1. Let $A = \begin{bmatrix} a & -b & c \\ b & c & -a \\ -c & a & b \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.

What is the second column of $A^T B$?

a) $\begin{bmatrix} b+c \\ a-c \\ a-b \end{bmatrix}$ b) $\begin{bmatrix} a-c \\ a-b \\ c+b \end{bmatrix}$ c) $\begin{bmatrix} a+b \\ -b+c \\ -a+c \end{bmatrix}$ d) $\begin{bmatrix} b-c \\ a+c \\ -a+b \end{bmatrix}$

2. Let $A = \begin{bmatrix} k & 5 & 7 \\ 2 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. For which value of k is the matrix A not invertible?

a) 14 b) 10 c) 7 d) -10

3. What is the standard form of the complex number $\dfrac{3-2i}{1+3i}$?

a) $\frac{3}{10} - \frac{11}{10}i$ b) $-\frac{3}{10} - \frac{11}{10}i$ c) $\frac{3}{10} + \frac{11}{10}i$ d) $-\frac{3}{10} + \frac{11}{10}i$

4. Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $B = \begin{bmatrix} 2a & 2b & 2c \\ g & h & i \\ d+3g & e+3h & f+3i \end{bmatrix}$.

If $\det A = 4$, what is $\det B$?

a) -8 b) 8 c) -12 d) 12

5. Let $T : R^3 \longrightarrow R^4$ be the linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 - x_3 \\ 2x_1 + x_2 - x_3 \\ x_1 - x_3 \\ x_2 - x_3 \end{bmatrix}.$$

Which one of the following statements is TRUE?

a) T is one-to-one and onto.
b) T is one-to-one but not onto.
c) T is onto but not one-to-one.
d) T is neither one-to-one nor onto.

6. If $z = \sqrt{3} + i$, what is the standard form of z^{30}?

a) 2^{30} b) $-i2^{30}$ c) -2^{30} d) $i2^{30}$

7. Let $\lambda = 5$ be an eigenvalue of a 3×3 matrix A.
Exactly one of the following statement is TRUE. Which one?

a) $\det(A - 5I) = 0$
b) $\det(A - 5I) \neq 0$
c) $Ax = 5x$ for all x in R^3.
d) The equation $(A - 5I)x = 0$ has a unique solution.

Solutions:							
1. d	2. a	3. b	4. a	5. b	6. c	7. a	8. d
9. b	10. d	11. c	12. b	13. c	14. d	15. a	16. c

8. Consider

$$\begin{aligned} x_1 - 5x_2 + 2x_3 &= 0 \\ x_1 + x_2 + 8x_3 &= 0 \\ -x_1 + 3x_2 - 4x_3 &= 0. \end{aligned}$$

What is the general solution of the above system of linear equations?

a) $\begin{bmatrix} t \\ 7t \\ t \end{bmatrix}, t \in R$ b) $\begin{bmatrix} t \\ -7t \\ -t \end{bmatrix}, t \in R$ c) $\begin{bmatrix} t \\ -t \\ 7t \end{bmatrix}, t \in R$ d) $\begin{bmatrix} -7t \\ -t \\ t \end{bmatrix}, t \in R$

9. Let $T : R^3 \longrightarrow R^3$ be a linear transformation such that

$$T\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 6 \\ 8 \\ 8 \end{bmatrix} \text{ and } T\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}.$$

What is $T\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right)$=?

a) $\begin{bmatrix} 10 \\ 10 \\ 14 \end{bmatrix}$ b) $\begin{bmatrix} 14 \\ 14 \\ 10 \end{bmatrix}$ c) $\begin{bmatrix} 14 \\ 10 \\ 10 \end{bmatrix}$ d) $\begin{bmatrix} 14 \\ 10 \\ 14 \end{bmatrix}$

10. Let A be a 3×3 matrix with the characteristic polynomial $p(\lambda) = \lambda(\lambda + 4)(\lambda - 5)$.
Exactly one of the following statement is FALSE. Which one?

a) Each eigenspace of A is one-dimensional.
b) A is not invertible.
c) A is diagonalizable.
d) The equation $Ax = b$ is consistent for every vector b in R^3.

11. Exactly one of the following is NOT an elementary matrix. Which one?

a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 7 & 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

12. What are the eigenvalues of the matrix $\begin{bmatrix} 4 & -5 \\ 1 & 0 \end{bmatrix}$?

a) $\lambda_1 = 1 + 2i$, $\lambda_2 = 1 - 2i$ b) $\lambda_1 = 2 + i$, $\lambda_2 = 2 - i$
c) $\lambda_1 = 2i$, $\lambda_2 = -2i$ d) $\lambda_1 = 3 + 2i$, $\lambda_2 = 3 - 2i$

13. Let $H = \left\{ \begin{bmatrix} a+b \\ b+c \\ c+d \\ c+d \end{bmatrix} \middle| \, a,\ b,\ c,\ d \in R \right\}$. What is the dimension of the subspace H?

a) 1 b) 2 c) 3 d) 4

14. If $\det A = 2$, $\det B = -3$ and $\det C = -6$ then, what is $\det(A^{-1}B^2C^T)$?

a) 18 b) -18 c) 27 d) -27

15. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}^{-1}$, what is A^{1107}?

a) $\begin{bmatrix} -3 & 2 \\ -4 & 3 \end{bmatrix}$ b) $\begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$ c) $\begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$ d) $\begin{bmatrix} 3 & -2 \\ 4 & -3 \end{bmatrix}$

16. What is the angle between the vectors $x = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ and $y = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$?

a) $\pi/2$ b) $\pi/3$ c) $\pi/4$ d) $\pi/6$