Q. 1. (i) The PDE $\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$ is known as :

(i) wave equation **(ii) heat equation**

(iii) Laplace equation **(iv) none of these**

Ans. (i) Wave equation

Q. 1. (j) The PDE $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, is :

(i) parabolic **(ii) elliptic**

(iii) hyperbolic **(iv) circular**

Ans. (ii) Elliptic

Section B

Note : *Attempt* ***any three*** *parts from this section. Each part carry equal marks :* $(3 \times 10 = 30)$

Q. 2. (a) A particle of mass m moves in a straight line under the action of force mn^2x which is always directed towards a fixed point O on the line. Determine the displacement $x(t)$ if the resistance to the motion is $2\lambda nv$ given that initially $x = 0$, $\frac{dx}{dt} = x_0$ $(0 < \lambda < 1)$.

Ans. The differential equation describing this simple harmonic motion is

$$m\frac{d^2x}{dt^2} = -2\lambda\, mnv - mn^2x, \quad \Rightarrow \quad m\frac{d^2x}{dt^2} = -2\lambda\, mn\frac{dx}{dt} - mn^2x$$

$$\frac{d^2x}{dt^2} + 2\lambda n\frac{dx}{dt} + n^2x = 0 \qquad \ldots(1)$$

The auxiliary equation is $M^2 + 2\lambda nM + n^2 = 0$

$$M = \frac{-2\lambda n \pm \sqrt{4\lambda^2n^2 - 4n^2}}{2} = -\lambda n \pm in\sqrt{1-\lambda^2}$$

The general solution is $x = e^{-\lambda nt}[c_1 \cos n\omega t + c_2 \sin n\omega t]$ ...(2)

where $\omega = \sqrt{1-\lambda^2}$,

using I.C. $x = 0, t = 0 \Rightarrow c_1 = 0$

Differentiating (2), we get

$$\frac{dx}{dt} = e^{-\lambda nt}[-c_1 n\,\omega \sin n\omega t + c_2 n\omega \cos n\omega t] - \lambda ne^{-\lambda nt}[c_1 \cos n\omega t + c_2 \sin n\omega t]$$

Again using I.C. $\frac{dx}{dt} = x_0, t = 0$

$$x_0 = -c_1 \cdot 0 + c_2 n\omega - \lambda n[c_1 + c_2 \cdot 0] \qquad c_2 = \frac{x_0}{n\omega}$$

From (2), its solution is $x = e^{-\lambda nt}\left(\frac{x_0}{n\omega}\right)\sin n\omega t$

Q. 2. (b) Using Frobenius method, obtain a series solution in powers of x for differential equation : $2x(1-x)\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + 3y = 0$

Ans. $2x(1-x)\dfrac{d^2y}{dx^2}+(1-x)\dfrac{dy}{dx}+3y=0$ ...(1)

$\because x=0$ is a regular singular point

Let $$y=x^m\sum_{n=0}^{\infty}a_n x^n=\sum_{m=0}^{\infty}a_n x^{m+n}$$ ...(2)

$$\frac{dy}{dx}=\sum_{n=0}^{\infty}a_n(m+n)x^{m+n-1}$$

$$\frac{d^2y}{dx^2}=\sum_{n=0}^{\infty}a_n(m+n)(m+n-1)x^{m+n-2}$$

Substituting these values in equation (1), we get

$$2x[1-x]\sum_{n=0}^{\infty}a_n(m+n)(m+n-1)x^{m+n-2}+(1-x)\sum_{n=0}^{\infty}a_n(m+n)x^{m+n-1}$$
$$+3\sum_{n=0}^{\infty}a_n x^{m+n}=0$$

$$\sum_{n=0}^{\infty}a_n[2(m+n)(m+n-1)+(m+n)]x^{m+n-1}$$
$$+\sum_{n=0}^{\infty}a_n[-2(m+n)(m+n-1)-(m+n)+3]x^{m+n}=0$$

$$\sum_{m=0}^{\infty}a_n(m+n)(2m+2n-1)x^{m+n-1}-\sum_{n=0}^{\infty}a_n(m+n+1)(2m+2n-3)x^{m+n}=0$$ ...(3)

Coefficient of $x^{m-1}=0$

$$a_0(m)(2m-1)=0\Rightarrow a_0\neq 0$$

$$m(2m-1)=0 \quad\Rightarrow\quad \boxed{m=0,\frac{1}{2}}$$

It is the root of indicial equation.

Coefficient of $x^m=0$

$$a_1(m+1)(2m+1)-a_0(m+1)(2m-3)=0 \Rightarrow \boxed{a_1=\left(\frac{2m-3}{2m+1}\right)a_0}$$

Coefficient of $x^{m+n}=0$

$$a_{n+1}(m+n+1)(2m+2n+1)-a_n(m+n+1)(2m+2n-3)=0$$

$$\boxed{a_{n+1}=\left(\frac{2m+2n-3}{2m+2n+1}\right)a_n}$$ ...(4)

$n=1$, $$a_2=\left(\frac{2m-1}{2m+3}\right)a_1=\frac{(2m-1)(2m-3)}{(2m+1)(2m+3)}a_0$$

$n=2$, $$a_3=\left(\frac{2m+1}{2m+5}\right)a_2=\frac{(2m-1)(2m-3)}{(2m+3)(2m+5)}a_0$$

Substituting these value of a_1, a_2, a_3 ..... in equation (2), we get

$$y=a_0x^m\left[1+\left(\frac{2m-3}{2m+1}\right)x+\frac{(2m-1)(2m-3)}{[illegible]}x^2+\frac{(2m-1)(2m-3)}{[illegible]}x^3+\ldots\right]$$ ...(5)