# FP2 Revision Notes



***Inequalities***

*Key Words: sketch, positive*

- *Use a sketch to best evaluate points of intersection*
- *Only multiply by POSITIVE values*

***Series***

*Key Words: method of differences, partial fractions, sigma notation rules*

- *When evaluating* $\sum_1^n f(r)$ *consider r=1, r=2, r=3 … then sum and terms will cancel!*
- *If the general term* $u_r = f(r) - f(r+1)$ *then* $\sum_1^n u_r = \sum_1^n f(r) - f(r+1)$

***Further Complex Numbers***

*Key Words: modulus-argument form, principal argument, complex exponential form, de Moivre's theorem, binomial expansion, locus of points: circle; perpendicular bisector,*

- *If* $z = x + iy$ *then the complex number can be written as* $z = r(cos\theta + isin\theta)$
- *Principal argument:* $-\pi < \theta \leq \pi$
- $e^{i\theta} = cos\theta + isin\theta$ *(can be proved using Maclaurin expansion of sinx, cosx and* $e^{x}$*)*
- *Thus a complex number can be written in complex exponential form* $z = re^{i\theta}$
- $cos(x) = cos(-x), -sin(x) = sin(-x)$
- *For* $z_1 = r_1(cos\theta_1 + isin\theta_1)$ *and* $z_2 = r_2(cos\theta_2 + isin\theta_2)$
    - $z_1 z_2 = r_1 r_2 (cos(\theta_1 + \theta_2) + isin(\theta_1 + \theta_2))$
    - $z_1/z_2 = r_1/r_2 (cos(\theta_1 - \theta_2) + isin(\theta_1 - \theta_2))$
    - *Can be proved using trig identities*
- *Modulus operation acts like a power thus* $|z_1 z_2| = |z_1||z_2|$ *and* $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$
- *Argument operation acts like a logarithm thus* $arg(z_1 z_2) = arg(z_1) + arg(z_2)$
- $z^n = r(cos\theta + isin\theta) = r^n(cosn\theta + isinn\theta)$ *(can be proved using induction)*
- *Remember the following identities*
    - $z + \frac{1}{z} = 2\cos\theta$
    - $z^n + \frac{1}{z^n} = 2cosn\theta$
    - $z - \frac{1}{z} = 2i\ sin\theta$
    - $z^n - \frac{1}{z^n} = 2isinn\theta$
    - *Can be proved using* $z = r(cos\theta + isin\theta)$
- $z = r(cos\theta + isin\theta) = r(cos(\theta + 2k\pi) + isin(\theta + 2k\pi))$
- *To remove a modulus (using Pythagoras' theorem):*
    - $|z| = k$
    - $\therefore |x + iy| = k$
    - $\therefore x^2 + y^2 = k^2$
- *To remove an argument:*
    - $arg(z) = \theta$
    - $arg(x + iy) = \theta$
    - $\frac{y}{x} = \tan\theta$ *(adjust accordingly depending on quadrant)*
- *For a complex number w,* $w = u + iv$
- *For a transformation T from the z-plane to the w-plane:*
    - $w = z + a + ib$ *is a translation* $\binom{a}{b}$
    - $w = kz$ *is an enlargement scale factor k centre (0,0)*
    - $w = Kz + a + ib$ *is an enlargement scale factor k centre (0,0) followed by translation* $\binom{a}{b}$

***First order differential equations***

*Key Words: family of solution curves, separating the variables, integrating factor, transformations*

- *If* $\frac{dy}{dx} = f(x)g(y)$*, then* $\int \frac{1}{g(y)} dy = \int f(x)\, dx + c$
- *For a 1st order D.E. in the form* $\frac{dy}{dx} + Py = Q$ *where P and Q are functions of x, multiply through by the integrating factor to obtain general solution*
- *When using substitutions get y and* $\frac{dy}{dx}$ *in terms of other variables and it should drop out!*

***Second order differential equations***

*Key Words: auxiliary quadratic, general solution, complementary function, particular integral*

- *For 2nd order D.E.* $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$ *aux equation is* $am^2 + bm + c = 0$
- *For roots to the aux equation, the general solution to the 2nd order D.E. is…*
    - $y = Ae^{\alpha x} + Be^{\beta x}$ *(distinct roots* $\alpha$ *and* $\beta$*)*
    - $y = (A + Bx)e^{\alpha x}$ *(repeated root* $\alpha$*)*
    - $y = Acos\omega x + Bsin\omega x$ *(imaginary roots* $\pm i\omega$*)*
    - $y = e^{px}(Acosqx + Bsinqx)$ *(complex roots* $p \pm iq$*)*
- *For* $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$
    - *Solve for complementary function* $\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$
    - *Then solve for particular integral*
    - *If* $f(x)$ *is in the form… then try…*
        - $k \to a$
        - $kx \to ax + b$
        - $kx^2 \to ax^2 + bx + c$
        - $ke^{px} \to Ae^{px}$
        - $mcos\omega x \to acos\omega x + bsin\omega x$
        - $msin\omega x \to acos\omega x + bsin\omega x$
        - $mcos\omega x + nsin\omega x \to acos\omega x + bsin\omega x$
    - *General solution is* $y = C.F. + P.I.$
- *When using substitutions get y,* $\frac{dy}{dx}$ *and* $\frac{d^2y}{dx^2}$ *in terms of other variables and it should drop out!*

***Maclaurin and Taylor Series***

*Key Words: look at formula booklet*

***Polar coordinates***

*Key Words: polar, Cartesian, converting*

- $rcos\theta = x$
- $rsin\theta = y$
- $x^2 + y^2 = r^2$
- $\theta = arctan\frac{y}{x}$
- $Area = \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2 d\theta$
- *For tangents parallel to initial line* $\frac{d}{d\theta}(rsin\theta) = 0$
- *For tangents perpendicular to initial line* $\frac{d}{d\theta}(rcos\theta) = 0$
- *For* $r = p + qcos\,\theta$*: conditions for a 'dimple'* $q \leq p < 2q$