# FP3 Revision Notes

## *Hyperbolic Functions*

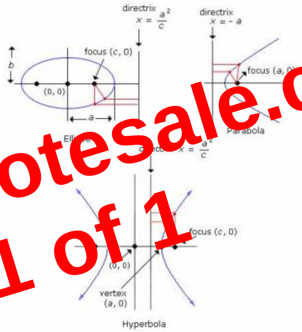
*Key Words: sinh, cosh, Osborn's rule*

- $\sinh x = \frac{e^x - e^{-x}}{2}$, $\cosh x = \frac{e^x + e^{-x}}{2}$
- *Osborn's rule (all trig identities go to hyperbolic except…)*
  - $\sin^2 A \to -\sinh^2 A$

## *Further Coordinate Systems*

*Key Words: USE FORAMULA BOOKELT!! Ellipse, major/ minor axis, hyperbola, eccentricity, parabola, focus, Directrix, locus*

- *Ellipse:* $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $x = a\cos t$ and $y = b\sin t$
- *Hyperbola:* $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $x = a\cosh t$ and $y = b\sinh t$, $x = a\sec\theta$ and $y = b\tan\theta$
  - *Asymptotes at* $y = \pm\frac{b}{a}x$
- *Eccentricity, e, describes the ratio of the shortest distance of a general point P from the Focus to the Directrix* $e = \frac{FP}{FD}$
  - *If* $0 < e < 1$ *= ellipse*
  - *If* $e = 1$ *= parabola*
  - *If* $e > 1$ *= hyperbola*
- *Each curve has its own definition of b (because of the following above)*
  - *Ellipse:* $b^2 = a^2(1 - e^2)$
  - *Hyperbola:* $b^2 = a^2(e^2 - 1)$






## *Integration*

*Key Words: learn results, integration by parts x1 trick, reduction formulae, arc length, surface area of revolution*

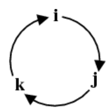
- *For an integral involving…use…*
  - $\sqrt{1-x^2}$ $x = \sin u$
  - $1+x^2$ $x = \tan u$
  - $\sqrt{1+x^2}$ $x = \sinh u$
  - $\sqrt{x^2-1}$ $x = \cosh u$
- *Arc Length between* $A(x_a, y_a)$ *and* $B(x_b, y_b)$
  - $\int_{x_a}^{x_b} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$ *or* $\int_{y_a}^{y_b} \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$ *or* $\int_{t_a}^{t_b} \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$
- *Surface Area between* $A(x_a, y_a)$ *and* $B(x_b, y_b)$
  - *x-Axis:* $2\pi\int_{x_a}^{x_b} y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$ *or* $2\pi\int_{t_a}^{t_b} y\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$

    *y-Axis:* $2\pi\int_{y_a}^{y_b} x\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$ *or* $2\pi\int_{x_a}^{x_b} x\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$ *or* $2\pi\int_{t_a}^{t_b} x\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$

## *Vectors*

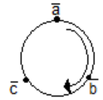
*Key Words: vector, cross, applications, determinant, alternating signs, triple scalar, planes, line, parallel, normal, angles, distances,*

- $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$
- *Scalar Product (dot product):* $a \cdot b = |a||b|\cos\theta$
- *Vector Product (cross product)*
  - *Use: to find a vector normal to a plane*
  - $a \times b = |a||b|\sin\theta\,\hat{n}$ *(angle between a and b, unit vector, perpendicular to a and b)*
  - $a \times b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = i\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - j\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + k\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$
  - *The vector product of any 2 parallel vectors is zero*
  - *The vector product is anti-commutative as* $a \times b = -b \times a$




- *Applications of the Vector Product*
  - $Area\ of\ Parallelogram\ ABCD = |\overrightarrow{AB} \times \overrightarrow{AC}|$
  - $Area\ of\ Triangle\ ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$
- *Triple Scalar Product*
  - $a \cdot (b \times c) = (a_1 i + a_2 j + a_3 k) \cdot \begin{vmatrix} i & j & k \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$
  - $Volme\ of\ Parallelepiped = |a \cdot (b \times c)|$
  - $Volme\ of\ Tetrahedron = \frac{1}{6}|a \cdot (b \times c)|$

- $[\bar{a}\ \bar{b}\ \bar{c}] = [\bar{b}\ \bar{c}\ \bar{a}] = [\bar{c}\ \bar{a}\ \bar{b}]$
- $[\bar{a}\ \bar{b}\ \bar{c}] = -[\bar{b}\ \bar{a}\ \bar{c}]$
- $[k\bar{a}\ \bar{b}\ \bar{c}] = k[\bar{a}\ \bar{b}\ \bar{c}]$
- $[\bar{a} + \bar{b}\ \bar{c}\ \bar{d}] = [\bar{a}\ \bar{c}\ \bar{d}] + [\bar{b}\ \bar{c}\ \bar{d}]$




- *Vector equation of a Line*
  - $r = a + \mu(b - a)$
  - *Alternative form:* $(r - a) \times d = 0$ $\quad \therefore r \times d = a \times d$
  - *Cartesian equation:* $\mu = \frac{x - a_1}{d_1} = \frac{y - a_2}{d_2} = \frac{z - a_3}{d_3}$
- *Vector equation of a Plane*
  - $r = r_p + \mu d_1 + \gamma d_2$
  - *If n is a vector normal to the plane then* $(r - r_p) \cdot n = 0$ $\quad \therefore r \cdot n = p$
  - *Cartesian equation:* $n_1 x + n_2 y + n_3 z = p$
- *Intersections (no intersection if parallel or skew)*
  - *Intersecting lines have a common point*
  - *Intersecting planes have a commons line*
  - *An intersecting plane with a line has a common point*
- *Angles (if >90 subtract from 180)*
  - *Angle between 2 lines:* $\cos\theta = \left|\frac{a \cdot b}{|a||b|}\right|$
  - *Angle between 2 planes:* $\cos\theta = \left|\frac{n \cdot m}{|n||m|}\right|$ *(n and m are normal directions vectors to planes)*
  - *Angle between a line and a plane:* $\sin\theta = \left|\frac{n \cdot d}{|n||d|}\right|$
- *Distances*
  - *Shortest distance from point* $(\alpha, \beta, \gamma)$ *to a plane* $(ax + by + cz = d)$*:* $\frac{|a\alpha + b\beta + c\gamma - d|}{\sqrt{a^2 + b^2 + c^2}}$
  - *Shortest distance between skew lines:* $\left|\frac{(a - a_1) \cdot (d \times d_1)}{|d \times d_1|}\right|$

## *Further Matrix Algebra*

*Key Words: Transpose identity and zero matrix, non-singular, inverse, matrix of minors, E-values and vectors, orthogonal, diagonal*

- $A^T$ *= Transpose of matrix A (obtained by switching rows and columns)*
  - *If* $A = A^T$ *then A is symmetric*
- *If A and B have dimensions* $(m \times p$ *and* $p \times n)$ *then* $(AB)^T = B^T A^T$
- *If A is non-singular then* $AA^{-1} = A^{-1}A = I$
- *To find the inverse of a matric A*
  - *Find* $\det(A)$
  - *Form M: the matric of the minors of A*
  - *Form C: by changing signs of elements of M according to the rule of alternating signs*
  - $A^{-1} = \frac{1}{\det(A)} C^T$
- *If A and B are non-singular then* $(AB)^{-1} = B^{-1}A^{-1}$
- *A transformation UT before A represents a transformation of T then a transformation of U on A*
- *Characteristic equation of A is* $\det(A - \lambda I)$*=0 – solve this to find Eigenvalues of A (remember normalising)*
- *For a Matrix A, a valid eigenvector x satisfies* $Ax = kx$ *(where k is a scalar constant)*
- *If M is square and* $MM^T = I$ *then M is orthogonal*
  - *If orthogonal then* $M^{-1} = M^T$
  - *If orthogonal then with normalised column vectors* $(x_1, x_2, x_3)$ *then:* $x_1.x_2 = x_3.x_2 = x_1.x_3 = 0$
- *A diagonal Matrix is* $\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$ *and* $\begin{pmatrix} e & 0 \\ 0 & f \end{pmatrix}$
- *To reduce a symmetric matrix A to a diagonal matrix D*
  - *Normalise eigenvectors of A*
  - *For an orthogonal matrix P with columns consisting of the normalised eigenvectors of A*
  - $D = P^T A P$
  - *When symmetric matrix A is reduced to a diagonal matrix D, the elements on the diagonal are the eigenvalues of A*