From the figure, $r_1 = PF$, $r_2 = FQ$ and $r_1 + r_2 = 2a$

$$\therefore a = \frac{r_1 + r_2}{2}$$

$$\therefore T^2 \propto a^3 \text{ or } T^2 = \left(\frac{r_1 + r_2}{2}\right)^3$$

Applying the law of conservation of angular momentum [$L = mvr$] for the motion of the planet at P and Q, we have $mv_P r_P = mv_Q r_Q$.

$$\therefore \frac{v_P}{v_Q} = \frac{r_Q}{r_P} = \frac{a + c}{a - c}$$

and $\because$ eccentricity $e = \frac{c}{a}$ $\quad \therefore c = ae$

$$\therefore \frac{v_P}{v_Q} = \frac{a + ae}{a - ae} = \frac{1 + e}{1 - e}$$

Thus the velocities are expressed in terms of eccentricity.

- An astronaut or any body inside a satellite feels weightless as there is no reaction of the satellite upon the astronaut. In this case, both the astronaut and the satellite are in the free fall towards the earth. Hence $R = m(g - a)$ but $g = a$.

  $\therefore R = 0$.

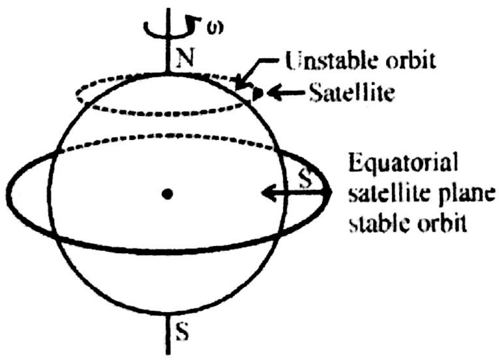
- For a communication or a geosynchronous or a geostationary satellite, T = 24 hours. It moves in the equatorial plane from west to east with a velocity of about 3.1 km/s.

  The geostationary satellite always appears stationary relative to the earth. Its orbit is known as the parking orbit.

$$\because T^2 = \frac{4\pi^2 (R + h)^3}{gR^2}$$

$$\therefore r = (R + h) = \sqrt[3]{\frac{T^2 g R^2}{4\pi^2}} \quad \therefore h = r - R$$

The height of the communication satellite above the surface of the earth is about 36000 km.



Satellite in the Equatorial Plane

- **Gravitational Field Intensity, Potential and Gravitational P.E. :** Similar to electric field we define a gravitational field and its intensity (I) and gravitational potential (V).

(a) If F is the gravitational force of attraction acting on a test mass m_0, then $\vec{I} = \frac{\vec{F}}{m_0}$ ... (1)

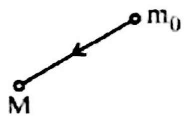
It is a vector quantity.

If m_0 is free to move, it will be accelerated and its acceleration $a = \frac{F}{m_0}$ ... (2)

$\therefore$ From (1) and (2), $I = a$

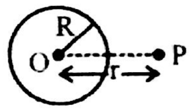
For a body lying on the surface of the earth $a = g = I$. The S.I. unit of I is m/s^2 or N/kg and its dimensional formula is $[I] = [M^0 L^1 T^{-2}]$.

Gravitational field intensity (I)

(1) 

Intensity due to a point mass M at a distance r,

$$I = \frac{GM}{r^2} \text{ i.e. } I \propto \frac{1}{r^2}$$

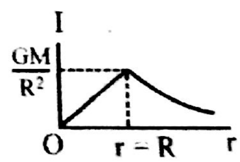
(2) 

Intensity due to a uniform solid sphere of radius R.

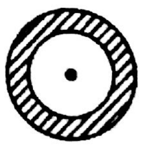
(i) For $r > R$, $I_1 = \frac{GM}{r^2}$

(ii) For $r = R$, $I_2 = \frac{GM}{R^2}$

(iii) For $r < R$, $I_3 = \frac{GMr}{R^3}$



Graph of I against r

(3) 

Intensity due to spherical shell

(i) Outside the shell $r > R$, $I_1 = \frac{GM}{r^2}$

(ii) On the surface, $r = R$, $I_2 = \frac{GM}{R^2}$

(iii) Inside the surface, $r < R$, $I = 0$