T is independent of the mass of the satellite. It depends upon the mass of the central body and the radius of the orbit (mass of the earth and the radius of the circular orbit).

- If the satellite is orbiting very close to the surface of the earth, then

$$T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R}{g}} \quad [\because GM = gR^2 \text{ and } h = 0]$$

Using $R = 64 \times 10^5$ m and $g = 9.8$ m/s^2,

we get T = 84.6 minute.

- A satellite, performing a U.C.M., around the earth, possesses both P.E. and K.E. Its

(a) P.E. $= -\dfrac{GMm}{R + h}$

(b) K.E. $= \dfrac{1}{2}mV_C^2 = \dfrac{GMm}{2(R + h)}$

(c) Total Energy (E) $= -\dfrac{GMm}{2(R + h)}$

Thus, $E = -K = \dfrac{U}{2}$

The –ve sign for E, indicates that the satellite is bound to the earth due to gravitational force of attraction.

(d) Binding Energy $= \dfrac{GMm}{2(R + h)} = -E$

For a body at rest on the surface of the earth,

$$B.E. = \frac{GMm}{R}$$

- The escape velocity of a body projected from the surface of the earth,

$$V_E = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} = R\sqrt{\frac{8\pi G\rho}{3}}$$

$$V_E = \sqrt{\frac{2GM}{R}} = \sqrt{2}\sqrt{\left(\frac{GM}{R}\right)} = \sqrt{2}V_C$$

V_E does not depend upon the mass of the projected body and the direction of projection. It depends upon the mass (M) and radius (R) of the earth.

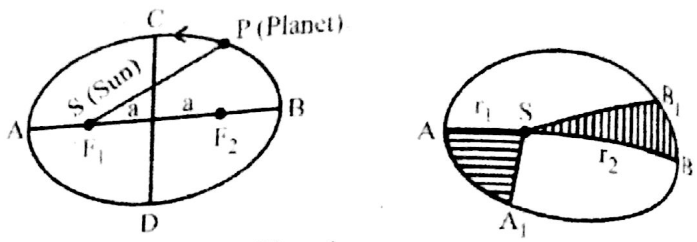
For a body to escape from the earth's gravitational influence, K.E. of projection = Binding energy

- **Kepler's three laws of planetary motion**

(i) Each planet revolves around the sun in an elliptical orbit, with the sun at one of the foci of the ellipse. (Law of orbit)

(ii) The straight line joining the sun and the planet or the radius vector, sweeps out equal areas in equal intervals of time. (Law of equal areas) or the areal velocity of the radius vector is constant. But the linear velocity of the planet goes on changing.

(iii) The squares of the periodic times (T^2) of the planets about the sun are proportional to the cubes of the semimajor axis (a) of the elliptical orbits i.e., $T^2 \propto a^3$ [Law of periods].



Elliptical Orbit of a Planet

- Period of a Satellite

$$T = 2\pi\sqrt{\frac{(R + h)^3}{GM}} \quad \therefore T^2 = \frac{4\pi^2}{GM}(R + h)^3$$

$$\because GM = gR^2 \quad \therefore T^2 = \frac{4\pi^2(R + h)^3}{gR^2}$$

$\therefore T^2 \propto (R + h)^3$ or $T^2 \propto r^3$

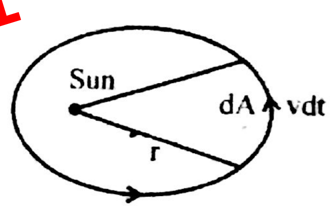
This is Kepler's third law.

Note : Gravitational force is a central force. Hence the torque on a planet relative to the sun is zero. Hence the angular momentum of the planet or satellite is conserved.

- **For Kepler's Second Law**

Areal velocity $= \dfrac{dA}{dt}$

$$= \frac{1}{2}\frac{rvdt}{dt} = \frac{1}{2}rv \qquad \text{... (1)}$$



But the angular momentum of the planet

$L = mvr \quad \therefore rv = \dfrac{L}{m}$ ... (2)

$\therefore$ from (1) and (2)

Areal velocity $\dfrac{dA}{dt} = \dfrac{L}{2m}$

- **Law of periods**

When the planet moves in the elliptical orbit, the sun is at the focus.

a - Semimajor axis of the ellipse and r_1 and r_2 are the shortest and largest distances of the planet from the sun.