Ref: Date:

Sol 1) $\nabla \times \nabla \phi = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ \frac{d\phi}{dx} & \frac{d\phi}{dy} & \frac{d\phi}{dz} \end{vmatrix}$

= mixed partial derivatives

= 0

$\therefore \nabla \times \nabla \phi = 0 \quad \therefore$ irrational.

Sol 2) $\vec{v} = (xyz)i + (3x^2y)j + (xz^2 - y^2z)k$

div $v = \nabla \cdot v$

$= yz + 3x^2 + (2xz - y^2)$

Sub $(2, -1, 1)$ 9

b) find grad of $(x^3 + y^3 + z^3 - 3xyz)$ the div x ie $\frac{d}{dx}$ $\frac{d}{dy}$ $\frac{d}{dz}$ at (2

2b) find div & curl grad of $\vec{F} = (x^3 + y^3 + z^3 - 3xyz)$

$\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$

$\vec{F} = i(3x^2 - 3yz) + j(3y^2 - 3xz) + k(3z^2 - 3xy)$

$\nabla \cdot F = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$

$= 6x + 6y + 6z$

$\nabla \cdot \vec{F}$ at $(2, -1, 1)$

$\therefore \nabla \cdot f = 12 - 6 + 6$

$= 12$

Curl $\vec{F} = \nabla \times F$

$= \nabla \times \nabla \phi$

$= 0 \quad \therefore \vec{F}$ is irrational.

Check: For $\nabla \cdot \vec{F} = \begin{vmatrix} \overset{+}{i} & \overset{-}{j} & \overset{+}{k} \\ d/dx & d/dy & d/dz \\ 3x^2-3yz & 3y^2-3xz & 3z^2-3xy \end{vmatrix}$

$= i\left[\frac{d}{dy}(3z^2-3xy) - \frac{d}{dz}(3y^2-3xz)\right] - j\left[\frac{d}{dx}(3z^2-3xy) - \frac{d}{dz}(3x^2-3yz)\right]$

$+ k\left[\frac{d}{dx}(3y^2-3xy) - \frac{d}{dy}(3x^2-3yz)\right]$

$= i[-3x+3x] - j(-3y+3y) + k(-3z+3z)$

$= 0j + 0j + 0k \leftarrow$

$\nabla \times F = 0 \quad \therefore$ F is irrota~~tional~~tional vector.

3a) $(-3x^2z + z^3)i + (3x^2-z)j + (3xz^2-y)k$

F is Solenoidal $\therefore \nabla \cdot F = 0$

$\nabla \cdot F = -6xz + 6xz$

$= 0$ $\therefore$ F is Solenoidal.

26/03/15

Integration of Vector functions

$\frac{d}{dt}[\vec{F}(t)] = \vec{f}(t)$

int on both sides:

$\int \frac{d}{dt}[F(t)]\, dt = \int f(t)\, dt$

$\cancel{\int \frac{d}{dt}} (F(t)) + c = \int f(t)\, dt$

If t between t_1 & t_2 then

$\int_{t_1}^{t_2} f(t)\, dt = (F(t) + c)_{t_1}^{t_2}$

$= \vec{F}(t_2) + c - \vec{F}(t_1) - c$

$$\boxed{\int_{t_1}^{t_2} f(t)\,dt = \vec{F}(t_2) - \vec{F}(t_1)}$$

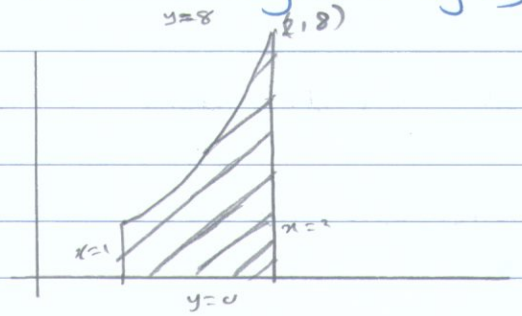
Ref: Date:

prob (3) $F = 5xy\,i + 2y\,j$, evaluate $F \cdot dr$, C is the part of the curve $y = x^3$ btwn $x=1$ to $x=2$
$y=1$ $y=8$

Sol: $F = 5xy\,i + 2y\,j$

$dr = dx\,i + dy\,j$

$F \cdot dr = 5xy\,dx + 2y\,dy$



$y = x^3$

$dy = 3x^2\,dx$

$F \cdot dr = 5xy\,dx + 2y\,dy$

$= 5x \cdot x^3\,dx + 2x^3 \cdot 3x^2\,dx$

$= 5x^4\,dx + 6x^5\,dx$

$\int_c F \cdot dr = \int_1^2 5x^4\,dx + \int_1^2 6x^5\,dx$

$= 94$ ←

prob (4) If $F = (3x^2+6y)\,i - 14yz\,j + 20xz^2\,K$, evaluate $\int_c F \cdot dr$ from $(0,0,0)$ to $(1,1,1)$ along curve $x = t$, $y = t^2$, $z = t^3$

Sol: $F \cdot dr = (3x^2+6y)dx - 14yz\,dy + 20xz^2\,dz$

$x = t$, $y = t^2$, $z = t^3$

$dx = dt$ $dy = 2t\,dt$ $dz = 3t^2\,dt$

$F \cdot dr = (3x^2+6y)dx - 14yz\,dy + 20xz^2\,dz$

$= (3t^2 + 6t^2)dt - (14t^2 \cdot t^3 \cdot 3t^2)dt + 20 \cdot t \cdot t^6 \cdot 3t^2\,dt$

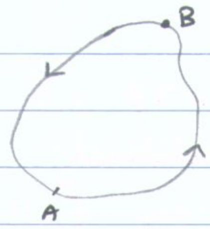
$= (9t^2\,dt - 28t^6 + 60t^9)\,dt$

$\int_c F \cdot dr = \int_0^1 (9t^2 dt - 28t^6 + 60t^9 dt)$

$= 5$ ←

- Work done by force.

$\vec{F}(x, y, z)$



- Total work done by force is $\int_A^B \vec{F} \cdot dr$

Conservative vector field:

$\int_C \vec{F} \cdot dr$ is independent of path iff $\vec{F} = \nabla \phi$ (if and only if)

$$\therefore \int_C \vec{F} \cdot dr = \int_C \nabla \phi \cdot d\vec{r}$$

$$= \int_C \left(i \frac{\partial \phi}{\partial x} + j \frac{\partial \phi}{\partial y} + k \frac{\partial \phi}{\partial z} \right) \cdot \left(dx\, i + dy\, j + dz\, k \right)$$

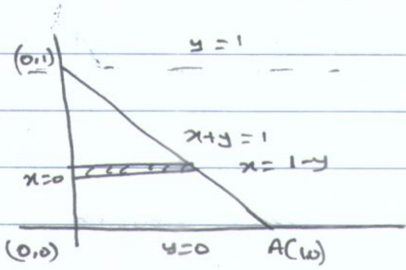
$$= \int_C \left(\frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy + \frac{\partial \phi}{\partial z} dz \right)$$

$$\Rightarrow \int_C d\phi = \left[\phi \right]_A^B$$

$$= \phi(B) - \phi(A) \leftarrow$$

Note: the total differential $\boxed{d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy + \frac{\partial \phi}{\partial z} dz}$

RHS: of ① : $\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dy\,dx = -6y - (-16y)\, dx\,dy$



$= \iint_R (-6y + 16y)\, dy\,dx$

$= \int_{y=0}^{y=1} \int_{x=0}^{x=1-y} 10y \, dx\, dy$

$= \int_0^1 10y \,(x)_0^{1-y} \, dy$

$= \int_0^1 10y\,(1-y)\, dy$

$= \int_0^1 10\left(y - y^2\right) dy$

$= 10\left(\frac{y^2}{2} - \frac{y^3}{3}\right)_0^1$

$= 10\left(\frac{1}{2} - \frac{1}{3}\right)$

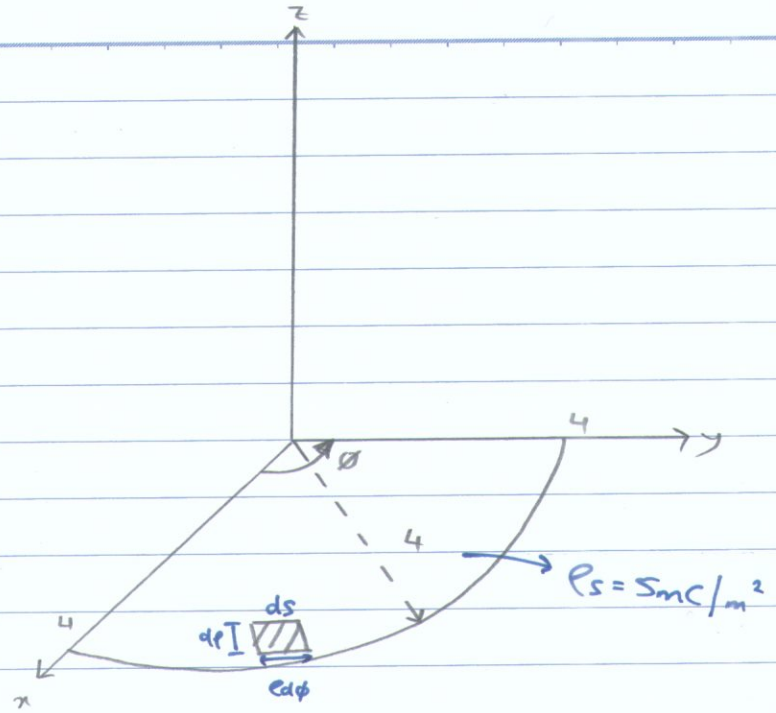
$= \frac{5}{3}$ ←

Ex2) Verify that Green theorem for : $\int_C (xy + y^2)\, dx + x^2 dy$ where C is area between $y = x^2$ and $y = x$.

$$\int_C (P\,dx + Q\,dy) = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy \longrightarrow ①$$

Ref: Date:

$Q_B =$



$$Q_B = \iint_S \rho_s \, ds$$

$ds = \rho \, d\rho \, d\phi$

$$Q_B = \int_{\rho=0}^{4} \int_{\phi=0}^{\frac{\pi}{2}} \rho \, d\rho \, d\phi$$

$$= 5 \int_0^4 \rho \, d\rho \int_0^{\frac{\pi}{2}} d\phi$$

$$= 5 \cdot \left(\frac{\rho^2}{2} \right)_0^4 \cdot \left(\phi \right)_0^{\frac{\pi}{2}}$$

$$= 5 \cdot \left(\frac{16}{2} \right) \cdot \frac{\pi}{2}$$

$$= 62.83 \text{ mC}$$

Ref: Date:

$E_T = E_{Q_B} + E_{Q_C} + E_{Q_D}$

$$= \frac{Q_A Q_B}{4\pi \varepsilon_0 |R_1|^3} + \frac{Q_A Q_C}{4\pi \varepsilon_0 |R_2|^3} + \frac{Q_A Q_D}{4\pi \varepsilon_0 |R_3|^3}$$

$R_1 = (2a_x)$

$R_2 = (1a_x - a_y)$

$R_3 = (a_x + a_y)$

$\therefore |R_1| = 2$

$|R_2| = \sqrt{2}$

$|R_3| = \sqrt{2}$

$$E = \frac{Q_A Q_B}{4\pi \varepsilon_0 (2)^3}(2a_x) + \frac{Q_A Q_C}{4\pi \varepsilon_0 (2)^{\frac{3}{2}}}(a_x - a_y) + \frac{Q_A Q_D}{4\pi \varepsilon_0 (2)^{\frac{3}{2}}}(a_x + a_y)$$

$= (56.25 \cdot 2a_x) + 159.1\, a_x - 159.1\, a_y + 159.1\, a_x + 159.1\, a_y$

$= 112.5\, a_x + 159.1\, a_x + 159.1\, a_x - 159.1\, a_y + 159.1\, a_y$

$= 430.7\, a_x + 0\, a_y$

$= \cancel{(430.7\, a_x)\, mN}\ (2.813 \times 10^{-6}\, a_x)\ N$

$$= \frac{50n \cdot 50n}{4\pi \varepsilon_0\ 2^3}(2a_x) + \frac{50n\, 50n}{4\pi \varepsilon_0 (2)^{\frac{3}{2}}}(a_x) + \frac{50n \cdot 50n}{4\pi \varepsilon_0\, 2(\frac{3}{2})}(a_x)$$

$= 5.625 \times 10^{-6}\, a_x + 2(7.9545 \times 10^{-6})$ $\therefore F =$

$= \cancel{1.3579 \times 10^{5}}$

$= 2.15 \times 10^{-6}$ ← Ans good!

$\boxed{Ans = 21.5 \times 10^{-6}\, a_x.}$ ~~Re-do~~

Ref: Date:

$$= 25 \int \frac{\sec^2 x}{(25\sec^2 x)^{\frac{3}{2}}}\, dx$$

$$= 25 \int \frac{\sec^2 x}{5^3 \sec^2 x}\, dx$$

$$= \frac{1}{5} \int \frac{1}{\sec x}\, dx$$

$$= \frac{1}{5} \int_{x=?}^{?} \cos x \, dx$$

when $y = -5$

$$x = \tan^{-1}\left(\frac{-5}{5}\right)$$

$$= \frac{-\pi}{4}$$

when $y = 0$, $x = 0$

$$\therefore \frac{1}{5} \int_{-\frac{\pi}{4}}^{0} \cos x \, dx$$

$$= \frac{1}{5} \left(\sin x \right)_{-\frac{\pi}{4}}^{0}$$

$$= 0.141 \leftarrow$$

Second integration => -0.059.