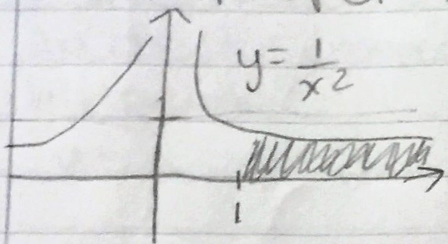
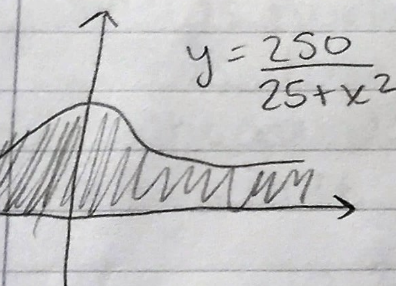
Improper Integrals 4.7



$$\int_1^{\infty} \frac{1}{x^2}\,dx = \lim_{n\to\infty} \int_1^{n} \frac{1}{x^2}\,dx$$

$$\lim_{n\to\infty} -\frac{1}{x}\Big|_1^n = \lim_{n\to\infty} \frac{-1}{n} - \frac{-1}{1} = \lim_{n\to\infty} 1 - \frac{1}{n} =$$

Improper Integrals with 2 infinite bounds



$$\int_{-\infty}^{\infty} \frac{250}{25+x^2}\,dx$$ → split it up into 2

$$\int_{-\infty}^{0} \frac{250}{25+x^2}\,dx + \int_{0}^{\infty} \frac{250}{25+x^2}\,dx =$$

$$\lim_{n\to-\infty} \int_n^0 \frac{250}{25+x^2}\,dx + \lim_{m\to\infty} \int_m^0 \frac{250}{25+x^2}\,dx$$ → find antiderivative

$$\int \frac{250}{25+x^2}\,dx \qquad x = 5\tan\theta \qquad dx = 5\sec^2\theta\,d\theta$$

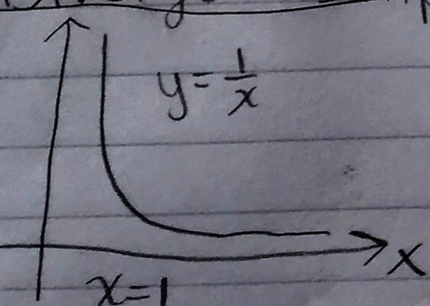
$$\frac{x}{5} = \tan\theta$$

$$\theta = \arctan\frac{x}{5}$$

$$\int \frac{250\cdot 5\sec^2\theta\,d\theta}{25+25\tan^2\theta} = \int \frac{250\cdot 5\sec^2\theta}{25(1+\tan^2\theta)}\,d\theta = 50\int d\theta = 50\theta$$

$$= 50\arctan\left(\frac{x}{5}\right)$$

Divergent Improper Integral



$$\int_1^{\infty} \frac{1}{x}\,dx = \lim_{n\to\infty} \int_1^{n} \frac{1}{x}\,dx = \lim_{n\to\infty} (\ln|x|)\Big|_1^n$$

$$= \lim_{n\to\infty} (\ln n - \cancel{\ln 1}^{\,0}) = \lim_{n\to\infty} \ln n =$$