A proof that $\sqrt{2}$ is irrational

Proof by contradiction

Here is a proof that there is no rational number whose square is 2.

The idea of the proof is to suppose that there is a rational number whose square is 2. You'll see that this leads to consequences that cannot be true. This means that there cannot, in fact, be any such number.

The proof depends on the following property of odd numbers: if you square an odd number, then the result is also odd. For example, $3^2 = 9$, $5^2 = 25$, $7^2 = 49$, and so on.

So, suppose that there is a rational number whose square is 2. Because it is rational, it can be written in the form

$$\frac{\boldsymbol{Interger}}{\boldsymbol{Interger}}$$

Let's cancel this fraction down to its simplest form, say:

$$\frac{\boldsymbol{m}}{\boldsymbol{n}}$$

Since this fraction is in its simplest form, the integer's *m* and *n* have no common factors.

Because: