

The minor (M_{ij}) of a_{ij} is the determinant of the matrix $(n-1) \times (n-1)$ and is found by canceling row i and column j from a_{ij} .

Cofactor A_{ij} of a_{ij} is $A_{ij} = (-1)^{i+j} M_{ij}$

Examples of Minors and Cofactors

1. Find M_{12} and A_{12} for the Matrix $\begin{bmatrix} 15 & 20 \\ 7 & -5 \end{bmatrix}$

To find M_{12} and A_{12} delete row 1 and column 2

$$\begin{array}{|c|} \hline \begin{array}{|c|} \hline 15 & 20 \\ \hline \end{array} \\ \hline 7 & -5 \\ \hline \end{array}$$

$$M_{12} = 7, A_{12} = (-1)^{1+2} M_{12} = (-1)^3 (7) = -7$$

$$A_{12} = -7$$

2. Find M_{23} and A_{23} for the Matrix $\begin{bmatrix} 8 & 2 & 3 \\ 16 & 4 & 6 \\ 36 & 8 & 12 \end{bmatrix}$

To find M_{23} delete row 2 and column 3

$$\begin{array}{|c|c|c|} \hline 8 & 2 & 3 \\ \hline 16 & 4 & 6 \\ \hline 36 & 8 & 12 \\ \hline \end{array}$$

$$M_{23} = \begin{vmatrix} 8 & 2 \\ 36 & 8 \end{vmatrix} = 8(8) - 36(2) = -8$$

$$A_{23} = (-1)^{2+3} M_{23} = (-1)^5 (8(8) - 36(2)) = 72 - 64 = 8$$

$$A_{23} = 8$$

Determinant of a 3 x 3 Matrix

The determinant of a 3 x 3 matrix is found by multiplying each value in row 1 by its cofactor then adding the sums. This is referred to as “expanding by the first row”.