DEFINING ADDITION + MULTIPLICATION

- Let $f(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$,

  $g(x) = b_0 + b_1x + b_2x^2 + \dots + b_mx^m$.

  → $f(x) + g(x) = (a_0 + b_0) + (a_1 + b_1)x + \dots + (a_n + b_n)x^n + \dots$

  → $f(x) \cdot g(x) = (a_0b_0) + (a_1b_0 + a_0b_1)x + \dots + \sum_{i=0}^{n} a_i b_{n-i} \dots$

- **THEOREM**: The set $R[x]$ of all polynomials with coefficients in a ring R is a ring under polynomial addition and multiplication.

  ↳ SOME FEATURES:

  - If R is commutative, so is $R[x]$.
  - If R has unity $1 \neq 0$, then 1 is also unity for $R[x]$.

- **EXAMPLE**: Consider $\mathbb{Z}_2[x]$.

  → $(x+1)^2 = (x+1)(x+1) = x^2 + (1+1)x + 1 = x^2 + 1$

  → $(x+1) + (x+1) = (1+1)x + (1+1) = 0x + 0 = 0$

→ TWO INDETERMINATES

- $(R[x])[y]$ is a ring of polynomials in y with coefficients that are polynomials in x.
- $(R[x])[y] \simeq (R[y])[x]$
- **NOTATION**: $(R[x])[y] = R[x, y]$

→ RANDOM NOTES

- If D is an integral domain, so is $D[x]$.
- If D is a field, $D[x]$ is an integral domain.