2 GRAVITATION

POINTS TO REMEMBER

- **Newton's Law of Gravitation:** The magnitude of the gravitational force of attraction between two particles of masses m_1 and m_2 separated by a distance r is given by

$$F = \frac{G m_1 m_2}{r^2}$$

where G is called the universal gravitational constant. G is a scalar quantity.

$G = 6.67 \times 10^{-11}$ Nm^2/kg^2 and its dimensional formula is $[G] = [M^{-1} L^3 T^{-2}]$. The value of G was first determined by Cavendish by using a torsional balance.

Gravitational forces are conservative forces. As they act along the line joining the centres of the two interacting bodies, they are central forces. They form action reaction pairs. Gravitational force is the weakest force among the four fundamental forces. Newton's law of gravitation is applicable for all bodies lying at very large as well as very small distances but fails for interatomic distances as well as for particles in the nucleus. The value of G does not depend upon the nature and size of the bodies as well as the medium between the two bodies.

It is useful to explain (i) motion of the earth and planets around the sun (ii) formation of tides due to attraction between the moon and the water in the ocean (iii) motion of artificial satellites.

In vector form it is written as $\vec{F}_{12} = -\frac{Gm_1m_2}{r^2}\hat{r}_{21}$ the

–ve sign indicates that the direction of $\vec{F}_{12}$ is opposite to the direction of $\hat{r}_{21}$. Similarly we can write the relation

for $\vec{F}_{21} = -\frac{Gm_1m_2}{r^2}\hat{r}_{12}$.

- The magnitude of the acceleration due to gravity (g) at the surface of the earth is given by $g = \frac{GM}{R^2}$ where M is the mass of the earth of radius R.

$$F = \frac{GMm}{R^2} \text{ and } F = mg.$$

$$\therefore mg = \frac{GMm}{R^2} \text{ and } g = \frac{GM}{R^2}$$

The gravitational force F = mg $\quad \therefore g = \frac{F}{m}$

The S.I. unit of g is m/s^2 or N/kg. g is a vector and its dimensional formula is $[g] = [M^0 L^1 T^{-2}]$.

G is a universal constant but g is neither universal nor constant as it changes from place to place and is different for different planets.

$$g = \frac{GM}{R^2} = \frac{G}{R^2}\left(\frac{4}{3}\pi R^3 \rho\right] = \frac{4}{3}\pi\rho GR$$

$$\therefore \rho = \frac{3g}{4\pi GR} \text{ and } M = \frac{gR^2}{G}$$

From these relations, we can find the density and mass of the earth and of any planet.

- A body has two masses, Inertial mass (m_i) and gravitational mass m_G.

(i) $m_i = \frac{F}{a}$. This is obtained from Newton's law of motion. Gravity has no effect on m_i. A physical balance measures m_i.

(ii) m_G is defined by using Newton's law of gravitation.

$$\because F = \frac{GMm_G}{R^2} \quad \therefore m_G = \frac{F}{\left(\frac{GM}{R^2}\right)} = \frac{F}{g}$$