The Absolute Value Function, and its Properties

One of the most used functions in mathematics is the absolute value function. Its definition and some of its properties are given below.

Absolute Value Function The absolute value of a real number x, |x|, is

$$|x| = \begin{cases} x & \text{if } x \ge 0\\ -x & \text{if } x < 0 \end{cases}$$

The graph of the absolute value function is shown below



The following properties of the absolute value function need to be memorized.

Lemma 1. For any two real numbers x and y, we have

$$|xy| = |x| |y|.$$

This equality can be verified by considering cases. One of the four possible cases is checked as follows: Suppose x < 0 and $y \ge 0$. Then xy is ≤ 0 and we have

$$|xy| = -(xy) = (-x)y = |x||y|$$
.

The other three cases are similarly checked.