It is measured by a spring balance. Inertia has no effect on m_G.

- The magnitude of the acceleration due to gravity (g^1) at a point at a distance r (r > R) from the centre of the earth is given by

$$g^1 = \frac{GM}{r^2} = \frac{GM}{(R+h)^2}$$

where h = altitude or the height of the point from the surface of the earth.

$$\frac{g^1}{g} = \left(\frac{R}{r}\right)^2 = \left(\frac{R}{R+h}\right)^2$$

- **Variation of g :**

(a) With altitude :

$$\because g = \frac{GM}{R^2} \text{ and } g_h = \frac{GM}{(R+h)^2} \text{ (at a height h)}$$

$$\therefore g_h = g\left[\frac{R^2}{(R+h)^2}\right] = g\left[1+\frac{h}{R}\right]^{-2},$$

where R is the radius of the earth. This is true for any height, which is comparable to the radius of the earth.

but if $h << R$ (h is very small as compared to R)

then $g_h = g\left(1 - \frac{2h}{R}\right)$.

The value of g decreases, as we go above the surface of the earth.

(b) With depth : $g_d = g\left(1 - \frac{d}{R}\right)$

where d is the depth of the body below the surface of the earth.

At the centre of the earth, $g_d = 0$. $\because d = R$.

The weight of the body will be zero at the centre of the earth. But its mass will not be zero.

Thus the value of g decreases with increase in height as well as depth.

(c) With latitude : (effect of rotation of the earth)

The latitude (λ) at a place is the angle made by the line joining the place and the centre of the earth with the equatorial plane. Latitude is an angle while altitude is a distance.

$g' = g - \omega^2 R \cos^2\lambda$, where λ is the latitude.

At the equator $\lambda = 0$ $\therefore g_{equator} = g - R\omega^2$. This is the minimum value of g.

At the poles $\lambda = 90°$ $\therefore g_{pole} = g$

$\therefore g_P - g_E = R\omega^2 = 0.034 \text{ m/s}^2$

This is the maximum value of g. At the poles, the value of g remains unaffected whether the earth remains at rest or rotating. If $\omega = 0$, $g' = g$. Thus g increases at all places except at the poles. If ω is increased, g is decreased (except at poles).

(d) Variation due to the shape of the earth : The equatorial radius (R_E) > the polar radius (R_P)

$$\because g \propto \frac{1}{R} \quad \therefore g_P > g_E$$

$\therefore$ The weight of a body increases if it is taken from the equator to the pole.

- **Projection of a satellite :** A satellite is an object which revolves around a planet in an orbit. The moon is a natural satellite of the earth. For projection of an artificial satellite, minimum two stage rocket is used. By using the first stage rocket the satellite is taken to the desired height and then by remote control, the empty first stage rocket is detached. By igniting the fuel in the second stage rocket, the satellite is projected in the horizontal direction.

The minimum horizontal velocity with which a satellite should be projected in space, so that it describes a U.C.M. around the earth, at that height is called its critical velocity or orbital velocity (v_C).

The motion of the satellite in space depends upon the horizontal velocity (v) imparted to it.

(i) If $v < v_C$, it enters the atmosphere and strikes the earth.

(ii) If $v = v_C$, it moves in the desired circular orbit.

(iii) If $v > v_C$ but less than the escape velocity (v_e), it moves in an elliptical orbit and

(iv) If $v \geq v_e$ it will escape from the earth's gravitational influence and will disappear in outer space.

- The critical or orbital velocity (V_C) of a satellite, [moving in a circular orbit of radius r = R + h], is given by

$$V_C = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{gR^2}{R+h}} = \sqrt{g^1(R+h)}$$

If the satellite is orbiting very close to the surface of the earth, i.e. if $h << R$, then $V_C = \sqrt{\frac{GM}{R}} = \sqrt{gR}$

In this case $V_C \doteqdot 8$ km/s

The radius of the orbit $(R+h) = \frac{GM}{V_C^2}$

V_C decreases with increase in height. V_C is independent of the mass of the orbiting satellite.

- The period (T) of a satellite revolving round the earth in a circular orbit of radius r = R + h is given by

$$T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$$

$$\text{or } T = \frac{2\pi(R+h)}{V_C} = \frac{\text{Circumference of the orbit}}{\text{Critical velocity}}$$

In terms of g^1, $T = 2\pi\sqrt{\frac{R+h}{g^1}}$