Maths 2

$\rightarrow$ $\boxed{\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}}$

e.g $\int \sqrt{3-4x^2} = \sqrt{(\sqrt{3})^2-(2x)^2} = \frac{2x}{2}\sqrt{3-4x^2} + \frac{3}{2}\sin^{-1}\frac{2x}{\sqrt{3}}$

$\rightarrow$ $\boxed{\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\ln\left(\frac{x+\sqrt{x^2+a^2}}{a}\right)+c}$

$\rightarrow$ ① $\int e^x\left(f(x)+f'(x)\right)dx = e^x f(x)+c$ e.g $\int e^x\left(\underbrace{\frac{1}{x}}_{f'(x)}+\underbrace{\ln x}_{f(x)}\right) = e^x \ln x + c$

② $\int e^{ax}\left(a f(x)+f'(x)\right) = e^{ax} f(x)$

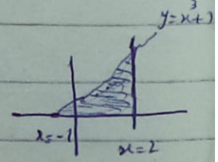
$\rightarrow$ e.g $\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx = \int e^x\left[\frac{1-2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}\right]$

$= \int e^x\left[\frac{1}{2\sin^2\frac{x}{2}} - \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}\right]$

$= \int e^x\left[\frac{1}{2}\operatorname{cosec}^2\frac{x}{2} - \cot\frac{x}{2}\right] \Rightarrow \int e^x\left[\underbrace{-\cot\frac{x}{2}}_{f(x)} + \underbrace{\frac{1}{2}\operatorname{cosec}^2\frac{x}{2}}_{f'(x)}\right]$

$= \boxed{e^x\left(-\cot\frac{x}{2}\right)+c}$

Q Find the area b/w the curve $y=x^3+1$ & the line $x=2$



put $y=0 \Rightarrow x^3+1=0$

$(x+1)(x^2-x+1)=0$

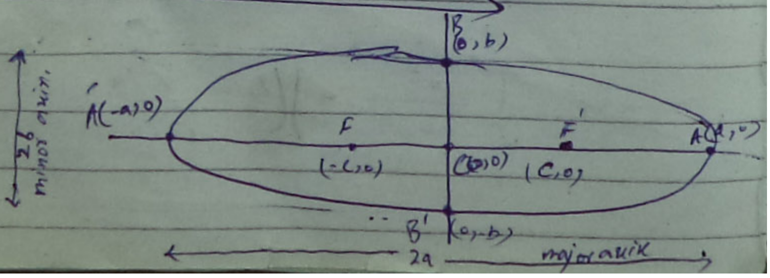
$x=-1$

so Area $= \int_{-1}^{2} x^3+1 = \left[\frac{x^4}{4}\right]_{-1}^{2} + [x]_{-1}^{2} = 27/4$

Since area under a curve is $A = \int_a^b f(x)\,dx$.

Q. Foci $(\pm 3, 0)$ $(c, 0)$ } given
$2b = 10$

find elips equation



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Page 11 of 16

Maths 2

→ Triangle area with 3 vertices given is: $A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

→ Angle b/w 2 lines with slopes m_1 & m_2 is $\tan\theta = \boxed{m_1 - m_2 / 1 + m_1 m_2}$

→ Equation $\boxed{ax^2 + 2hxy + by^2 = 0}$ represents a pair of lines & if $a + b = 0$ then lines will be $\perp$ar

Conic Section

→ circle ⇒ $\boxed{x^2 + y^2 = r^2}$ center $(0,0)$

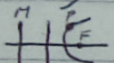
→ Standard circle equation ⇒ $x^2 + y^2 + 2gx + 2fy + c = 0$, center $(-g, -f)$

→ circle passing through 2 points (x_1, y_1) & (x_2, y_2) is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

→ Point $P(x_1, y_1)$ lies inside or outside circle? put point in standard circle equation if result < 0 then inside otherwise outside.

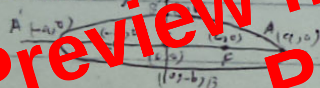
→ Line $y = \underline{m}x + \underline{c}$ is tangent to circle $x^2 + y^2 = \underline{a}^2$ iff $c^2 = a^2(1 + m^2)$

→ if eccentricity $e = 1$ (Parabol), $e > 1$ (hyperbola), $e < 1$ (elips)

→ Parabola  $|PF| = |PM|$, In parabola directrix is $y = -a$, then Focus is $y = a$

→ $\boxed{x^2/a + y^2/b^2 = 1}$ (elips) or $(x-h)^2/a^2 + (y-k)^2/b^2 = 1$ (elips on x-axis)

→ Directrix of ellipse $x = \pm c/e^2$ or $y = \pm c/e^2$



→ Find elips Eq if $C(0,0)$, Focus $(0,-3)$ & one vertex at $V(0,4)$ / sol: since $V(0,a)$ & $F(0, \pm c)$ so $a = 4$, $c = 3$

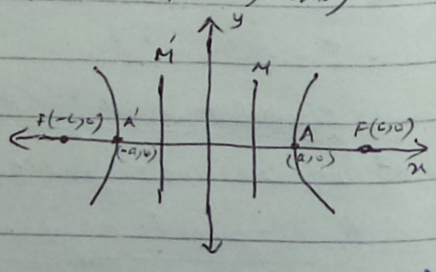
since $\boxed{b^2 = a^2 - c^2}$ ⇒ $b^2 = 16 - 9$ ⇒ $\boxed{b = \sqrt{7} \quad a = 4}$

so elips is $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ ⇒ $\frac{x^2}{7} + \frac{y^2}{16} = 1$ (since lies on y-axis)

→ hyperbola $\boxed{x^2/a - y^2/b^2 = 1}$ (x-axis)

→ hyperbola $\boxed{y^2/a^2 - x^2/b^2 = 1}$ (y-axis)

→ Directrix ⟶ $x = \pm c/e^2$ or $y = \pm c/e^2$



→ $\boxed{b^2 = c^2 - a^2}$

→ if 2 lines are perpendicular then $m_1 m_2 = -1$ (m_1 & m_2 are their slopes)

→ Equation of tangent to the circle $x^2 + y^2 = r^2$ at $P(4,3)$ (x_1, y_1) is $\boxed{y - y_1 = m(x - x_1)}$
where $m = \left.\frac{d}{dx}f(x)\right|_{P(4,3)}$

& $\boxed{y = mx \pm a\sqrt{1 + m^2}}$ if point not given $a = r$

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Page 16 of 16