipulated to express one variable in terms of the other. Then the expression is substituted in the other equation. For example, to solve the system of equations

$$\begin{aligned} 4x + 3y &= 13 \\ x + 2y &= 2 \end{aligned}$$

you can express x in the second equation in terms of y as $x = 2 - 2y$. Then substitute $2 - 2y$ for x in the first equation to find the value of y.

and the vertical number line is called the ***y*-axis**. The point where the two axes intersect is called the **origin**, denoted by O. The positive half of the x-axis is to the right of the origin, and the positive half of the y-axis is above the origin. The two axes divide the plane into four regions called **quadrants I, II, III**, and **IV**, as shown in the figure below.



Each point P in the xy-plane can be identified with an ordered pair (x, y) of real numbers and is denoted by $P(x, y)$. The first number is called the ***x*-coordinate**, and the second number is called the ***y*-coordinate**. A point with coordinates (x, y) is located $|x|$ units to the right of the y-axis if x is positive or to the left of the y-axis if x is negative. Also, the point is located $|y|$ units above the x-axis if y is positive or below the x-axis if y is negative. If $x = 0$, the point lies on the y-axis, and if $y = 0$, the point lies on the x-axis. The origin has coordinates $(0, 0)$. Unless otherwise noted, the units used on the x-axis and the y-axis are the same.

In the figure above, the point $P(4, 2)$ is 4 units to the right of the y-axis and 2 units above the x-axis, and the point $P‴(-4, -2)$ is 4 units to the left of the y-axis and 2 units below the x-axis.

Note that the three points $P'(4, -2)$, $P″(-4, 2)$, and $P‴(-4, -2)$ have the same coordinates as P except for the sign. These points are geometrically related to P as follows.

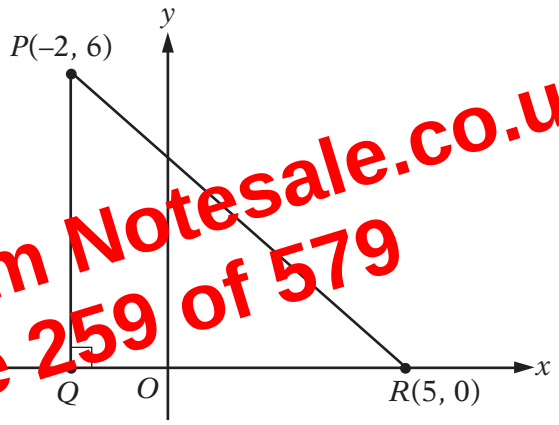
- P' is the **reflection of *P* about the *x*-axis**, or P' and P are **symmetric about the *x*-axis**.
- $P″$ is the **reflection of *P* about the *y*-axis**, or $P″$ and P are **symmetric about the *y*-axis**.
- $P‴$ is the **reflection of *P* about the origin**, or $P‴$ and P are **symmetric about the origin**.

The distance between two points in the xy-plane can be found by using the Pythagorean theorem. For example, the distance between the two points $Q(-2, -3)$ and $R(4, 1.5)$ in the figure at the top of the following page is the length of line segment QR. To find this distance, construct a right triangle (indicated by the dashed lines) and then note that the two shorter sides of the triangle have lengths $QS = 4 - (-2) = 6$ and $RS = 1.5 - (-3) = 4.5$.

15. A group can charter a particular aircraft at a fixed total cost. If 36 people charter the aircraft rather than 40 people, then the cost per person is greater by $12.
    (a) What is the fixed total cost to charter the aircraft?
    (b) What is the cost per person if 40 people charter the aircraft?

16. An antiques dealer bought c antique chairs for a total of x dollars. The dealer sold each chair for y dollars.
    (a) Write an algebraic expression for the profit, P, earned from buying and selling the chairs.
    (b) Write an algebraic expression for the profit per chair.

17. In the coordinate system below, find the following.
    (a) Coordinates of point Q
    (b) Lengths of PQ, QR, and PR
    (c) Perimeter of $\triangle PQR$
    (d) Area of $\triangle PQR$
    (e) Slope, y-intercept, and equation of the line passing through points P and R



18. In the xy-plane, find the following.
    (a) Slope and y-intercept of the line with equation $2y + x = 6$
    (b) Equation of the line passing through the point $(3, 2)$ with y-intercept 1
    (c) The y-intercept of a line with slope 3 that passes through the point $(-2, 1)$
    (d) The x-intercepts of the graphs in (a), (b), and (c)

19. For the parabola $y = x^2 - 4x - 12$ in the xy-plane, find the following.
    (a) The x-intercepts
    (b) The y-intercept
    (c) Coordinates of the vertex

20. For the circle $(x - 1)^2 + (y + 1)^2 = 20$ in the xy-plane, find the following.
    (a) Coordinates of the center
    (b) Radius
    (c) Area

15. (a) \$4,320 (b) \$108

16. (a) $P = cy - x$ (b) Profit per chair: $\frac{P}{c} = \frac{cy - x}{c} = y - \frac{x}{c}$

17. (a) $(-2, 0)$
(b) $PQ = 6$, $QR = 7$, $PR = \sqrt{85}$
(c) $13 + \sqrt{85}$
(d) 21
(e) Slope: $-\frac{6}{7}$; y-intercept: $\frac{30}{7}$; equation of line: $y = -\frac{6}{7}x + \frac{30}{7}$, or $7y + 6x = 30$

18. (a) Slope: $-\frac{1}{2}$; y-intercept: 3
(b) $y = \frac{x}{3} + 1$
(c) 7
(d) $6, -3, -\frac{7}{3}$

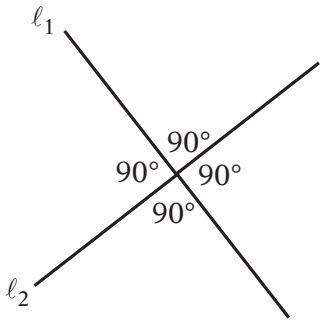
19. (a) $x = -2$ and $x = 6$
(b) $y = -12$
(c) $(2, -16)$

20. (a) $(1, -1)$
(b) $\sqrt{20}$
(c) 20π

21. (a) Domain: the set of all real numbers. The graph is a horizontal line with y-intercept -4 and no x-intercept.
(b) Domain: the set of all real numbers. The graph is a line with slope -900, y-intercept 100, and x-intercept $\frac{1}{9}$.
(c) Domain: the set of all real numbers. The graph is a parabola opening downward with vertex at $(-20, 5)$, line of symmetry $x = -20$, y-intercept -395, and x-intercepts $-20 \pm \sqrt{5}$.
(d) Domain: the set of numbers greater than or equal to -2. The graph is half a parabola opening to the right with vertex at $(-2, 0)$, x-intercept -2, and y-intercept $\sqrt{2}$.
(e) Domain: the set of all real numbers. The graph is two half-lines joined at the origin: one half-line is the negative x-axis and the other is a line starting at the origin with slope 2. Every nonpositive number is an x-intercept, and the y-intercept is 0. The function is equal to the following piecewise-defined function

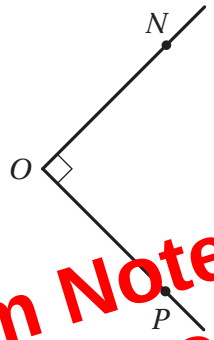
$$f(x) = \begin{cases} 2x, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

Two lines that intersect to form four congruent angles are called **perpendicular lines**. Each of the four angles has a measure of 90°. An angle with a measure of 90° is called a **right angle**. The figure below shows two lines, ℓ_1 and ℓ_2, that are perpendicular, denoted by $\ell_1 \perp \ell_2$.



$\ell_1 \perp \ell_2$

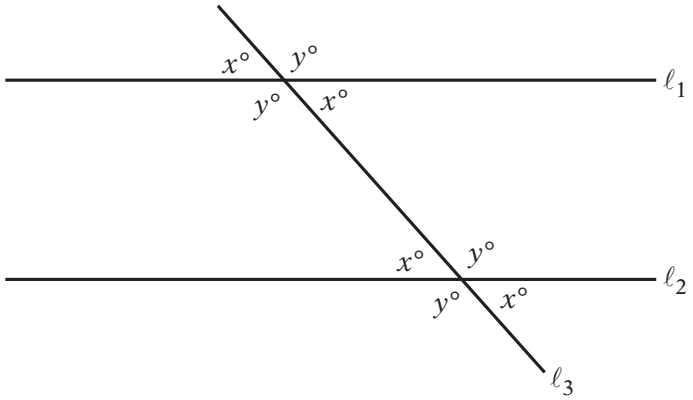
A common way to indicate that an angle is a right angle is to draw a small square at the vertex of the angle, as shown below, where *PON* is a right angle.



$ON \perp OP$

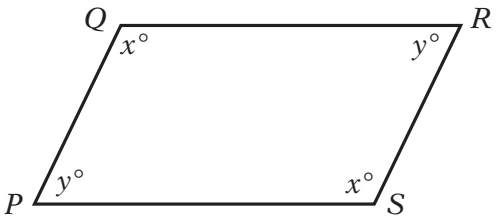
An angle with a measure less than 90° is called an **acute angle**, and an angle with a measure between 90° and 180° is called an **obtuse angle**.

Two lines in the same plane that do not intersect are called **parallel lines**. The figure below shows two lines, ℓ_1 and ℓ_2, that are parallel, denoted by $\ell_1 \| \ell_2$. The two lines are intersected by a third line, ℓ_3, forming eight angles. Note that four of the angles have the measure $x°$, and the remaining four angles have the measure $y°$, where $x + y = 180$.



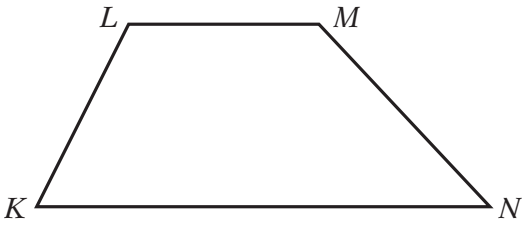
$\ell_1 \| \ell_2$

- A quadrilateral in which both pairs of opposite sides are parallel is called a **parallelogram**. In a parallelogram, opposite sides are congruent and opposite angles are congruent.



$PQ \| SR$ and $PS \| QR$
$PQ = SR$ and $PS = QR$

- A quadrilateral in which two opposite sides are parallel is called a **trapezoid**.

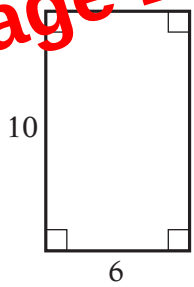


$KN \| LM$

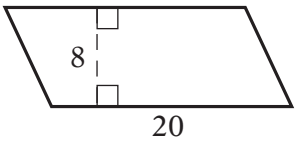
For all parallelograms, including rectangles and squares, the **area** A equals the product of the length of a base b and the corresponding height h; that is,

$$A = bh$$

Any side can be used as a base. The height corresponding to the base is the perpendicular line segment from any point of a base to the opposite side (or to an extension of that side). Below are examples of finding the areas of a rectangle and a parallelogram.



$A = (6)(10) = 60$

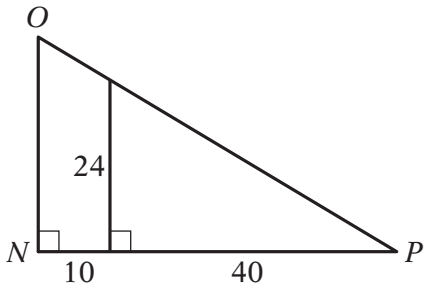


$A = (20)(8) = 160$

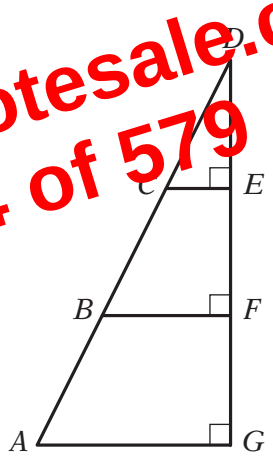
The **area** A of a trapezoid equals half the product of the sum of the lengths of the two parallel sides b_1 and b_2 and the corresponding height h; that is,

$$A = \frac{1}{2}(b_1 + b_2)(h)$$

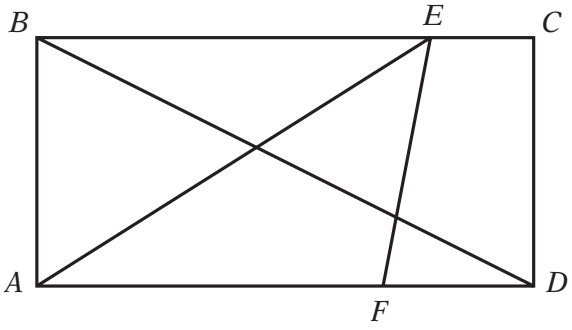
4. What is the sum of the measures of the interior angles of a decagon (10-sided polygon)?
5. If the decagon in exercise 4 is regular, what is the measure of each interior angle?
6. The lengths of two sides of an isosceles triangle are 15 and 22, respectively. What are the possible values of the perimeter?
7. Triangles PQR and XYZ are similar. If $PQ = 6, PR = 4$, and $XY = 9$, what is the length of side XZ?
8. What are the lengths of sides NO and OP in triangle NOP below?



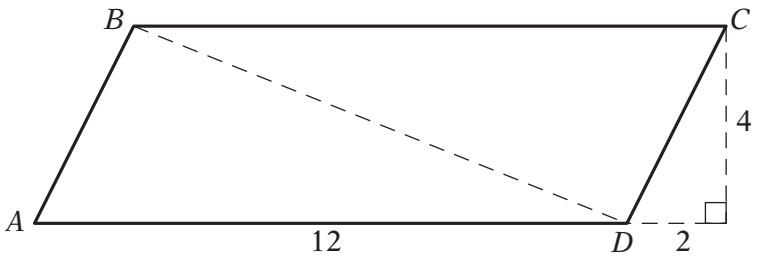
9. In the figure below, $AB = BC = CD$. If the area of triangle CDE is 42, what is the area of triangle ADG?



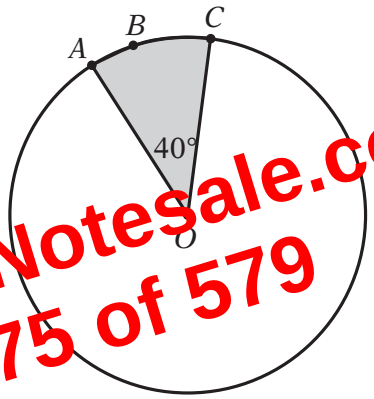
10. In rectangle $ABCD$ below, $AB = 5, AF = 7$, and $FD = 3$. Find the following.
    (a) Area of $ABCD$
    (b) Area of triangle AEF
    (c) Length of BD
    (d) Perimeter of $ABCD$



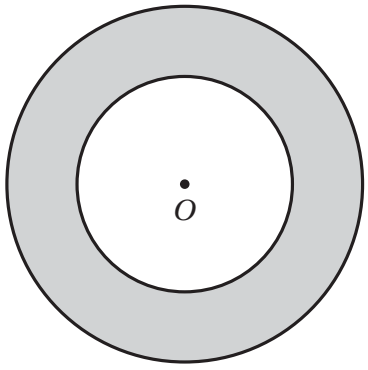
11. In parallelogram *ABCD* below, find the following.
    (a) Area of *ABCD*
    (b) Perimeter of *ABCD*
    (c) Length of diagonal *BD*



12. The circle with center *O* below has radius 4. Find the following.
    (a) Circumference of the circle
    (b) Length of arc *ABC*
    (c) Area of the shaded region



13. The figure below shows two concentric circles, each with center *O*. Given that the larger circle has radius 12 and the smaller circle has radius 7, find the following.
    (a) Circumference of the larger circle
    (b) Area of the smaller circle
    (c) Area of the shaded region



To calculate the **mean** of n numbers, take the sum of the n numbers and divide it by n.

Example 4.2.1: For the five numbers 6, 4, 7, 10, and 4, the mean is

$$\frac{6+4+7+10+4}{5} = \frac{31}{5} = 6.2$$

When several values are repeated in a list, it is helpful to think of the mean of the numbers as a **weighted mean** of only those values in the list that are *different*.

Example 4.2.2: Consider the following list of 16 numbers.

$$2, 4, 4, 5, 7, 7, 7, 7, 7, 7, 8, 8, 9, 9, 9, 9$$

There are only 6 different values in the list: 2, 4, 5, 7, 8, and 9. The mean of the numbers in the list can be computed as

$$\frac{1(2)+2(4)+1(5)+6(7)+2(8)+4(9)}{1+2+1+6+2+4} = \frac{109}{16} = 6.8125$$

The number of times a value appears in the list, or the frequency, is called the **weight** of that value. So the mean of the 16 numbers is the weighted mean of the values 2, 4, 5, 7, 8, and 9, where the respective weights are 1, 2, 1, 6, 2, and 4. Note that the sum of the weights is the number of numbers in the list, 16.

The mean can be affected by just a few values that lie far above or below the rest of the data, because these values contribute directly to the sum of the data and therefore to the mean. By contrast, the **median** is a measure of central tendency that is fairly unaffected by unusually high or low values relative to the rest of the data.

To calculate the median of n numbers, first order the numbers from least to greatest. If n is odd, then the median is the middle number in the ordered list of numbers. If n is even, then there are *two* middle numbers, and the median is the average of these two numbers.

Example 4.2.3: The five numbers in example 4.2.1 listed in increasing order are 4, 4, 6, 7, 10, so the median is 6, the middle number. Note that if the number 10 in the list is replaced by the number 24, the mean increases from 6.2 to

$$\frac{4+4+6+7+24}{5} = \frac{45}{5} = 9$$

but the median remains equal to 6. This example shows how the median is relatively unaffected by an unusually large value.

The median, as the "middle value" of an ordered list of numbers, divides the list into roughly two equal parts. However, if the median is equal to one of the data values and it is repeated in the list, then the numbers of data above and below the median may be rather different. See example 4.2.2, where the median is 7, but four of the data are less than 7 and six of the data are greater than 7.

The **mode** of a list of numbers is the number that occurs most frequently in the list.

the mean and then we divided by the standard deviation. The number of standard deviations that a rating of 30 is away from the mean is

$$\frac{30 - 32.5}{7.1} = \frac{-2.5}{7.1} \approx -0.4$$

where the negative sign indicates that the rating is 0.4 standard deviation *below* the mean.

The number of standard deviations that a rating of 20 is away from the mean is

$$\frac{20 - 32.5}{7.1} = \frac{-12.5}{7.1} \approx -1.8$$

where the negative sign indicates that the rating is 1.8 standard deviations *below* the mean.

To summarize:

- 48 points is 15.5 points above the mean, or approximately 2.2 standard deviations above the mean.
- 30 points is 2.5 points below the mean, or approximately 0.4 standard deviation below the mean.
- 20 points is 12.5 points below the mean, or approximately 1.8 standard deviations below the mean.

One more instance, which may seem trivial, is important to note:

- 32.5 points is 0 points from the mean, or 0 standard deviations from the mean.

Example 4.2.9 shows that for a group of data, each value can be located with respect to the mean by using the standard deviation as a ruler. The process of subtracting the mean from each value and then dividing the result by the standard deviation is called **standardization**. Standardization is a useful tool because for each data value, it provides a measure of position relative to the rest of the data independently of the variable for which the data was collected and the units of the variable.

Note that the standardized values 2.2, −0.4, and −1.8 from example 4.2.9 are all between −3 and 3; that is, the corresponding ratings 48, 30, and 20 are all within 3 standard deviations above or below the mean. This is not surprising, based on the following fact about the standard deviation.

*In **any group of data**, most of the data are within about **3** standard deviations above or below the mean.*

Thus, when *any group of data* are standardized, most of the data are transformed to an interval on the number line centered about 0 and extending from about −3 to 3. The mean is always transformed to 0.

4.3 Counting Methods

Uncertainty is part of the process of making decisions and predicting outcomes. Uncertainty is addressed with the ideas and methods of probability theory. Since elementary probability requires an understanding of counting methods, we now turn to a discussion of counting objects in a systematic way before reviewing probability.

When a set of objects is small, it is easy to list the objects and count them one by one. When the set is too large to count that way, and when the objects are related in a

plying the multiplication principle, the number of ways to select and order k objects from a set of n objects is $n(n-1)(n-2)\cdots(n-k+1)$. It is useful to note that

$$n(n-1)(n-2)\cdots(n-k+1) = n(n-1)(n-2)\cdots(n-k+1)\frac{(n-k)!}{(n-k)!}$$
$$= \frac{n!}{(n-k)!}$$

This expression represents the number of **permutations of *n* objects taken *k* at a time**, that is, the number of ways to select and order k objects out of n objects.

Example 4.3.5: How many different five-digit positive integers can be formed using the digits 1, 2, 3, 4, 5, 6, and 7 if none of the digits can occur more than once in the integer?

Solution: This example asks how many ways there are to order 5 integers chosen from a set of 7 integers. According to the counting principle above, there are (7)(6)(5)(4)(3) = 2,520 ways to do this. Note that this is equal to $\frac{7!}{(7-5)!} = \frac{(7)(6)(5)(4)(3)(2!)}{2!} = (7)(6)(5)(4)(3)$.

Combinations

Given the five letters A, B, C, D, and E, suppose that you want to determine the number of ways in which you can select 3 of the 5 letters, but unlike before, you do not want to count different orders for the 3 letters. The following is a list of all of the ways in which 3 of the 5 letters can be selected without regard to the order of the letters.

ABC ABD ABE ACD ACE ADE BCD BCE BDE CDE

There are 10 ways of selecting the 3 letters without order. There is a relationship between selecting with order and selecting without order.

The number of ways to select 3 of the 5 letters without order, which is 10, *multiplied by* the number of ways to order the 3 letters, which is 3!, or 6, *is equal to* the number of ways to select 3 of the 5 letters and order them, which is $\frac{5!}{2!} = 60$. In short,

(number of ways to select without order) × (number of ways to order)
= (number of ways to select with order)

This relationship can also be described as follows.

$$\text{(number of ways to select without order)} = \frac{\text{(number of ways to select with order)}}{\text{(number of ways to order)}}$$
$$= \frac{\frac{5!}{2!}}{3!} = \frac{5!}{3!2!} = 10$$

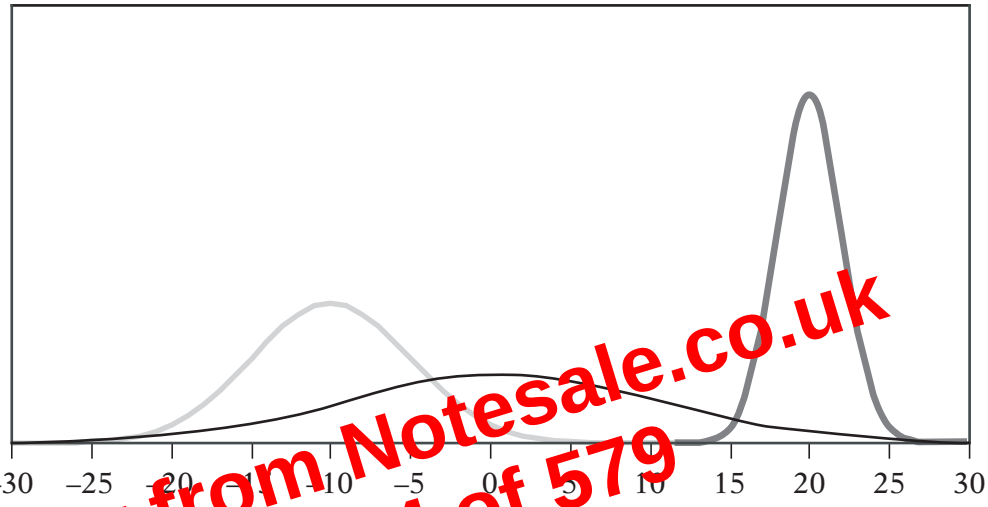
More generally, suppose that k objects will be chosen from a set of n objects, where $k \le n$, but that the k objects will *not* be put in order. The number of ways in which this can be done is called the number of **combinations of *n* objects taken *k* at a time** and is given by the formula $\frac{n!}{k!(n-k)!}$.

Another way to refer to the number of combinations of n objects taken k at a time is ***n* choose *k***, and two notations commonly used to denote this number are ${}_nC_k$ and $\binom{n}{k}$.

Just as a data distribution has a mean and standard deviation, the normal probability distribution has a mean and standard deviation. Also, the properties listed above for the approximately normal distribution of data hold for the normal distribution, except that the mean, median, and mode are exactly the same and the distribution is perfectly symmetric about the mean.

A normal distribution, though always shaped like a bell, can be centered around any mean and can be spread out to a greater or lesser degree, depending on the standard deviation. Below are three normal distributions that have different centers and spreads. From left to right, the means of the three distributions are −10, 1, and 20; and the standard deviations are 5, 10, and 2.

Three Normal Distributions



As mentioned earlier, areas of the bars in a histogram for a discrete random variable correspond to probabilities for the values of the random variable; the sum of the areas is 1 and the sum of the probabilities is 1. This is also true for a continuous probability distribution: the area of the region under the curve is 1, and the areas of vertical slices of the region—similar to the bars of a histogram—are equal to probabilities of a random variable associated with the distribution. Such a random variable is called a **continuous random variable**, and it plays the same role as a random variable that represents a randomly chosen value from a distribution of data. The main difference is that we seldom consider the event in which a continuous random variable is equal to a single value like $X = 3$; rather, we consider events that are described by intervals of values such as $1 < X < 3$ and $X > 10$. Such events correspond to vertical slices under a continuous probability distribution, and the areas of the vertical slices are the probabilities of the corresponding events. (Consequently, the probability of an event such as $X = 3$ would correspond to the area of a line segment, which is 0.)

Example 4.5.4: If W is a random variable that is normally distributed with a mean of 5 and a standard deviation of 2, what is $P(W > 5)$? Approximately what is $P(3 < W < 7)$? Which of the four numbers 0.5, 0.1, 0.05, or 0.01 is the best estimate of $P(W < -1)$?

(c) The set described here is represented by the part of the rectangle that is *not in either circle*. Let N be the number of these travelers. Note that the entire rectangular region has two main nonoverlapping parts: the part *outside* the circles and the part *inside* the circles. The first part represents N travelers and the second part represents $93 + 155 - 70 = 178$ travelers (from question (b)). Therefore,

$$250 = N + 178$$

and solving for N yields

$$N = 250 - 178 = 72$$

DATA ANALYSIS EXERCISES

1. The daily temperatures, in degrees Fahrenheit, for 10 days in May were 61, 62, 65, 65, 65, 68, 74, 74, 75, and 77.
   (a) Find the mean, median, mode, and range of the temperatures.
   (b) If each day had been 7 degrees warmer, what would have been the mean, median, mode, and range of those 10 temperatures?

2. The numbers of passengers on 9 airline flights were 22, 33, 21, 28, 22, 31, 44, 50, and 19. The standard deviation of these numbers is approximately equal to 10.2.
   (a) Find the mean, median, mode, range, and interquartile range of the 9 numbers.
   (b) If each flight had had 3 times as many passengers, what would have been the mean, median, mode, range, interquartile range, and standard deviation of the 9 numbers?
   (c) If each flight had had 2 fewer passengers, what would have been the interquartile range and standard deviation of the 9 numbers?

3. A group of 20 values has a mean of 85 and a median of 80. A different group of 30 values has a mean of 75 and a median of 72.
   (a) What is the mean of the 50 values?
   (b) What is the median of the 50 values?

4. Find the mean and median of the values of the random variable X, whose relative frequency distribution is given in the table below.

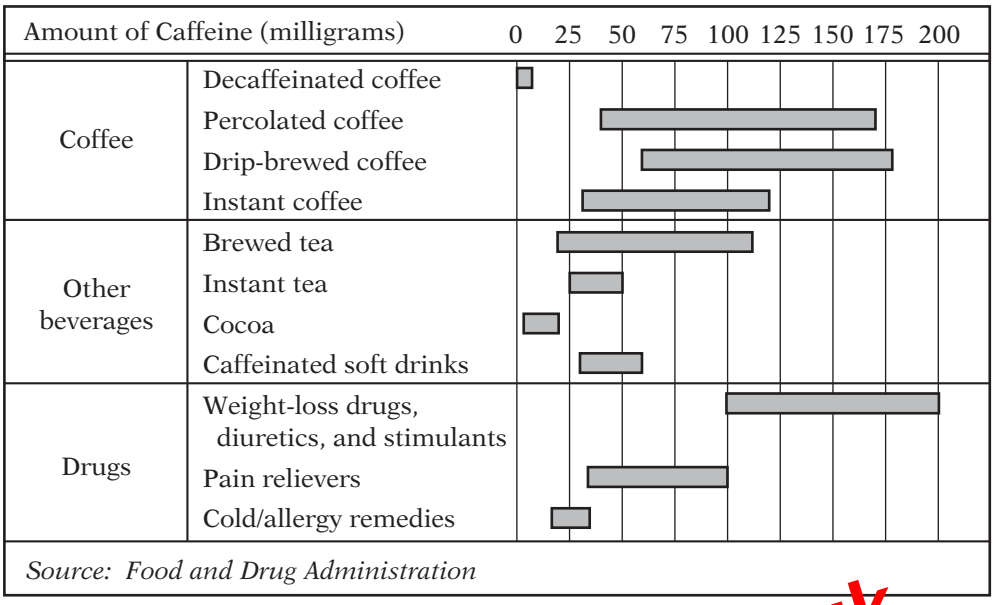
X	Relative Frequency
0	0.18
1	0.33
2	0.10
3	0.06
4	0.33

6. $5! = 120$
7. 24
8. 288
9. 210
10. (a) 336 (b) 56
11. (a) $\frac{1}{9}$ (b) $\frac{1}{5}$ (c) $\frac{4}{5}$
12. (a) $\frac{4}{5}$ (b) $\frac{1}{45}$
13. (a) $\frac{21}{40}$ (b) $\frac{7}{10}$ (c) $\frac{9}{40}$
14. (a) 0.4 (b) 0.2
15. (a) 0.56 (b) 0.94 (c) 0.06
16. (a) 1,440 (b) 0.15
17. (a) 1998 (b) 19%
18. (a) Three (b) 9 to 14, or $\frac{9}{14}$ (c) I, II, and III
19. (a) $17,550 (b) Miscellaneous

NO TEST MATERIAL ON THIS PAGE

Questions 17 to 20 are based on the following data.

VARIATION IN THE AMOUNT OF CAFFEINE IN COMMON BEVERAGES AND DRUGS*



*Based on 5-ounce cups of coffee, tea, and cocoa; 12-ounce cups of soft drinks; and single doses of drugs.

17. The least amount of caffeine in a 5-ounce cup of drip-brewed coffee exceeds the greatest amount of caffeine in a 5-ounce cup of cocoa by approximately how many milligrams?

(A) 160
(B) 80
(C) 60
(D) 40
(E) 20

For the following question, enter your answer in the box.

18. For how many of the 11 categories of beverages and drugs listed in the graph can the amount of caffeine in the given serving size be less than 50 milligrams?

[ ] categories

19. Approximately what is the minimum amount of caffeine, in milligrams, consumed per day by a person who daily drinks two 10-ounce mugs of percolated coffee and one 12-ounce cup of a caffeinated soft drink?

(A) 230
(B) 190
(C) 140
(D) 110
(E) 70

Questions 10 to 25 have several different formats. Unless otherwise directed, select a single answer choice. For Numeric Entry questions, follow the instructions below.

Numeric Entry Questions
Enter your answer in the answer box(es) below the question.

- **Your answer may be an integer, a decimal, or a fraction, and it may be negative.**
- **If a question asks for a fraction, there will be two boxes—one for the numerator and one for the denominator.**
- **Equivalent forms of the correct answer, such as 2.5 and 2.50, are all correct. Fractions do not need to be reduced to lowest terms.**
- **Enter the exact answer unless the question asks you to round your answer.**

10. The fabric needed to make 3 curtains sells for \$8.00 per yard and can be purchased only by the full yard. If the length of fabric required for each curtain is 1.6 yards and all of the fabric is purchased as a single length, what is the total cost of the fabric that needs to be purchased for the 3 curtains?

(A) \$40.00
(B) \$38.40
(C) \$24.00
(D) \$16.00
(E) \$11.20

For the following question, select all the answer choices that apply.

11. In the xy-plane, line k is a line that does not pass through the origin.

Which of the following statements individually provide(s) sufficient additional information to determine whether the slope of line k is negative?

Indicate all such statements.

A The x-intercept of line k is twice the y-intercept of line k.
B The product of the x-intercept and the y-intercept of line k is positive.
C Line k passes through the points (a, b) and (r, s), where $(a - r)(b - s) < 0$.

For the following question, select all the answer choices that apply.

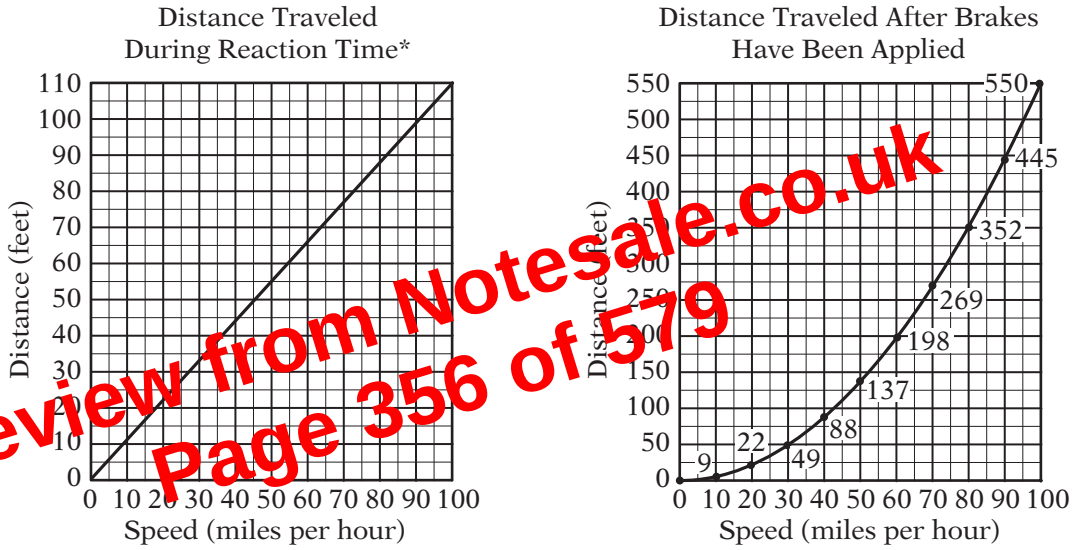
16. The integer v is greater than 1. If v is the square of an integer, which of the following numbers must also be the square of an integer?

Indicate all such numbers.

A $81v$
B $25v+10\sqrt{v}+1$
C $4v^2+4\sqrt{v}+1$

Questions 17 to 20 are based on the following data.

DISTANCE TRAVELED BY A CAR ACCORDING TO THE CAR'S SPEED WHEN THE DRIVER IS SIGNALED TO STOP



*Reaction time is the time period that begins when the driver is signaled to stop and ends when the driver applies the brakes.

Note: Total stopping distance is the sum of the distance traveled during reaction time and the distance traveled after brakes have been applied.

17. The speed, in miles per hour, at which the car travels a distance of 52 feet during reaction time is closest to which of the following?

(A) 43
(B) 47
(C) 51
(D) 55
(E) 59

Once you determine your scaled scores, you will need to evaluate your performance. To get a sense of how test takers are scoring on the Verbal Reasoning and Quantitative Reasoning measures of the actual test, you can review Verbal Reasoning and Quantitative Reasoning percentile ranks on the GRE website at **www.ets.org/gre/percentile** (PDF). A percentile rank for a score indicates the percentage of examinees who took that test and received a lower score. Updated annually in July, this table includes the Verbal Reasoning and Quantitative Reasoning scores on the 130–170 scale in one-point increments and the corresponding percentile ranks. For each score you earned on Practice Test 1, note the percent of GRE test takers who earned lower scores. This is a reasonable indication of your rank among GRE revised General Test examinees if you took Practice Test 1 under standard timed conditions.

a negative manner. Perhaps the child's parents are in the middle of a divorce and the child is outwardly expressing his frustration in the classroom. Or the academic content of the class may not be challenging enough for the child and so he is misbehaving out of boredom. Whatever the reason behind the behavior, the key factor is that the teacher works with the child to try and identify it. Simply punnishing or ignoring the child would not solve the problem, whereas working to create a plan for success in the classroom would. Likewise, rather than punnishing and defeating the child, the teacher is working with and empowering the child; a much more positive outcome to the situation.

Reader Commentary

This strong response presents a thoughtful and well-developed analysis of the issue. In this case the writer argues that teachers need to modify their approach based on context and observation, meaning that a blanket approach cannot be successful. The writer supports this position with relevant reasons and examples that present logically sound support. Note that the task instructions ask writers to discuss circumstances in which adopting the recommendation might or might not prove advantageous, and this response does that quite clearly. In the second paragraph, the writer gives an example of a student who completes an assignment on time and receives positive feedback, showing how the recommendation could prove advantageous. Other examples show circumstances in which adopting the recommendation would not be a good idea, and these various points are brought together to support the writer's position that teachers have to look at the context of the situation and cannot rely on simply ignoring negative actions. This response also demonstrates facility with language, using appropriate vocabulary and sentence variety. Sentences like this one demonstrate the writer's command of the conventions of standard written English: "If the child does have an attentional problem, then the teacher can work with a related service, such as occupational therapy, to alter the classroom environment in order to cater to the needs of the child." There are some minor errors, but overall the response demonstrates strong control of language. Although the response is clearly stronger than a 4, which would simply present a clear position on the issue according to the task instructions, it does not reach the level of a 6 because it does not develop its points in a way that creates a cogent and insightful position. It does, however, present a generally thoughtful and well-developed analysis of the issue, leading to a score of 5.

Score 4 Response

I absolutely agree with the first section of the statement above, but find fault with the latter half.

There is no doubt that praising positive actions is an excellent way to teach, and this method is most clearly exemplified when dealing with much younger children. When a young child is learning basic social behavior, it is imperative that he is encouraged to repeat positive actions. For example, when a child voluntarily shares his toys with another, if a teacher rewards that behavior, the child will understand that this is a good practice, and likely share again in the future.

In contrast, if a child displays negative behavior by stealing a toy away from his playmate, it would be very dangerous for the teacher to ignore this action, for then the child may never recognize that this is unacceptable. In this instance, the child has not

learned from the situation at all. So what should a teacher do when faced with such a situation? Punishment is not necessarily the optimal choice, either. Rather than scolding a child for mistreating his playmates and sending him off to a corner, a teacher would be wise to demonstrate the positive alternative: to share his toys instead. In this case, rather than ignoring or punishing negative actions, the teacher could seize the opportunity to reinforce positive behavior, and further extend the child's learning experience.

In summary, positive reinforcement is certainly an excellent method for teaching new methods or behaviors, and encouraging a student to learn more. However to ignore, rather than recognize and correct negative actions, would be a disservice to the student, for he would not know what conclusion to draw from his action.

Reader Commentary

This adequate response follows the task directions and presents a clear position on the issue, supporting its main points with examples that are relevant, if only adequately developed. For instance, the discussion in the second paragraph of a teacher who reinforces the positive behavior of sharing a toy is certainly relevant and on-task (i.e., it describes a situation in which adopting the recommendation would be advantageous). However, the development of this idea does not lead to generally thoughtful or insightful analysis. Instead, it is simply presented as an example. In addition to its adequate development, this response also demonstrates sufficient control of the conventions of standard written English, and its main points are made with reasonable clarity. Some of the sentences demonstrate the syntactical variety normally seen in responses that receive higher scores (e.g., "Rather than scolding a child for mistreating his playmates and sending him off to a corner, a teacher would be wise to demonstrate the positive alternative: to share his toys instead"). However, the overall use of language in this response is merely adequate.

Score 3 Response

Praising postive actions and ignoring negative ones may be a good way to teach but not the best way. Ignoring negative actions could negate all the postive praises given to an individual, having negative actions go unchecked will lead to habits formed that would overwhelm any positive actions that are complementary to an individuals learning process.

For instance, in a classroom full of eight-year old kids; if during a lesson they are making alot of noise, having this ignored would tell the kids that it is okay to be disruptive in class. The individuals in that class would develop the habit of being distruptive hence hindering their learning process. However if the eight-year old kids were immediately told to stop the distruption then it will never become a habit.

Every action needs to have a related consequence follow in a learning environment. In the early years of education, the way they are taught becomes a lifelong habit which is hard to change in later years. If negative actions are not assigned a related consequences then teaching becomes ineffective because the students negative actions soon diminish the ability to do well in school. The way postive actions are dealt with should also be done with negative actions rather than being ignored which in turn enhance the learning environment.

Answers and Explanations

SECTION 3
Verbal Reasoning
25 Questions with Explanations

For each of Questions 1 to 4, select one answer choice unless otherwise instructed.

Questions 1 to 3 are based on the following reading passage.

Whether the languages of the ancient American peoples were used for expressing abstract universal concepts can be clearly answered in the case of Nahuatl. Nahuatl, like Greek and German, is a language that allows the formation of extensive compounds. By the combination of radicals or semantic elements, single compound words can express complex conceptual relations, often of an abstract universal character.

The *tlamatinime* (those who know) were able to use this rich stock of abstract terms to express the nuances of their thought. They also availed themselves of other forms of expression with metaphorical meaning, some probably original, some derived from Toltec coinages. Of these forms, the most characteristic in Nahuatl is the juxtaposition of two words that, because they are synonyms, associated terms, or even contraries, complement each other to evoke one single idea. Used metaphorically, the juxtaposed terms connote specific or essential traits of the being they refer to, introducing a mode of poetry as an almost habitual form of expression.

Description

This passage claims that Nahuatl was used to express abstract universal concepts, by combining semantic elements, and goes on to explain that the *tlamatinime* used these terms to express subtle distinctions.

For the following question, consider each of the choices separately and select all that apply.

1. Which of the following can be inferred from the passage regarding present-day research relating to Nahuatl?

   A Some record or evidence of the thought of the *tlamatinime* is available.

   B For at least some Nahuatl expressions, researchers are able to trace their derivation from another ancient American language.

   C Researchers believe that in Nahuatl, abstract universal concepts are always expressed metaphorically.

Explanation

Choices A and B are correct.

Choice A is correct: the *tlamatinime* are mentioned in the first sentence of the second paragraph, where it says they were able to use Nahuatl's stock of abstract terms "to express the nuances of their thought." This suggests that there is some evidence of what those thoughts were, and therefore Choice A can be inferred.

Choice B is correct: according to the next sentence, Nahuatl speakers used "forms of expression with metaphorical meaning," some of which were probably "original" and others "derived from Toltec coinages." That researchers know certain Nahuatl expressions are derived from Toltec suggests that they are able to trace the derivation of some Nahuatl expressions from another language besides Nahuatl, and therefore Choice B may be inferred.

Choice C is incorrect: the passage says that in Nahuatl there are single compound words that can express conceptual relations of an "abstract universal character" and mentions "other forms of expression with metaphorical meaning," but it does not indicate whether metaphorical words or phrases are the only way that abstract universal concepts are expressed in Nahuatl, or whether researchers believe this about Nahuatl. Therefore Choice C cannot be inferred.

2. Select the sentence in the passage in which the author introduces a specific Nahuatl mode of expression that is not identified as being shared with certain European languages.

Explanation

The passage introduces two specific Nahuatl modes of expression. One is the formation of single compound words that are capable of expressing complex conceptual relations (first paragraph); the other is the juxtaposition of two related words to evoke a single idea (second paragraph). In the formation of compounds Nahuatl is described as being "like Greek and German," but the second mode is not identified as being shared with other languages. Therefore the **sixth sentence** ("All of these forms . . . one single idea") is the best choice.

3. In the context in which it appears, "coinages" (line 9) most nearly means
   - (A) adaptations
   - (B) creations
   - (C) idiosyncrasies
   - (D) pronunciations
   - (E) currencies

Explanation

"Coinage" has two senses that are represented among the answer choices: in one sense it denotes coins and currency, while in the other it denotes things—especially words—that are invented. The fifth sentence draws a contrast between linguistic expressions original to Nahuatl and those derived from Toltec. In this context of original versus derived language, "coinages" means "inventions," not "currencies." Of the answer choices given, "creations" is the nearest equivalent of "coinages" in the sense of "inventions," and therefore **Choice B** is the best answer.

For the following question, consider each of the choices separately and select all that apply.

10. The passage implies that during the day before a night on which a male nightingale's song rate is high, that nightingale probably does which of the following?

[A] Expends less of its reserves on thermoregulation than on other days
[B] Stores more energy as body reserves than on other days
[C] Hides to avoid predators

Explanation

Choice B is correct.

Choice A is incorrect: the only reference to thermoregulation comes in line 24 and discusses nighttime activity, not daytime activity.

Choice B is correct: the second paragraph explains that birds store energy as fat deposits that can be estimated by measuring body mass, and that body mass at dusk was significantly higher in nightingales on nights when their song rate was higher.

Choice C is incorrect: while the passage does say that singing exposes birds to predators (line 3), it says nothing to suggest that they make special efforts to hide before singing, and in fact it says that nightingales spend extra time foraging (line 20).

11. Select the sentence in the first or second paragraph that presents empirical results in support of a hypothesis about the energy costs of singing.

Explanation

Only two sentences in the relevant portion of the passage contain information that might be considered to be empirical results. The last sentence of the first paragraph contains information about increases in energy consumption but only the last sentence of the second paragraph provides results in support of the only hypothesis in the passage, that nightingales should lose more body mass on nights when their song rate is high. Thus, **sentence 9** ("Thomas found . . . high") is the correct choice.

For the following question, consider each of the choices separately and select all that apply.

12. It can be inferred from the passage that compared with other costs of singing, which of the following is true of the energy costs of singing?

[A] They are the single greatest cost to an individual bird.
[B] They have generally received more attention from scientists.
[C] They vary less from one bird species to another.

Explanation

Choice B is correct.

Choice A is incorrect: you might infer that energy costs of singing are significant but no information is given to suggest that they are greater than other costs.

Choice B is correct: lines 4–5 say that discussions of the costs of singing have generally focused on energy costs.

Explanation

The point of the sentence is to emphasize contradictory aspects of Belanger's dancing: we are told, for example, that he seems "at once intensely present and curiously detached." Looking at the second blank with this point in mind, we can see that the sentence is saying that Belanger draws attention in some way that would not normally be a means of doing so. The only choice that fits, therefore, is "deflect"; focusing or overwhelming attention would certainly be expected to draw it. And since employing "unrestrained enthusiasm" or "unattractive gawkiness" would not be ways of deflecting attention, the correct choice for the first blank is "undemonstrative panache," another paradoxical term, since "panache" means "dash or flamboyance in style."

Thus, the correct answer is **undemonstrative panache** (Choice A) and **deflect** (Choice F).

17. The most striking thing about the politician is how often his politics have been (i)_____ rather than ideological, as he adapts his political positions at any particular moment to the political realities that constrain him. He does not, however, piously (ii)_____ political principles only to betray them in practice. Rather, he attempts in subtle ways to balance his political self-interest with a (iii) _____, viewing himself as an instrument of some unchanging higher purpose.

Blank (i)	Blank (ii)	Blank (iii)
(A) quixotic	(D) brandish	(G) profound cynicism
(B) self-righteous	(E) flout	(H) deeply felt moral code
(C) strategic	(F) follow	(I) thoroughgoing pragmatism

Explanation

Since the politician is portrayed as adapting political positions to political realities, blank (i) should be filled with "strategic," which is also the only choice that provides the required contrast with "ideological." The second blank, *brandishing* political principles is what a politician might do piously, while *flouting* is not pious and *following* principles does not make sense when combined with "betray[ing] them in practice." The third blank requires something that would have to be balanced against "political self-interest" and that would be embraced in service of an "unchanging higher purpose," making "deeply felt moral code" the only viable choice.

Thus, the correct answer is **strategic** (Choice C), **brandish** (Choice D), and **deeply felt moral code** (Choice H).

For each of Questions 18 to 20, select <u>one</u> answer choice unless otherwise instructed.

Questions 18 to 20 are based on the following reading passage.

The condition of scholarship devoted to the history of women in photography is confounding. Recent years have witnessed the posthumous inflation of the role of the hobbyist Alice Austen into that of a pioneering documentarian while dozens of notable

Explanation

Any of the offered words could possibly describe a conductor's choice of tempo. However, the phrase "without necessary relation to what had gone before" is presented as an elaboration on the word in the blank. Among the answer choices, only "arbitrary" and "capricious" could be elaborated that way; none of the other choices would be explained by the final phrase.

Thus, the correct answer is **arbitrary** (Choice A) and **capricious** (Choice B).

25. Because they had expected the spacecraft Voyager 2 to be able to gather data only about the planets Jupiter and Saturn, scientists were _____ the wealth of information it sent back from Neptune twelve years after leaving Earth.

A anxious for
B confident in
C thrilled about
D keen on
E elated by
F eager for

Explanation

In the sentence, the words "expected" and "only" imply that the data received from the spacecraft exceeded scientists' expectations. Therefore, the words that fill the blank should describe a reaction to results that are better than hoped for, and the choices "thrilled about" and "elated by" both express such a reaction. The scientists may well also have been eager for, or keen on, the information, but their eagerness is not well explained by the unexpectedness of the information.

Thus, the correct answer is **thrilled about** (Choice C) and **elated by** (Choice E).

18. The activists' energetic work in the service of both woman suffrage and the temperance movement in the late nineteenth century (i)_____ the assertion that the two movements were (ii)_____.

Blank (i)	Blank (ii)
(A) undermines	(D) diffuse
(B) supports	(E) inimical
(C) underscores	(F) predominant

Explanation

The sentence is about the implications of the activists' energetic work for some assertion about the woman suffrage and temperance movements. The second blank, however, obscures the nature of that assertion. But it is clear that the "energetic work" could either support an assertion that the two movements were similar, or undermine an assertion that the two movements were opposed. "Supports" is offered as a choice for the first blank (as is the somewhat similar "underscores"), but there is no corresponding term in the second blank, nothing along the lines of "similar" or "compatible." "Undermines" and "inimical" make for the only meaningful statement.

Thus, the correct answer is **undermines** (Choice A) and **inimical** (Choice E).

19. There is nothing quite like this movie, and indeed I am not altogether sure there is much more to it than its lovely (i) _____. In a moment when so many films strive to be as (ii)_____ as possible, it is gratifying to find one that is so subtle and puzzling.

Blank (i)	Blank (ii)
(A) peculiarity	(D) indirect
(B) pellucidity	(E) assertive
(C) conventionality	(F) enigmatic

Explanation

The two sentences provide the reader with quite a bit of information about the movie. There is "nothing quite like it" and it is "subtle and puzzling." "Peculiarity" is clearly a solid fit for the first blank, while "conventionality" clearly does not work, given the fact that there is "nothing quite like it." That leaves "pellucidity," which, while it could fit logically in the first sentence in isolation, does not fit the later claim that the movie is "subtle and puzzling." The second blank needs simply to provide a contrast with "subtle and puzzling." Of the choices offered, only "assertive" clearly does that.

Thus, the correct answer is **peculiarity** (Choice A) and **assertive** (Choice E).

Explanation

According to the last two sentences of the paragraph, Titan was able to acquire an atmosphere because of a prevailing low temperature, but Ganymede and Callisto could not because they were at a higher temperature. Because the reason for this difference in temperature was their respective distances from the Sun, **Choice B** is correct. The passage says nothing about differences in rate of heat loss, availability of methane and ammonia, or distance from their planets, and it explicitly states that the three moons are the same size.

Question 22 is based on the following reading passage.

Observations of the Arctic reveal that the Arctic Ocean is covered by less ice each summer than the previous summer. If this warming trend continues, within 50 years the Arctic Ocean will be ice free during the summer months. This occurrence would in itself have little or no effect on global sea levels, since the melting of ice floating in water does not affect the water level. However, serious consequences to sea levels would eventually result, because _____.

22. Which of the following most logically completes the passage?

(A) large masses of floating sea ice would continue to form in the wintertime

(B) significant changes in Arctic sea temperatures would be accompanied by changes in sea temperatures in more temperate parts of the world

(C) such a warm Arctic Ocean would trigger the melting of massive land-based glaciers in the Arctic

(D) an ice-free Arctic Ocean would support a very different ecosystem than it does presently

(E) in the spring, melting sea ice would cause more icebergs to be created and to drift south into shipping routes

Explanation

To logically complete the passage's open-ended "because," something is needed that will explain why the continuation of the warming trend would have serious consequences for sea levels. The passage explains that the melting of the Arctic Ocean ice will not affect sea levels because the contribution that the water contained in that ice makes to sea levels is the same whether the water is frozen or liquid. But Choice C points to a way in which increasing temperatures in the Arctic could add water to the ocean, namely by melting ice on the land. So **Choice C** logically completes the passage and is the correct answer.

Given that the passage has already explained that melting sea ice does not affect sea levels, the formation of sea ice described in Choice A does not explain why there would be consequences for sea levels.

Choices B, D, and E all describe possible consequences of increased temperatures in the Arctic, but none of these consequences suggests a mechanism by which sea levels would change. So none of these options provides a logical completion for the passage.

Runner A ran $\frac{4}{5}$ kilometer and Runner B ran 800 meters.

	Quantity A	Quantity B	
2.	The distance that A ran	The distance that B ran	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question you are asked to compare two measurements, one given in kilometers and the other in meters. It would be easier to compare these measurements if they were both given in meters or both given in kilometers.

If you choose to convert the distance that Runner B ran from meters to kilometers, you need to use the conversion 1 meter is equal to $\frac{1}{1{,}000}$ kilometer. Since B ran 800 meters, it follows that B ran $(800)\left(\frac{1}{1{,}000}\right)$, or $\frac{4}{5}$ kilometer, which is the same distance that A ran.

If you choose to convert the distance that Runner A ran from kilometers to meters, you need to use the conversion 1 kilometer is equal to 1,000 meters. Since A ran $\frac{4}{5}$ kilometer, it follows that A ran $\left(\frac{4}{5}\right)(1{,}000)$, or 800 meters, which is the same distance that B ran. Either way, A and B ran the same distance, and the correct answer is **Choice C**.

$x < y < z$

	Quantity A	Quantity B	
3.	$\frac{x+y+z}{3}$	y	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question you are given that $x < y < z$, and you are asked to compare $\frac{x+y+z}{3}$ with y.

Two approaches that you could use to solve this problem are:

1: Search for a mathematical relationship between the two quantities.
2: Plug in numbers for the variables.

Approach 1: Note that $\frac{x+y+z}{3}$ is the average of the three numbers x, y, and z and that y is the median. Is the average of 3 numbers always equal to the median? The average could equal the median, but in general they do not have to be equal. Therefore, the correct answer is **Choice D**.

Approach 2: When you plug in numbers for the variables, it is a good idea to consider what kind of numbers are appropriate to plug in and to choose numbers that are easy to work with, if possible.

Since $\frac{x+y+z}{3}$ is the average of the three numbers x, y, and z and you are comparing it to the median, you may want to try plugging in numbers that are evenly spaced and plugging in numbers that are not evenly spaced.

p is the probability that event E will occur, and s is the probability that event E will not occur.

	Quantity A	Quantity B	
6.	$p + s$	ps	(A) (B) (C) (D)

Explanation

Since event E will either occur or not occur, it follows that $p + s = 1$, and the value of Quantity A is always 1. Since Quantity B is the product of the two probabilities p and s, you need to look at its value for the cases $p = 1$, $p = 0$, and $0 < p < 1$.

If $p = 1$, then $s = 0$; similarly, if $p = 0$, then $s = 1$. In both cases, ps is equal to 0.

If $0 < p < 1$, both p and s are positive and less than 1, so ps is positive and less than 1. Since Quantity A is equal to 1 and Quantity B is less than 1, the correct answer is **Choice A**.

X is the set of all integers n that satisfy the inequality $2 \le |n| \le 5$.

	Quantity A	Quantity B	
7.	The absolute value of the greatest integer in X	The absolute value of the least integer in X	(A) (B) (C) (D)

Explanation

When comparing these quantities, it is important to remember that a nonzero number and its negative have the same absolute value. For example, $|-2| = |2| = 2$. Keeping this in mind, you can see that the positive integers 2, 3, 4, and 5 and the negative integers -2, -3, -4, and -5 all satisfy the inequalities $2 \le |n| \le 5$, and that these are the only such integers. Thus, the set X consists of the integers -5, -4, -3, -2, 2, 3, 4, and 5. The greatest of these integers is 5, and its absolute value is 5. The least of these integers is -5, and its absolute value is also 5. Therefore, Quantity A is equal to Quantity B. The correct answer is **Choice C**.

x and m are positive numbers, and m is a multiple of 3.

	Quantity A	Quantity B	
8.	$\frac{x^m}{x^3}$	$x^{m/3}$	(A) (B) (C) (D)

Explanation

Since $\frac{x^m}{x^3} = x^{m-3}$, you need to compare x^{m-3} with $x^{m/3}$. Since the base in both expressions is the same, a good strategy to use to solve this problem is to plug in numbers for m in both expressions and compare them.

You know that m is a multiple of 3, so the least positive integer you can plug in for m is 3.

than \$3.00, the total amount Mary paid for the book must have been greater than \$7.00. You can express this information algebraically by the inequality

$$7.00 < 1.04x < 10.00$$

Solving the inequality for x by dividing by 1.04, and rounding, you get

$$6.73 < x < 9.62$$

So you see that x, the price of the book, must be between \$6.73 and \$9.62. With this information, you can quickly examine the first two statements. Choice A is not necessarily true because the price could be as high as \$9.61, and Choice B is not necessarily true because the price could be as low as \$6.74.

To examine Choice C, you could compute the tax for the greatest possible price, which would be 4% of 9.61, or $(0.04)(9.61) = 0.38$. Since this greatest possible tax is less than \$0.45, Choice C must be true.

You can also quickly see that Choice C must be true if you note that 4% of \$10.00 would only be \$0.40, and since the price must be less than \$10.00, the tax must be less than \$0.40. The correct answer consists of **Choice C**.

16. If $\frac{1}{(2^{11})(5^{17})}$ is expressed as a terminating decimal, how many nonzero digits will the decimal have?

Ⓐ One
Ⓑ Two
Ⓒ Four
Ⓓ Six
Ⓔ Eleven

Explanation

To convert the fraction to a decimal, it is helpful to first write the fraction in powers of 10. Using the rules of exponents, you can write the following.

$$\begin{aligned}
\frac{1}{(2^{11})(5^{17})} &= \frac{1}{(2^{11})(5^{11+pl6})} \\
&= \frac{1}{(2^{11})(5^{11})(5^{6})} \\
&= \frac{1}{(10^{11})(5^{6})} \\
&= \left(\frac{1}{5}\right)^{6}(10^{-11}) \\
&= (0.2)^{6}(10^{-11}) \\
&= ((2)(10)^{-1})^{6}(10^{-11}) \\
&= (2^{6})(10^{-6})(10^{-11}) \\
&= (2^{6})(10^{-17}) \\
&= (64)(10^{-17})
\end{aligned}$$

So the decimal has two nonzero digits, 6 and 4. The correct answer is **Choice B**.

	Distance from Centerville (miles)
Freight train	$-10t + 115$
Passenger train	$-20t + 150$

12. The expressions in the table above give the distance from Centerville to each of two trains t hours after 12:00 noon. At what time after 12:00 noon will the trains be equidistant from Centerville?

(A) 1:30
(B) 3:30
(C) 5:10
(D) 8:50
(E) 11:30

Explanation

The distance between the freight train and Centerville at t hours past noon is $-10t + 115$. The distance between the passenger train and Centerville at t hours past noon is $-20t + 150$. To find out at what time the distances will be the same you need to equate the two expressions and solve for t as follows:

$$-10t + 115 = -20t + 150$$
$$10t + 115 = 150$$
$$10t = 35$$
$$t = 3.5$$

The trains will be the same distance from Centerville at 3.5 hours past noon, or at 3:30. The correct answer is **Choice B**.

13. The company at which Mark is employed has 80 employees, each of whom has a different salary. Mark's salary of $43,700 is the second-highest salary in the first quartile of the 80 salaries. If the company were to hire 8 new employees at salaries that are less than the lowest of the 80 salaries, what would Mark's salary be with respect to the quartiles of the 88 salaries at the company, assuming no other changes in the salaries?

(A) The fourth-highest salary in the first quartile
(B) The highest salary in the first quartile
(C) The second-lowest salary in the second quartile
(D) The third-lowest salary in the second quartile
(E) The fifth-lowest salary in the second quartile

Explanation

In this question you are told that Mark's salary is the second-highest in the first quartile. From this you can conclude that the word *quartile* refers to one of the four groups that are created by listing the data in increasing order and then dividing the data into four groups of equal size. When the salaries of the 80 employees are listed in order, the 20 lowest salaries (that is, the salaries in the first quartile) are the first 20 salaries in the list. Since Mark's salary is the

speed corresponding to a distance of 52 feet is a little less than 50 miles per hour. The correct answer is **Choice B**.

18. Approximately what is the total stopping distance, in feet, if the car is traveling at a speed of 40 miles per hour when the driver is signaled to stop?

Ⓐ 130
Ⓑ 110
Ⓒ 90
Ⓓ 70
Ⓔ 40

Explanation

Since the total stopping distance is the sum of the distance traveled during reaction time and the distance traveled after the brakes have been applied, you need information from both graphs to answer this question. At a speed of 40 miles per hour, the distance traveled during reaction time is a little less than 45 feet, and the distance traveled after the brakes have been applied is 88 feet. Since 45 + 88 = 133, the correct answer is **Choice A**.

19. Of the following, which is the greatest speed, in miles per hour, at which the car can travel and stop with a total stopping distance of less than 200 feet?

Ⓐ 50
Ⓑ 55
Ⓒ 60
Ⓓ 65
Ⓔ 70

Explanation

Since the total stopping distance is the sum of the distance traveled during reaction time and the distance traveled after the brakes have been applied, you need information from both graphs to answer this question. A good strategy for solving this problem is to calculate the total stopping distance for the speeds given in the options. For a speed of 50 miles per hour, the distance traveled during reaction time is about 55 feet, and the distance traveled after the brakes have been applied is 137 feet; therefore, the total stopping distance is about 55 + 137, or 192 feet. For a speed of 55 miles per hour, the distance traveled during reaction time is about 60 feet, and the distance traveled after the brakes have been applied is about 170 feet; therefore, the total stopping distance is about 60 + 170, or 230 feet. Since the speeds in the remaining choices are greater than 55 miles per hour and both types of stopping distances increase as the speed increases, it follows that the total stopping distances for all the remaining choices are greater than 200 feet. The correct answer is **Choice A**.

The explanation above uses a process of elimination to arrive at Choice E, which is sometimes the most efficient way to find the correct answer. However, one can also show directly that the correct answer is 58. For if a positive integer n is not a factor of 25!, then one of the following must be true:

(i) n is a prime number greater than 25, like 29 or 31, or a multiple of such a prime number, like 58 or 62;
(ii) n is so great a multiple of some prime number less than 25, that it must be greater than 58.

To see that (i) or (ii) is true, recall that every integer greater than 1 has a unique prime factorization, and consider the prime factorization of 25!. The prime factors of 25! are 2, 3, 5, 7, 11, 13, 17, 19, and 23, some of which occur more than once in the product 25!. For example, there are 8 positive multiples of 3 less than 25, namely 3, 6, 9, 12, 15, 18, 21, and 24. The prime number 3 occurs once in each of these multiples, except for 9 and 18, in which it occurs twice. Thus, the factor 3 occurs 10 times in the prime factorization of 25!. The same reasoning can be used to find the number of times that each of the prime factors occur, yielding the prime factorization $25! = (2^{22})(3^{10})(5^6)(7^3)(11^2)(13)(17)(19)(23)$. Any integer whose prime factorization is a combination of one or more of the factors in the prime factorization of 25!, perhaps with lesser exponents, is a factor of 25!. Equivalently, if the positive integer n is not a factor of 25!, then, restating (i) and (ii) above, the prime factorization of n must

(i) include a prime number greater than 23, or
(ii) have a greater exponent for one of the prime numbers in the prime factorization of 25!.

For (ii), the least possibilities are 2^{23}, 3^{11}, 5^7, 7^4, 11^3, 13^2, 17^2, 19^2, and 23^2. Clearly all of these are greater than 58. The least possibility for (i) that is not a prime number is 58, and the least possibility for (ii) is greater than 58, so **58** is the correct answer.

22. If $0 < a < 1 < b$, which of the following is true about the reciprocals of a and b ?

(A) $1 < \frac{1}{a} < \frac{1}{b}$

(B) $\frac{1}{a} < 1 < \frac{1}{b}$

(C) $\frac{1}{a} < \frac{1}{b} < 1$

(D) $\frac{1}{b} < 1 < \frac{1}{a}$

(E) $\frac{1}{b} < \frac{1}{a} < 1$

Explanation

To answer this question, you must first look at the answer choices. Note that all of the choices are possible orderings of the quantities $\frac{1}{a}$, $\frac{1}{b}$, and 1 from least to greatest. So to answer the question, you must put the three quantities in order

The Graduate Record Examinations®

SECTION 2

Analytical Writing

ANALYZE AN ARGUMENT

30 minutes

You will be given a short passage that presents an argument and specific instructions on how to respond to that passage. You will have 30 minutes to plan and compose a response in which you evaluate the passage according to the specific instructions. A response to any other argument will receive a score of zero.

Note that you are NOT being asked to present your own views on the subject. Make sure that you respond according to the specific instructions and support your evaluation with relevant reasons and/or examples.

Trained GRE readers will evaluate your response for its overall quality, based on how well you:

- Respond to the specific task instructions
- Identify and analyze features of the argument relevant to the assigned task
- Organize, develop, and express your ideas
- Support your analysis with relevant reasons and/or examples
- Control the elements of standard written English

Before you begin writing, you may want to think for a few minutes about the argument passage and the specific task instructions and then plan your response. Be sure to develop your response fully and organize it coherently, but leave time to reread what you have written and make any revisions you think are necessary.

SECTION 3
Verbal Reasoning
Time—35 minutes
25 Questions

For questions 1 to 8, select <u>one</u> entry for each blank from the corresponding column of choices. Fill all blanks in the way that best completes the text.

1. Although plant and animal species that become established in ecosystems where they did not originate are sometimes referred to by the alarming term "invasive species," many such species are _____ in their new environments.

(A) innocuous
(B) conspicuous
(C) robust
(D) menacing
(E) distinctive

2. Far from being _____ the corporate world because of cutbacks, serious researchers are playing a growing role in innovation at many firms.

(A) lured to
(B) enchanted with
(C) banished from
(D) protected by
(E) immured in

3. The brief survey, published under the title *The Work of Nature: How the Diversity of Life Sustains Us*, is surprisingly (i)_____. Indeed it makes several longer treatments of the effects of lost biodiversity seem (ii)_____.

Blank (i)	Blank (ii)
(A) distorted	(D) redundant
(B) objective	(E) pithy
(C) comprehensive	(F) premature

4. The government has no choice but to (i)_____ the incessant demands for land reform, and yet any governmental action that initiated land reform without requisite attention to agrarian reform would (ii)_____ the overall goal of economic modernization.

Blank (i)	Blank (ii)
(A) anticipate	(D) delineate
(B) heed	(E) condone
(C) silence	(F) compromise

GO ON TO NEXT PAGE

For each of questions 9 to 14, select one answer choice unless otherwise instructed.

Questions 9 and 10 are based on the following reading passage.

Fossil bones of the huge herbivorous dinosaurs known as sauropods were first discovered and studied between 1840 and 1880, providing evidence for the gargantuan dimensions of the adults. The shape of sauropod teeth suggested what they ate. But aside from trackways, or series of fossilized footprints—which established that sauropods at least occasionally lived in herds — fossils incorporating direct evidence of other behavior, such as reproductive behavior, have been almost nonexistent. Because no modern land animals even approach sauropod size, scientists have also lacked a living analogue to use as a guide to possible sauropod behavior. Until the recent discovery of fossilized sauropod nesting grounds, scientists were thus uncertain whether sauropods laid eggs or gave birth to live young.

For the following question, consider each of the choices separately and select all that apply.

9. Which of the following can be inferred from the passage regarding the evidence provided by sauropod teeth?

   A The teeth allow inferences to be made about sauropod social behavior.
   B The shape of the teeth indicates that sauropods were herbivorous.
   C The teeth have no resemblance to those of any modern land animal.

For the following question, consider each of the choices separately and select all that apply.

10. Which of the following can be inferred from the passage regarding the recently discovered fossilized sauropod nesting grounds?

   A They are among the few fossils incorporating direct evidence of sauropod behavior.
   B They confirm the evidence provided by trackways about sauropod behavior.
   C They have forced a reevaluation of theories regarding the nature of sauropod herd behavior.

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For each of questions 20 to 25, select one answer choice unless otherwise instructed.

Question 20 is based on the following reading passage.

Astronomers found a large body orbiting close to the star Upsilon Andromedae. The standard theory of planet formation holds that no planet that large could be formed so close to a star, leading to the suggestion that the body is a companion star. A subsequent discovery puts that suggestion in doubt: two other large bodies were found orbiting close to Upsilon Andromedae, and the standard theory of companion stars allows for at most one companion star.

20. Which of the following, if true, most helps to resolve the status of the orbiting body without casting doubt on the two standard theories mentioned?

(A) The smaller a planet orbiting a star is, and the farther away it is from the star, the less likely it is to be discovered.

(B) If a planet's orbit is disturbed, the planet can be drawn by gravity toward the star it is orbiting.

(C) The largest of the bodies orbiting Upsilon Andromedae is the farthest away from the star, and the smallest is the nearest.

(D) It is likely that there are many stars, in addition to Upsilon Andromedae and the Sun, that are orbited by more than one smaller body.

(E) In most cases of companion stars, the smaller companion is much fainter than the larger star.

Question 21 is based on the following reading passage.

In Gilavia, the number of reported workplace injuries has declined 16 percent in the last five years. However, perhaps part of the decline results from injuries going unreported: many employers have introduced safety-incentive programs, such as prize drawings for which only employees who have a perfect work-safety record are eligible. Since a workplace injury would disqualify an employee from such programs, some employees might be concealing injury, when it is feasible to do so.

21. Which of the following, if true in Gilavia, most strongly supports the proposed explanation?

(A) In the last five years, there has been no decline in the number of workplace injuries leading to immediate admission to a hospital emergency room.

(B) Employers generally have to pay financial compensation to employees who suffer work-related injuries.

(C) Many injuries that happen on the job are injuries that would be impossible to conceal and yet would not be severe enough to require any change to either the employee's work schedule or the employee's job responsibilities.

(D) A continuing shift in employment patterns has led to a decline in the percentage of the workforce that is employed in the dangerous occupations in which workplace injuries are likely.

(E) Employers who have instituted safety-incentive programs do not in general have a lower proportion of reported workplace injuries among their employees than do employers without such programs.

GO ON TO NEXT PAGE

5. Unlike the problems in recent financial scandals, issues raised by the regulators in this case appear largely to pertain to unwieldy accounting rules that are open to widely divergent interpretations—not to (i)_____ transactions designed to (ii)_____ corporate malfeasance.

Blank (i)	Blank (ii)
(A) sham	(D) cloak
(B) unpremeditated	(E) ameliorate
(C) justifiable	(F) illuminate

6. Everyone has routines that govern their work. The myth is that artists are somehow different, that they reject (i)_____, but of course that's not true: most artists work as the rest of us do, (ii) _____, day by day, according to their own customs.

Blank (i)	Blank (ii)
(A) latitude	(D) impetuously
(B) habit	(E) ploddingly
(C) materialism	(F) sporadically

7. Repression of painful memories is sometimes called "willed forgetting." Yet true forgetting is (i)_____ than the phenomenon of repressed memory. In spite of the effort that it (ii)_____, repressing unwanted memories is less (iii) _____ than truly forgetting them, for repressed memories are prone to come back.

Blank (i)	Blank (ii)	Blank (iii)
(A) less controlled	(D) aids	(G) permanent
(B) different in its effect	(E) conveys	(H) arduous
(C) far more common	(F) entails	(I) immediate

8. Rather than viewing the Massachusetts Bay Colony's antinomian controversy as the inevitable (i)_____ of the intransigent opposing forces of radical and (ii)_____ beliefs, male and female piety, (iii)_____ and secular power, and the like, as other critics have, Winship argues that the crisis was not "fixed and structural."

Blank (i)	Blank (ii)	Blank (iii)
(A) dissolution	(D) revolutionary	(G) clerical
(B) melding	(E) orthodox	(H) civil
(C) collision	(F) questionable	(I) cerebral

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Ⓐ	**Quantity A is greater.**
Ⓑ	**Quantity B is greater.**
Ⓒ	**The two quantities are equal.**
Ⓓ	**The relationship cannot be determined from the information given.**

	Quantity A	Quantity B				
1.	$\frac{3^{-1}}{4^{-1}}$	$\frac{4}{3}$	Ⓐ	Ⓑ	Ⓒ	Ⓓ

$x < 1$ and $x \neq 0$

	Quantity A	Quantity B				
2.	$x^2 + 1$	$x^3 + 1$	Ⓐ	Ⓑ	Ⓒ	Ⓓ

$x > 0$

	Quantity A	Quantity B				
3.	0.5% of x	$\frac{1}{2}x$	Ⓐ	Ⓑ	Ⓒ	Ⓓ

The median income of a group of College *C* graduates six months after graduation was $3,000 higher than the median income of a group of College *D* graduates six months after graduation.

	Quantity A	Quantity B				
4.	The 75th percentile of the incomes of the group of College *C* graduates six months after graduation	The 75th percentile of the incomes of the group of College *D* graduates six months after graduation	Ⓐ	Ⓑ	Ⓒ	Ⓓ

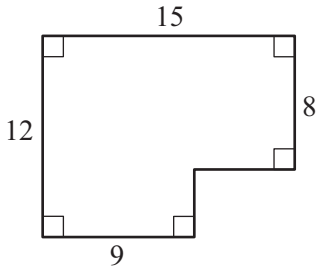
GO ON TO THE NEXT PAGE

$$b-3,\ b-1,\ b+2,\ b+3,\ b+4$$

11. The median of the five terms listed above is 5, where b is a constant. What is the average (arithmetic mean) of the five terms?

(A) 3
(B) 4
(C) 5
(D) 6
(E) 7

For the following question, enter your answer in the box.



12. What is the area of the region shown above?

13. During a one-year study, biologists observed the number of fish in a certain pond as well as the percent of the fish that were catfish. At the beginning of the year, there were 300 fish in the pond, of which 15 percent were catfish; and at the end of the year, there were 400 fish in the pond, of which 10 percent were catfish. From the beginning of the year to the end of the year, the number of catfish in the pond

(A) decreased by more than 5%
(B) decreased by 5%
(C) did not change
(D) increased by 5%
(E) increased by more than 5%

For the following question, enter your answer in the box.

14. On a radio tower, a red light flashes every 6 seconds and a blue light flashes every 10 seconds. If both lights flash together at a certain time, how many seconds later will both lights flash together the next time?

_____ seconds

Evaluating Your Performance

Now that you have completed Practice Test 2, it is time to evaluate your performance.

Analytical Writing Measure

One way to evaluate your performance on the Issue and Argument topics you answered on this practice test is to compare your essay responses with the scored sample essay responses for these topics and review the rater commentary. Scored sample essay responses and rater commentary are presented starting on page 481 for the one Issue topic and one Argument topic presented in the Analytical Writing sections of Practice Test 2. The Issue and Argument scoring guides start on page 37.

To better understand the analytical writing abilities characteristic of particular score levels, you should review the score level descriptions on page 41.

Verbal Reasoning and Quantitative Reasoning Measures

The tables that follow contain information to help you evaluate your performance on the Verbal Reasoning and Quantitative Reasoning measures of Practice Test 2. An answer key with the correct answers to the questions in the Verbal Reasoning and Quantitative Reasoning sections in this practice test begins on page 475. Compare your answers with the correct answers given in the table, crossing out questions you answered incorrectly or omitted. Partially correct answers should be treated as incorrect. Knowing which questions you answered incorrectly or omitted can help you identify content areas in which you need more practice or review.

The answer key contains additional information to help you evaluate your performance. With each answer, the key provides a number, the P+. The P+ is the percent of a group of actual GRE takers who were administered that same question at a previous test administration and who answered it correctly. P+ is used to gauge the relative difficulty of a test question. The higher the P+, the easier the test question. You can use the P+ to compare your performance on each test question to the performance of other test takers on that same question. For example, if the P+ for a question is 89, that means that 89 percent of GRE test takers who received this question answered it correctly. Alternatively, if the P+ for a question is 14, that means that 14 percent of GRE test takers who received this question answered it correctly. A question with a P+ of 89 may be interpreted as a relatively easy question, and a question with a P+ of 14 may be interpreted as a difficult question.

To calculate your scores on Practice Test 2:

- Add the number of correct answers in Sections 3 and 4 to obtain your raw Verbal Reasoning score.
- Add the number of correct answers in Sections 5 and 6 to obtain your raw Quantitative Reasoning score.
- Once you have calculated your raw scores, refer to the Practice Test 2 score conversion table on pages 479–480. Find the scores on the 130–170 score scales that correspond to your Verbal Reasoning and Quantitative Reasoning raw scores. Note the scaled scores provided.

Answer Key

Section 6. Quantitative Reasoning

Question Number	P+	Correct Answer
1	73	**Choice A:** Quantity A is greater.
2	69	**Choice D:** The relationship cannot be determined from the information given.
3	64	**Choice B:** Quantity B is greater.
4	41	**Choice C:** The two quantities are equal.
5	36	**Choice D:** The relationship cannot be determined from the information given.
6	42	**Choice D:** The relationship cannot be determined from the information given.
7	72	**Choice C:** The two quantities are equal.
8	28	**Choice C:** The two quantities are equal.
9	36	**Choice B:** Quantity B is greater.
10	69	**Choice D:** 1
11	69	**Choice B:** 4
12	78	**156**
13	66	**Choice A:** decreased by more than 10%
14	72	**30**
15	58	**Choice D:** $a^2 - b^2$ AND **Choice C:** ab
16	46	**Choice C:** 6
17	9[illegible]	**Choice D:** 28
18	82	**Choice C:** 24
19	56	**Choice C:** 150%
20	66	$\frac{\mathbf{44}}{\mathbf{68}}$ (or any equivalent fraction)
21	58	**Choice B:** 2,100
22	45	**Choice A:** $\frac{x}{80}$
23	37	**Choice E:** $d(1 - 0.01t)$ dollars
24	53	**Choice B:** 03 AND **Choice D:** 6
25	32	**Choice B:** $\frac{1}{35}$

Raw Score	Verbal Reasoning Scaled Score	Quantitative Reasoning Scaled Score
16	144	143
15	143	143
14	142	142
13	141	141
12	140	140
11	139	139
10	138	138
9	137	137
8	135	136
7	134	135
6	132	134
5	130	132
4	130	130
3	130	130
2	130	130
1	130	130
0	130	130

Analytical Writing Sample Responses and Reader Commentaries

SECTION 1
Analytical Writing

ANALYZE AN ISSUE

Some people believe that corporations have a responsibility to promote the well-being of the societies and environments in which they operate. Others believe that the only responsibility of corporations, provided they operate within the law, is to make as much money as possible.

Write a response in which you discuss which view more closely aligns with your own position and explain your reasoning for the position you take. In developing and supporting your position, you should address both of the views presented.

Score 6 Response*

It is not uncommon for some to argue that in the world in which we live, corporations have a responsibility to society and to the environment in which they operate. Proponents of this view would argue that major environmental catastrophes (e.g., the oil spill in the Gulf) are key examples of the damage that can be wrought when corporations are allowed to operate unchecked. Yet within that very statement lies a contradiction that undermines this kind of thinking—it is necessary for outside forces to check the behavior of corporations, because we do not expect corporations to behave in such a manner. In fact, the expectation is simply that corporations will follow the law, and in the course of doing so, engage in every possible tactic to their advantage in the pursuit of more and greater profit. To expect otherwise from corporations is to fail to understand their purpose and their very structure.

The corporation arose as a model of business in which capital could be raised through the contributions of stockholders; investors purchases shares in a company, and their money is then used as the operating capital for the company. Shareholders buy stock not because they are hoping to better make the world a better place or because they have a desire to improve the quality of life but because they expect to see a return in their investment in this company. The company may itself have generally altruistic goals (perhaps it is a think tank that advises the government on how to improve relations with the Middle East, or perhaps it is a company built around finding alternative forms of energy), but the immediate expectation of the investor is that he himself will see dividends, or profits, from the investment he has made. This is even more true in the case of companies that are purely profit driven and which do not have

*NOTE: All responses are reproduced exactly as written, including errors, misspellings, etc., if any.

goals that are particularly directed toward social improvement—a description that applies to the vast majority of corporations.

Is it a bad thing to have a corporation negatively affect the environment (and by extentsion, its inhabitants)? To pump noxious fumes into the atmosphere as a by-product of its manufacturing processes? Of course, and this is why agencies such as the EPA were established and why governments—federal, state, and local—are expected to monitor such companies to ensure that such practices fall within the boundaries of legal expectations. Any and all corporations should be expected to temper their pursuit of profit with the necessity of following those safeguards that have been legislated as protections. But the assumption that corporations have an inherent obligation or responsibility to go above and beyond that to actively PROMOTE the environment and the well-being of society is absurd.

Engaging in practices to adhere to legal expectations to protect society and the environment is costly to corporations. If the very purpose of a corporation is to generate profits, and the obligation to adhere to safety expectations established by law cuts into those profits, then to expect corporations to embrace such practices beyond what is required is to presume that they willingly engage in an inherently self-destructive process: the unnecessary lowering of profits. This is antithetical to the very concept of the corporation. Treehuggers everywhere should be pleased that environmental protections exist, but to expect corporations to "make the world a better place" is to embrace altruism to the point that it becomes delusion.

This is not to say that we should reject efforts to hold corporations accountable. In fact, the opposite is true—we should be vigilant with the business world and maintain our expectations that corporations do not make their profits at the EXPENSE of the well-being of society. But that role must be fulfilled by a watchdog, not the corporation itself, and those expectations must be imposed UPON the corporations, not expected FROM them.

Reader Commentary

This response receives a 6 for developing an insightful position on the issue in accordance with the assigned task, skillfully weaving a position that takes into consideration both of the statements in the prompt. Beginning in the first paragraph, the writer rejects the idea that corporations themselves "have a responsibility to promote the well-being of the societies and environments in which they operate." In the second paragraph, the writer offers compelling reasons for this rejection by discussing the purpose and structure of corporations. The writer then considers the role of government in promoting corporations' social and environmental responsibility, developing the position fully. A cogent statement of the writer's position appears at the conclusion of the response: "we should be vigilant with the business world and maintain our expectations that corporations do not make their profits at the EXPENSE of the well-being of society. But that role must be fulfilled by a watchdog, not the corporation itself." The response as a whole is logically organized, with each paragraph serving as a stepping stone in the development of the writer's position. It also demonstrates the writer's ability to convey ideas fluently and precisely, using effective vocabulary and sentence variety. This sentence demonstrates the level of language facility seen throughout the response: "If the very purpose of a corporation is to generate profits, and the obligation to adhere to safety expectations established by law cuts into those profits, then to expect corporations to embrace such practices beyond what is required is to presume that they willingly engage in an inherently self-destructive process: the unnecessary

from around the tungsten wire, you'd basically have an exposed electrical wire that could hurt anyone who touched it. Makers of light bulbs know and understand all these dangers. They want consumers to purchase their products, so the first and smartest way to make that happen is to ensure that the products are safe and thus more attractive to the customer base. If everyone who used light bulbs was afraid of getting zapped profits would obviously go down and light bulbs would not be a very profitable enterprise.

This same thinking applies to all major products. The automobile is one of the most dangerous tools man uses. Tens of thousands of automobile drivers die every year in accidents. Insuring that the vehicles contain designs and parts that promote customer safety is a main focus of car manufacturers. Certain parts of of cars were built with promoting driver's well-being in mind. For instance, air bags, anti-lock braking systems, online crash reporting. These features are considered standard now, and they were all developed to increase the safety of consumers. These features were not cheap to develop, but car manufacturers improved their profits anyway because they developed products with public safety in mind, which is what customers expect. If this symbiosis relationship wasn't true, then we would still have cars without airbags or even seatbelts. Worrying about the safety and actually improving it for customers is not just a basic responsibility of corporations, but it drives their profits, too.

In conclusion, its pretty clear that a corporation's desire to make more profits is in line with a corporation's responsibility to consumers. Increasing the focus on consumers, worrying about taking care of them and the environment, can only lead to bigger profits and success for corporations in the long run.

Reader Commentary

This adequate response follows the task directions and presents a clear position on the issue. It supports and develops its position competently, using relevant examples. In accordance with the assigned task, the response addresses both of the competing positions. Specifically, its position and the examples it develops argue that businesses can care about both profits and ethical responsibility through the ways they develop products. The development of examples and ideas, while adequate, is not as thoughtful or compelling as would be needed to earn higher scores. For instance, both of the examples the response uses are about product safety; the discussion of automobile design does not advance the position much more than the prior discussion of lightbulb production. Language control in the response is also competent. It demonstrates sufficient control of the conventions of standard written English, and its main points are made with acceptable clarity. The response features a few grammatical and mechanical errors (e.g., "Certain parts of of cars…" and "symbiosis relationship") and some awkward sentences. However, for the 4 range, GRE raters allow for minor errors in responses like this one that holistically demonstrate sufficient clarity and control. Overall, then, this response demonstrates adequate development and control of language, making the score of 4 appropriate.

Finally, to what degree did each of the two changes made, closing early and getting rid of older movies, affect profits? To determine if such a change would be helpful, it is important to understand how each variable contributed to the end result, assuming that it was effective. Perhaps closing early resulted in such a decline in the operating costs as employees did not have to be paid, that the reduction in their stock was unnecessary. It could be the the stock reduction actually decreased profits but this was masked by the increased profits caused by closing early. A more in depth analysis of the variables involved is necessary.

To accept such an extreme change in the practices of these stores, the preceeding recommendations should be followed. Specifically, the necessity of the reduction in other stores shoudl be determined, data regarding the effectiveness of the reduction in operating expenses in the Marston store should be analyzed, and an analysis of the components of this reduction should be completed.

Reader Commentary

This adequate response presents a competent examination of the argument and conveys meaning with acceptable clarity. In accordance with the task directions, the response raises appropriate questions that could help to evaluate the recommendation and its predicted result. Unlike the thoughtful development of a 5-level response, however, this response develops its ideas (i.e., answers to the questions it raises) unevenly, sometimes underdeveloping key claims. For example, the relatively brief second paragraph supports the assertion that the downtown Marston store may not be comparable to the chain's other stores, but it does so with minimal reasoning. Other body paragraphs more satisfactorily develop questions about the timing of the recommendation and the profits that have actually been generated, while the conclusion merely recapitulates the assertion made earlier. A basic organizational structure, aided by the use of simple transitions between paragraphs and sufficient sentence variety within paragraphs, are other qualities of this response that underscore its adequacy. The language control is also adequate, demonstrating control, but not facility, with the conventions of standard written English. There are some minor grammatical errors and typos (e.g., there is a tense error in paragraph 3: "what effect did the reduction in operating expenses had on store profits in Marston?"; there is also vague diction in the same paragraph: "Because the declines in profit are termed to be 'recent' and that the reduction of operating expenses happened within the last month, this is unclear"), but the response manages to convey ideas with acceptable clarity overall. Because of its adequate analytical development and language control, this response earns a score of 4.

Score 3 Response

It is imperative that "Movies Galore" must find a way to reduce operating expenses without jepardizing its popularity with the customer. The option of reducing operating hours and reducing its stock of avalible movies is a good start, however these two ideas need some revising in order for them to be successful in turning the company's profits around.

The reduction of hours needs to be reversed. Instead of closing earlier they should open later. People go to the video store to rent movies more frequently in the evening hours than in the morning. In the morning is when most customers return movies. The adjustment in hours can be structured so that the store opens later in the morning, and

For each of questions 9 to 14, select one answer choice unless otherwise instructed.

Questions 9 and 10 are based on the following reading passage.

Fossil bones of the huge herbivorous dinosaurs known as sauropods were first discovered and studied between 1840 and 1880, providing evidence for the gargantuan dimensions of the adults. The shape of sauropod teeth suggested what they ate. But aside from trackways, or series of fossilized footprints—which established that sauropods at least occasionally lived in herds—fossils incorporating direct evidence of other behavior, such as reproductive behavior, have been almost nonexistent. Because no modern land animals even approach sauropod size, scientists have also lacked a living analogue to use as a guide to possible sauropod behavior. Until the recent discovery of fossilized sauropod nesting grounds, scientists were thus uncertain whether sauropods laid eggs or gave birth to live young.

line 5 10

Description

The passage outlines what was learned about sauropods after the discovery of their fossilized bones in the nineteenth century, including what has been inferred about their behavior from the fossil record.

For the following question, consider each of the choices separately and select all that apply.

9. Which of the following can be inferred from the passage regarding the evidence provided by sauropod teeth?

A The teeth allow inferences to be made about sauropod social behavior.
B The shape of the teeth indicates that sauropods were herbivorous.
C The teeth have no resemblance to those of any modern land animal.

Explanation

Choice B is the only correct answer.

Choice A is incorrect: the passage mentions that fossilized footprints permit the inference that sauropods exhibited herd behavior, but there is no indication that this or any other social behavior can be inferred from sauropod teeth.

Choice B is correct: the passage states that sauropods were herbivorous (feeding on plants) and that "the shape of sauropod teeth suggested what they ate."

Choice C is incorrect: the passage says that there are no modern land animals similar in size to sauropods, not that there are no such animals with similar teeth.

Questions 11 to 14 are based on the following reading passage.

Some researchers contend that sleep plays no role in the consolidation of declarative memory (i.e., memory involving factual information). These researchers note that people with impairments in rapid eye movement (REM) sleep continue to lead normal lives, and they argue that if sleep were crucial for memory, then these individuals would have apparent memory deficits. Yet the same researchers acknowledge that the cognitive capacities of these individuals have never been systematically examined, nor have they been the subject of studies of tasks on which performance reportedly depends on sleep. Even if such studies were done, they could only clarify our understanding of the role of REM sleep, not sleep in general.

These researchers also claim that improvements of memory overnight can be explained by the mere passage of time, rather than attributed to sleep. But recent studies of memory performance after sleep—including one demonstrating that sleep stabilizes declarative memories from future interference caused by mental activity during wakefulness—make this claim unsustainable. Certainly there are memory consolidation processes that occur across periods of wakefulness, some of which neither depend on nor are enhanced by sleep. But when sleep is compared with wakefulness, and performance is better after sleep, then some benefit of sleep for memory must be acknowledged.

Description

The passage presents and then rebuts two arguments made by researchers who question the contribution of sleep to the consolidation of declarative memory (memory involving factual information). The first argument is that people with impairments to REM sleep continue to lead normal lives. In response, the passage says that these researchers themselves acknowledge the absence of systematic study of such individuals' cognitive abilities, study that would be necessary in order to fully support the researchers' claim. The passage also points out that the researchers' claim applies only to REM sleep rather than to sleep in general. The second claim is that improvements of memory that occur overnight might be explained merely by the passage of time. In response, the passage cites research findings that demonstrate the role of sleep in stabilizing declarative memory.

11. The primary purpose of the passage is to
    - (A) present the evidence that supports a particular claim regarding REM sleep and memory
    - (B) describe how various factors contribute to the effect of sleep on memory
    - (C) argue against a particular position regarding sleep's role in memory
    - (D) summarize the most prevalent theory regarding sleep and memory
    - (E) defend the importance of the consolidation of declarative memory

Explanation

As described above, the purpose of the passage as a whole is to argue against the view held by some researchers that sleep plays no role in the consolidation of declarative memory. Therefore, **Choice C** is correct. Choice A is incorrect: the passage does mention REM sleep twice in the first paragraph, but its primary purpose is not to examine REM sleep in particular, and it does not present evidence related to REM sleep. Choice B is incorrect: the passage is concerned with the effect of sleep on memory, but not with any factors that contribute to that effect. Choice D is incorrect: the passage does

SECTION 4
Verbal Reasoning
25 Questions with Explanations

For questions 1 to 8, select <u>one</u> entry for each blank from the corresponding column of choices. Fill all blanks in the way that best completes the text.

1. The unexplained digressions into the finer points of quantum electrodynamics are so _____ that even readers with a physics degree would be wise to keep a textbook handy to make sense of them.

(A) uninteresting
(B) controversial
(C) unsophisticated
(D) frustrating
(E) humorless

Explanation

An initial reading of this sentence might suggest that the blank should be filled with a word like "complex" that indicates how hard it is to "make sense of" the digressions. However, there is no such word among the answer choices. Focusing on the second half of the sentence suggests a different interpretation. According to the sentence, it would be "wise to" make sense of the digressions, and a textbook would help the reader to do so. If the digressions are "uninteresting," "unsophisticated," or "humorless," the sentence provides no reason to think it would be wise to make sense of them, and if they are "controversial," it provides no reason to think that a textbook would help. Only if the digressions are "frustrating" does the sentence make a coherent whole.

Thus, the correct answer is **frustrating** (Choice D).

2. The belief that politicians might become _____ after their election to office led to the appointment of ethics officers at various levels of government.

(A) scrupulous
(B) entrenched
(C) venal
(D) puzzled
(E) artificial

Explanation

If a certain belief led to the appointment of ethics officers, that belief must concern some ethical issue. Of the choices provided, only "venal" fits that context. Although several of the other choices are not necessarily positive characteristics, none of them involves ethics.

Thus, the correct answer is **venal** (Choice C).

20. In the context in which it appears, "robust" (line 8) most nearly means

(A) crude
(B) demanding
(C) productive
(D) vigorous
(E) rich

Explanation

In discussing the advantages of less vulnerable crops, the author describes corn as "robust." Of the choices presented, "vigorous" is most similar in meaning to "robust." Neither "crude" nor "demanding" is an advantage, and although being "productive" or "rich" might be desirable, neither matches the meaning of "robust" in this context. Therefore, **Choice D** is the correct answer.

Question 21 is based on the following reading passage.

In 1998 the United States Department of Transportation received nearly 10,000 consumer complaints about airlines; in 1999 it received over 20,000. Moreover, the number of complaints per 100,000 passengers also more than doubled. In both years the vast majority of complaints concerned flight delays, cancellations, mishandled baggage, and customer service. Clearly, therefore, despite the United States airline industry's serious efforts to improve performance in these areas, passenger dissatisfaction with airline service increased significantly in 1999.

21. Which of the following, if true, most seriously weakens the argument?

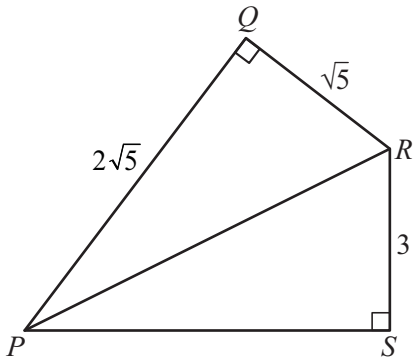
(A) Although the percentage of flights that arrived on time dropped slightly overall, from 77 percent in 1998 to 76 percent in 1999, some United States airlines' 1999 on-time rate was actually better than their 1998 on-time rate.
(B) The number of passengers flying on United States airlines was significantly higher in 1999 than in 1998.
(C) Fewer bags per 1,000 passengers flying on United States airlines were lost or delayed in 1999 than in 1998.
(D) The appearance in 1999 of many new Internet sites that relay complaints directly to the Department of Transportation has made filing a complaint about airlines much easier for consumers than ever before.
(E) Although the number of consumer complaints increased for every major United States airline in 1999, for some airlines the extent of the increase was substantial, whereas for others it was extremely small.

Explanation

The passage describes two different year-over-year increases in airline passenger complaints: both the absolute number of complaints and the rate of complaints more than doubled from 1998 to 1999. From these facts, the author of the passage concludes that passenger dissatisfaction with airline service significantly increased in the same period.

Choice D is the correct answer: it weakens the argument because it presents a scenario in which the increase in complaints and in the rate of complaints could merely be the result of an easier means of filing complaints, not an actual increase in passenger dissatisfaction.

Thus the average of the 50 values, $\frac{217}{50}$, or 4.34, is less than the median of the 50 values, 5. The correct answer is **Choice B**.



Quantity A	Quantity B	A	B	C	D

6. The area of triangle *PQR* | The area of triangle *PSR*

Explanation

In this question, you are asked to compare the area of triangle *PQR* with the area of triangle *PSR*. Note that both triangles are right triangles and that line segment *PR* is the hypotenuse of both triangles. Recall that the area of a triangle is equal to one-half the product of a base and the height corresponding to the base. Also, for any right triangle, the lengths of the two legs of the triangle are a base and the corresponding height.

The area of triangle PQR: In the figure, it is given that the length of leg *PQ* is $2\sqrt{5}$ and the length of leg *QR* is $\sqrt{5}$. Therefore, you can conclude that the area of triangle *PQR* is $\frac{1}{2}(2\sqrt{5})(\sqrt{5})$, or 5.

The area of triangle PSR: To calculate the area of triangle *PSR*, you need to know the lengths of the legs *PS* and *RS*. From the figure, you know that the length of *RS* is 3, but you do not know the length of *PS*. How can you determine the length of *PS* ? If, in addition to the length of *RS*, you knew the length of hypotenuse *PR*, you could use the Pythagorean theorem to determine the length of *PS*. So, to find the length of *PS*, you first need to find the length of hypotenuse *PR*.

Recall that *PR* is also the hypotenuse of triangle *PQR*. The lengths of legs *PQ* and *QR* of triangle *PQR* are $2\sqrt{5}$ and $\sqrt{5}$, respectively. By the Pythagorean theorem,

$$\begin{aligned}(PR)^2 &= (PQ)^2 + (QR)^2\\ &= \left(2\sqrt{5}\right)^2 + \left(\sqrt{5}\right)^2\\ &= 20 + 5\\ &= 25\end{aligned}$$

Thus, the length of *PR* is $\sqrt{25}$, or 5.

Returning to triangle PSR, you now know that the length of hypotenuse PR is 5 and the length of leg RS is 3. Therefore, by the Pythagorean theorem,

$$\begin{aligned} 3^2 + (PS)^2 &= 5^2 \\ 9 + (PS)^2 &= 25 \\ (PS)^2 &= 25 - 9 \\ (PS)^2 &= 16 \end{aligned}$$

and the length of PS is 4.

Since legs PS and RS have lengths 4 and 3, respectively, the area of triangle PSR is $\frac{1}{2}(4)(3)$, or 6. Recall that you have already determined that the area of triangle PQR is 5. So Quantity B, the area of triangle PSR, is greater than Quantity A, the area of triangle PQR, and the correct answer is **Choice B**.

	Quantity A	Quantity B	
7.	The sum of the odd integers from 1 to 199	The sum of the even integers from 2 to 198	Ⓐ Ⓑ Ⓒ Ⓓ

Explanation

In this question, you are asked to compare the sum of the odd integers from 1 to 199 with the sum of the even integers from 2 to 198. Both of these sums involve many integers. How many integers are in each sum? Note that there are 200 integers from 1 to 200, where 100 of them are even and 100 of them are odd. The 100 odd integers are precisely the odd integers in Quantity A, whereas the 100 even integers include one more integer, 200, than the even integers in Quantity B. So Quantity A is the sum of 100 integers and Quantity B is the sum of 99 integers.

It would be very time-consuming to write out all the terms in each sum and add them together. Therefore, it is reasonable to find a more efficient way to calculate the sums or to find a way to compare the sums without actually calculating them. To find a more efficient way to calculate the two sums, it is often useful to look for ways to rearrange the terms in the sum so that they can be added more easily. You can begin by writing a few terms from the beginning and the end of the sum.

For the sum of the 100 odd integers from 1 to 199, you could write

$$1 + 3 + 5 + \ldots + 195 + 197 + 199$$

You can pair the odd integers in the sum and add the two integers in each pair as follows.

$$1 + 3 + 5 + \ldots + 195 + 197 + 199$$

$$200 \quad 200 \quad 200$$

Note that the sum of the integers in each of the three pairs shown is 200. You can continue pairing terms in the sum in this way until all 100 terms have been rearranged in 50 pairs, where the sum of each pair is 200. It follows that

$$\begin{aligned} 1 + 3 + 5 + \ldots + 195 + 197 + 199 &= (1 + 199) + (3 + 197) + (5 + 195) + \ldots + (99 + 101) \\ &= 50(200) \\ &= 10{,}000 \end{aligned}$$

12. The numbers in data set S have a standard deviation of 5. If a new data set is formed by adding 3 to each number in S, what is the standard deviation of the numbers in the new data set?

Ⓐ 2
Ⓑ 3
Ⓒ 5
Ⓓ 8
Ⓔ 15

Explanation

Recall that the standard deviation of the numbers in a data set is a measure of the spread of the numbers about the mean of the numbers. The new data set is formed by adding the same number, 3, to each number in data set S. Thus, the mean of the numbers in the new data set is 3 more than the mean of the numbers in S, but the spread of the numbers in the new data set about the mean of the numbers in the new data set is the same as the spread of the numbers in S about the mean of the numbers in S. Because the standard deviation of the numbers in S is 5, the standard deviation of the numbers in the new data set is also 5. The correct answer is **Choice C**.

13. If $\frac{2y-3}{y} = \frac{3-y}{2}$, which of the following could be the value of y ?

Ⓐ 4
Ⓑ 1
Ⓒ −1
Ⓓ −3
Ⓔ −5

Explanation

One approach to answer the question is to solve the equation for y as follows.

$$\begin{aligned} 2(2y-3) &= y(3-y) \\ 4y-6 &= 3y-y^2 \\ y^2+y-6 &= 0 \\ (y+3)(y-2) &= 0 \end{aligned}$$

Since a product equals 0 only if at least one of the factors equals 0,

$$\begin{aligned} y+3=0 \text{ or } y-2&=0 \\ y=-3 \text{ or } y&=2 \end{aligned}$$

Thus, there are two values of y that satisfy the equation, -3 and 2. The value -3 is Choice D, and the value 2 is not among the answer choices. The correct answer is **Choice D**.

17. In the circle graphs, the degree measure of the central angle of the sector representing the number of workers unemployed for 11 to 14 weeks is how much greater in the manufacturing industry graph than in the service industry graph?

(A) 5°
(B) 10°
(C) 15°
(D) 18°
(E) 20°

Explanation

Recall that in a circle graph, the degree measure of the central angle of a sector representing n percent of the data is equal to n percent of 360°.

The degree measure of the sector representing the number of workers unemployed for 11 to 14 weeks is 10% of 360°, or 36°, for the manufacturing industry graph and is 5% of 360°, or 18°, for the service industry graph. Since $36° - 18° = 18°$, the measure of the central angle of that sector in the manufacturing industry graph is 18° greater than the measure of the central angle of the corresponding sector in the service industry graph. The correct answer is **Choice D**.

18. Which of the following could be the median length of unemployment, in weeks, for manufacturing industry workers who were unemployed for at least 1 week?

(A) 4
(B) 8
(C) 12
(D) 16
(E) 20

Explanation

Note that the sectors in the manufacturing industry circle graph separate the unemployed manufacturing industry workers into five groups by length of unemployment; also, the percent of workers within each of the five groups is given. Also note that there are 10 million lengths of unemployment, one length for each of the 10 million workers. Since the lengths are rounded to whole numbers of weeks, most of the 10 million lengths must be repetitions.

The median length of unemployment is the average of the two middle lengths when the lengths are listed in order from least to greatest; that is, the median is the number at which 50% of the lengths have been listed.

To find the median length, first note that the group with the shortest unemployment lengths, 1 to 4 weeks, accounts for the first 40% of the lengths in the ordered list. Then, because the group with the next longer lengths, 5 to 10 weeks, accounts for the next 20% of the lengths in the list, the number at which 50% of the lengths have been listed is in this group. So the median length is in the 5-to-10 week interval. Among the answer choices, the only choice that is in the 5-to-10 week interval is Choice B, 8. The correct answer is **Choice B**.

22. The operation $\otimes$ is defined for all integers x and y as $x \otimes y = xy - y$. If x and y are positive integers, which of the following CANNOT be zero?

(A) $x \otimes y$
(B) $y \otimes x$
(C) $(x - 1) \otimes y$
(D) $(x + 1) \otimes y$
(E) $x \otimes (y - 1)$

Explanation

In the formula $x \otimes y = xy - y$, the variables x and y are placeholders that can be replaced by integers or by expressions representing integers. Here are two examples.

If x is replaced by 3 and y is replaced by 4, then the formula gives

$$3 \otimes 4 = (3)(4) - 4 = 12 - 4 = 8$$

If x is replaced by $x - 1$ and y is replaced by 2, then the formula gives

$$(x - 1) \otimes 2 = ((x - 1)(2)) - 2 = 2x - 2 - 2 = 2x - 4$$

Scanning the answer choices, you can see that all of them are of the form

"first expression" $\otimes$ "second expression"

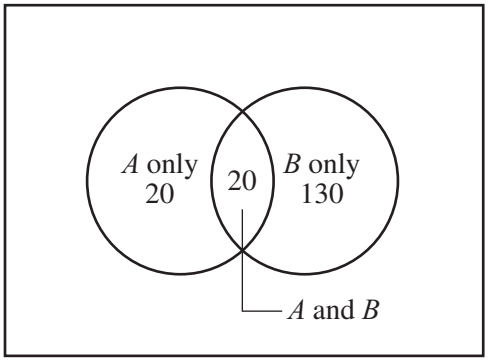
For each answer choice, you must determine whether the answer choice can be equal to 0 for some positive integers x and y. Are there positive integers x and y for which the answer choice is equal to 0? If not, then that answer choice is the correct answer.

Choice A: $x \otimes y$. Using the formula, try to find positive integers x and y for which $x \otimes y = 0$, that is, for which $xy - y = 0$. To solve this equation, note that factoring y out of the left-hand side of the equation $xy - y = 0$ gives the equation $(x - 1)y = 0$. So now you must find positive integers x and y such that the product of the two numbers $x - 1$ and y is 0. Since the product of two numbers is 0 only if at least one of the numbers is 0, it follows that the product of $x - 1$ and y will be 0 if $x = 1$, no matter what the value of y is. For example, if $x = 1$ and $y = 2$, then $x \otimes y = 1 \otimes 2 = (1)(2) - 2 = 0$, and both x and y are positive integers. Therefore, Choice A is not correct, since there are positive integers x and y for which $x \otimes y = 0$.

Choice B: $y \otimes x$. This is similar to Choice A, except the x and y are interchanged. Therefore, you might try the example in Choice A but with the values of x and y interchanged: $y = 1$ and $x = 2$. Using the formula, $y \otimes x = yx - x = (1)(2) - 2 = 0$. Therefore, Choice B is not correct, since there are positive integers x and y for which $y \otimes x = 0$.

Choice C: $(x - 1) \otimes y$. Using the formula, try to find positive integers x and y for which $(x - 1) \otimes y = 0$, that is, for which $(x - 1)y - y = 0$. Factoring y out of the left-hand side of the equation $(x - 1)y - y = 0$ yields $(x - 1 - 1)y = (x - 2)y = 0$. Here the product of the two numbers $x - 2$ and y is 0. So the product will be 0 if $x = 2$, no matter what the value of y is. For example, if $x = 2$ and $y = 10$, then $(x - 1) \otimes y = (2 - 1) \otimes 10 = 1 \otimes 10 = (1)(10) - 10 = 0$, and both x and y are positive

integers in B. It is a good idea, therefore, to redraw the Venn diagram so that the numbers are separated into three categories: the integers in A only, the integers in B only, and the integers in both A and B. The revised Venn diagram follows.



Observe that summing the numbers of integers in set A only, set B only, and both A and B yields the total number of integers that are in set A or set B, or both. Therefore, Quantity A is 20 + 130 + 20, or 170, and the correct answer is **Choice C**.

Another approach is to realize that if you listed the integers in set A and the integers in set B, you would have listed the integers that are in both A and B twice and all of the other integers once. So the total number of integers in set A or set B, or both, is equal to

(number in set A) + (number in set B) − (number in both sets)

Thus, the number of integers in set A or set B, or both, is 40 + 150 − 20, or 170, and the correct answer is **Choice C**.

x is a negative integer.

	Quantity A	Quantity B	
5.	2^x	3^{x+1}	(A) (B) (C) (D)

Explanation

In this question, you are asked to compare 2^x with 3^{x+1}, given that x is a negative integer. One way to approach this problem is to plug a value of x in both expressions and compare the results.

You are given that x is a negative integer, so the greatest integer you can plug in for x is -1.

For $x = -1$, it follows that $2^x = 2^{-1} = \frac{1}{2}$ and $3^{x+1} = 3^{-1+1} = 3^0 = 1$.

In this case, 2^x is less than 3^{x+1}. However, to conclude that Quantity B is greater, it is not sufficient for 2^x to be less than 3^{x+1} for one particular value of x; the relationship would need to be true for all negative integer values of x. To analyze this relationship further, plug in another value of x, for example, -2.

For $x = -2$, it follows that $2^x = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$ and $3^{x+1} = 3^{-2+1} = 3^{-1} = \frac{1}{3}$.

Again, 2^x is less than 3^{x+1}, but note that these values are closer together than the previous values of 2^x and 3^{x+1}. It appears that the relationship between the quantities may differ for smaller values of x, so now try plugging in -3 for x.

For $x = -3$, it follows that $2^x = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ and $3^{x+1} = 3^{-3+1} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$.