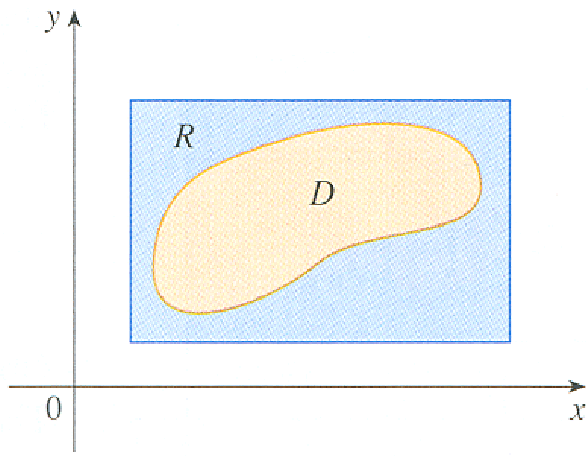

Figure 4.8: Bounded region in $\mathbb{R}^2$, enclosed in a rectangle.

Definition 330 *If* $\displaystyle\iint_R F(x,y)\,dA$ *exists, then we define*

$$\iint_D f(x,y)\,dA = \iint_R F(x,y)\,dA$$

This definition makes sense since, as we can see in figures 4.9 and 4.10, the portion of the graph of F which is not 0 is identical to the graph of f. The portion which is 0 will not contribute to the integral. In particular, this means that for our definition, it does not matter which rectangle R we select. In the case, $f(x,y) \geq 0$, $\displaystyle\iint_D f(x,y)\,dA$ corresponds to the volume of the solid which lies above D and below the graph of $z = f(x,y)$.

We still must be able to compute $\displaystyle\iint_R F(x,y)\,dA$. This is not always a simple task. But it is for certain regions, which we consider next.

4.3.2 Regions of Type I

When describing a region, one has to give the condition x and y must satisfy so that a point (x,y) lies in the region. A region is said to be of type I if x is between two constants, and y is between two continuous functions of x. More precisely, we have the following definition:

Definition 331 *A plane region D is said to be of **type I** if it is of the form*

$$D = \left\{(x,y) \in \mathbb{R}^2 \mid a \leq x \leq b \text{ and } g_1(x) \leq y \leq g_2(x)\right\}$$

Figure 4.15: Region between $y = x$ and $y = x^2$

Method 1 *Treating Ω as a type I region.*

$$\begin{aligned}
\iint\limits_{\Omega} \left(\sqrt{x} - y^2\right) dA &= \int_0^1 \int_{x^2}^{x^{\frac{1}{4}}} \left(\sqrt{x} - y^2\right) dydx \\
&= \int_0^1 \left[y\sqrt{x} - \frac{y^3}{3} \Big|_{x^2}^{x^{\frac{1}{4}}} \right] dx \\
&= \int_0^1 \left(\left(x^{\frac{3}{4}} - \frac{x^{\frac{3}{4}}}{3} \right) - \left(x^{\frac{5}{2}} - \frac{x^6}{3} \right) \right) dx \\
&= \int_0^1 \left(\frac{2}{3} x^{\frac{3}{4}} - x^{\frac{5}{2}} + \frac{x^6}{3} \right) dx \\
&= \left(\frac{8}{21} x^{\frac{7}{4}} - \frac{2}{7} x^{\frac{7}{2}} + \frac{x^7}{21} \right) \Big|_0^1 \\
&= \frac{8}{21} - \frac{2}{7} + \frac{1}{21} \\
&= \frac{1}{7}
\end{aligned}$$

Method 2 *Treating Ω as a type II region.*

$$\begin{aligned}
\iint\limits_{\Omega} \left(\sqrt{x} - y^2\right) dA &= \int_0^1 \int_{y^4}^{y^{\frac{1}{2}}} \left(\sqrt{x} - y^2\right) dxdy \\
&= \int_0^1 \left[\left(\frac{2}{3} x^{\frac{3}{2}} - xy^2 \right) \Big|_{y^4}^{y^{\frac{1}{2}}} \right] dy \\
&= \int_0^1 \left(\left(\frac{2}{3} y^{\frac{3}{4}} - y^{\frac{5}{2}} \right) - \left(\frac{2}{3} y^6 - y^6 \right) \right) dy \\
&= \int_0^1 \left(\frac{2}{3} y^{\frac{3}{4}} - y^{\frac{5}{2}} + \frac{y^6}{3} \right) dy \\
&= \left(\left(\frac{8}{21} y^{\frac{7}{4}} - \frac{2}{7} y^{\frac{7}{2}} - \frac{y^7}{21} \right) \right) \Big|_0^1 \\
&= \left(\frac{8}{21} - \frac{2}{7} + \frac{1}{21} \right) \\
&= \frac{1}{7}
\end{aligned}$$

Example 349 *Evaluate* $\iint\limits_{\Omega} \cos \frac{\pi x^2}{2} dA$ *where D is the region bounded by $x = 1$, $y = 0$ and $y = x$.*