

Algebra Cheat Sheet

Basic Properties & Facts

Arithmetic Operations

$$ab + ac = a(b + c) \quad a\left(\frac{b}{c}\right) = \frac{ab}{c}$$

$$\left(\frac{a}{b}\right) \frac{c}{d} = \frac{ac}{bd}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{a-b}{c-d} = \frac{b-a}{d-c}$$

$$\frac{ab+ac}{a} = b+c, \quad a \neq 0$$

$$\frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{ad}{bc}$$

Exponent Properties

$$a^n a^m = a^{n+m}$$

$$(a^n)^m = a^{nm}$$

$$(ab)^n = a^n b^n$$

$$a^{-n} = \frac{1}{a^n}$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$$

$$\frac{a^n}{a^m} = a^{n-m} = \frac{a^n}{a^{m-n}}$$

$$a^0 = 1, \quad a^{-n} = \frac{1}{a^n}$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$\frac{1}{a^{-n}} = a^n$$

$$a^{\frac{n}{m}} = \left(a^{\frac{1}{m}}\right)^n = \left(a^n\right)^{\frac{1}{m}}$$

Properties of Radicals

$$\sqrt[n]{a} = a^{\frac{1}{n}} \quad \sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} \quad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$\sqrt[n]{a^n} = a, \text{ if } n \text{ is odd}$$

$$\sqrt[n]{a^n} = |a|, \text{ if } n \text{ is even}$$

Properties of Inequalities

If $a < b$ then $a + c < b + c$ and $a - c < b - c$

If $a < b$ and $c > 0$ then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$

If $a < b$ and $c < 0$ then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$

Properties of Absolute Value

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

$$|a| \geq 0$$

$$|-a| = |a|$$

$$|ab| = |a||b|$$

$$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$$

$$|a+b| \leq |a| + |b| \quad \text{Triangle Inequality}$$

Distance Formula

If $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ are two points, the distance between them is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Complex Numbers

$$i = \sqrt{-1} \quad i^2 = -1 \quad \sqrt{-a} = i\sqrt{a}, \quad a \geq 0$$

$$(a+bi) + (c+di) = a+c + (b+d)i$$

$$(a+bi) - (c+di) = a-c + (b-d)i$$

$$(a+bi)(c+di) = ac - bd + (ad+bc)i$$

$$(a+bi)(a-bi) = a^2 + b^2$$

$$|a+bi| = \sqrt{a^2 + b^2} \quad \text{Complex Modulus}$$

$$\overline{(a+bi)} = a-bi \quad \text{Complex Conjugate}$$

$$\overline{(a+bi)}(a+bi) = |a+bi|^2$$