The minor (M_{ij}) of a_{ij} is the determinant of the matrix (n – 1) (n – 1) and is found by canceling row *i* and column *j* from a_{ij}.

Cofactor A_{ij} of a_{ij} is $A_{ij} = (-1)^{i+j} M_{ij}$

Examples of Minors and Cofactors

1. Find M_{12} and A_{12} for the Matrix $\begin{bmatrix} 15 & 20 \\ 7 & -5 \end{bmatrix}$

To find M_{12} and A_{12} delete row 1 and column 2

$$\begin{bmatrix} 15 & 20 \\ 7 & -5 \end{bmatrix}$$

$M_{12}=7$, $A_{12} = (-1)^{1+2} M_{12} = (-1)^3 (7) = -7$

$A_{12} = -7$

2. Find M_{23} and A_{23} for the Matrix $\begin{bmatrix} 8 & 2 & 3 \\ 16 & 4 & 6 \\ 36 & 8 & 12 \end{bmatrix}$

To find M_{23} delete row 2 and column 3

$$\begin{bmatrix} 8 & 2 & 3 \\ 16 & 4 & 6 \\ 36 & 8 & 12 \end{bmatrix}$$

$M_{23} = \begin{vmatrix} 8 & 8 \\ 36 & 2 \end{vmatrix} = 8(8)-36(2)= -8$

$A_{23} = (-1)^{2+3} M_{23} = (-1)^5 (8(8) – 36(2)=72-64 = 8$

$A_{23} = 8$

Determinant of a 3 x 3 Matrix

The determinant of a 3 x 3 matrix is found by multiplying each value in row 1 by its cofactor then adding the sums. This is referred to as *"expanding by the first row"*.